

Research

Exchequer motor game enhances geometric thinking and mood in first grade children

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Abstract

This study investigated the effect of the motor game 'Exchequer Motor Game' (EMG) on first-grade children's Level of Geometric Thinking (LGT) and their post-learning mood tracking (PLMT). Thirty children (age 6.1 ± 0.7 years; physical education experience: 0.6 ± 0.4 years), classified at the "Visualization" stage of van Hiele's geometric thinking, were randomly assigned to two groups. Both groups engaged in the EMG and the Conventional Geometry Course (CGC) in a counterbalanced, randomized cross-over design. LGT and PLMT were measured for all participants after the intervention using the Van Hiele Geometry Test (level 1) and a mood chart. Statistical analyses showed a significant increase in LGT after EMG ($p < 0.05$, Hedges' $g = -0.91$, large effect) compared to that recorded after CGC with a significant increase in LGT scores when switching from CGC to EMG ($p < 0.001$) and a significant decrease in scores when switching from EMG to CGC. Similarly, the PLMT was significantly higher after the EMG session than after the CGC session ($p < 0.001$) with significant increases in PLMT scores when switching from CGC to EMG ($p < 0.001$) and significant decreases when switching from EMG to CGC. Therefore, the results of the study suggest that practicing the EMG can positively contribute to improving the level of children's geometric thinking.

Keywords Exchequer motor game · Level of geometric thinking · Post-learning mood tracking · Childrens · Treasure game

1 Introduction

Geometry instruction begins as early as primary school, and there has been significant emphasis on researching effective teaching and learning methods for geometry over several decades [6]. Geometry has a unique position in the mathematics curriculum and is considered a fundamental mathematical skill [14, 44] because it involves many concepts such as logical, systematic and creative thinking [49]. These skills are essential for solving everyday problems.

However, researchers around the world have been able to prove that learning geometry is based on van Hiele's level of geometric thinking [34, 35, 39, 41].

For this reason, it is necessary to have a good ability to think geometrically. However, pupils still struggle with geometric thinking [3]. This is indicated by the fact that the level of mastery of geometric concepts is still not maximised in

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primary school [53]. Furthermore, it is suggested that teachers need to identify the level of geometric thinking of their pupils [30] and use new learning strategies that can stimulate and support pupils in developing their geometric thinking [2]. This fact is supported by the findings of [1] who suggested that it is possible to implement geometry activities for primary school pupils using van Hiele's geometry learning phases. In fact, van Hiele's phases and levels of geometric thinking are suitable as a basis for planning geometry learning activities [39]. Consequently, the researchers sought to assess students' geometric thinking stages by administering a test designed according to van Hiele's indicators of geometric thinking stages across middle school, secondary school, and primary school levels.

In primary school, it has been found that primary school pupils have a low level of Van Hiele geometric thinking, which is necessary for them to understand and interpret geometry in school [27, 44]. In fact, traditional approaches to learning geometry in primary school place more emphasis on how much pupils can remember and less on how well pupils can think and reason [43].

Several studies have associated the progression of geometric thinking with van Hiele's model [1, 23, 34, 39, 51, 54], which remains the primary and widely adopted framework in geometry education research [6, 10]. Notably, van Hiele derived various insights regarding the cognitive developmental stages in children's comprehension of geometry [9].

The model delineates five hierarchical levels, each characterized by specific behavioral traits closely aligned with different learning phases [19, 21, 29, 31, 54]. These levels are intricately linked to the geometry skills students possess at each stage. Van Hiele identified these levels of geometrical reasoning as follows: Level 1 (Visualization), Level 2 (Analysis), Level 3 (Informal Deduction), Level 4 (Deduction), and Level 5 (Rigor) [13, 34, 36, 43, 53]. As students progress through each phase gradually, the teacher's pivotal role lies in facilitating their advancement toward higher levels of geometric thinking and bolstering their comprehension of geometry. This understanding of geometric thinking equips educators with valuable insights into how students perceive and engage with geometry across primary, secondary, and university education levels.

However, the learning strategies of primary mathematics teachers have not been structured to support the development of geometric thinking [10]. As a result, the learning of geometry is forced and rarely brings satisfaction to the pupils [13]. This is due to the inability of primary school pupils to translate the problem into a mathematical model [36]. In other words, pupils were not able to use the geometric model to solve the problem [37], while using their prior knowledge to solve the problem [48]. Failure to consider the developmental level of students in learning can lead to difficulties as the material presented may not align with their cognitive abilities, potentially resulting in challenges in comprehension [20].

Geometric thinking is crucial from an early age, as it contributes to the development of children's spatial reasoning and mathematical skills. Understanding and applying geometric concepts not only supports academic success in mathematics but is also essential for solving everyday problems. Recent studies have shown that educational experiences that stimulate early geometric thinking are linked to a better understanding of spatial relationships and enhanced problem-solving abilities [53, 18]. In this context, our study aims to explore how motor activities, such as the Exchequer Motor Game (EMG), can provide opportunities to develop geometric thinking in primary school children. The game, integrating movement and spatial reasoning, represents a powerful tool to foster the acquisition of geometric skills in an engaging and practical way.

Similarly, the assessment tools used in primary schools do not provide a comprehensive description of the level of our pupils' geometric thinking to enable teachers to plan appropriate interventions to improve pupils' geometric thinking [4]. On the other hand, a primary pupils needs to think at a relatively high level in order to learn geometry formally. He needs to have a deeper experience of thinking at a lower level before learning formal geometric concepts in primary school [22].

Recent studies have emphasized the transformative potential of motor games, not only as tools for physical skill development but also as a medium to achieve broader educational goals in physical education settings. For instance, Mallén-Lacambra et al. [57] explored the impact of cooperative non-competitive motor games on fostering gender equity, highlighting their effectiveness in dismantling stereotypes and enhancing socio-affective dynamics. Similarly, Luchoro-Parrilla et al. [56] illustrated how traditional sporting games could be utilized to teach sustainability by integrating local cultural practices with environmental and social educational objectives. These games exemplify how physical activities can be leveraged to promote holistic learning experiences that extend beyond mere physical engagement.

For physical education teachers, these findings open up a new realm of possibilities. Motor games, especially when rooted in ethnomotor traditions, provide a practical and engaging way to address contemporary challenges such as inclusivity, sustainability, and emotional well-being in their curricula. By linking physical education to cultural heritage and sustainable practices, teachers can foster a sense of responsibility and cultural identity in their students. Furthermore, these activities provide opportunities for collaborative learning, involving families and communities, which can enhance the overall educational impact.

Building on this perspective, the "Exchequer Motor Game" is designed to align with these innovative approaches. The EMG integrates movement, spatial reasoning, and problem-solving into a structured yet playful framework, making it an attractive tool for educators. It enables physical education teachers to address key competencies outlined in modern educational frameworks, such as critical thinking, collaboration, and adaptability, while also nurturing students' socio-emotional and cognitive skills. By incorporating these elements, the game fosters a holistic development approach. Furthermore, the EMG provides a dynamic and interactive way to progress through the stages of the van Hiele model of geometric thinking. By facilitating movement and engaging with spatial reasoning, it helps students move from basic geometric visualization to more advanced stages of reasoning and deduction. The physical engagement with geometric concepts grounds abstract ideas in concrete, memorable experiences, thus making geometric thinking more accessible and engaging for young learners. This alignment with the van Hiele model highlights the game's potential as an effective tool in developing students' geometric thinking from the early stages of geometry learning, providing both a fun and educational experience that contributes to children's cognitive and spatial development.

Participation in the EMG sessions is expected to foster the growth of verbal, visual, drawing and logical reasoning skills among students, further strengthening their geometric thinking skills.

However, to identify the unique and original ideas that primary school children form regarding geometric figures, it is imperative to evaluate whether these children advance in the development of geometric thinking following the stages outlined in van Hiele's theory. To achieve this objective, we suggest evaluating the geometric thinking phase of first grade children through the administration of level 1 (visualization) of the van Hiele geometric thinking test. We then explored the potential impact of the EMG on improving children's abilities to understand, think critically, and reason about spatial relationships, pattern recognition, and numerical manipulation, among first-grade children.

2 Methods

2.1 Participants

Based on an a priori calculation conducted using G*Power 3.1.9.4 software (version 3.1, University of Düsseldorf, Düsseldorf, Germany), it was determined that a sample size of 30 participants would be required to detect a medium effect size of $d = 0.82$ with 95% power and an alpha level of 0.05. The paired samples t-test was identified as the primary statistical test for this study. Written informed consent was obtained from the parents of the students, explicitly granting permission for their children to participate in the study. Additionally, the research protocol obtained approval from the local research ethics committee of University of Manouba (CPP N° 0117/2023).

All participants were classified at level 1 of van Hiele's geometric thinking. In fact, all participants could not answer at least 3 out of 5 questions of level 1 of the Van Hiele Geometry Test [54], which was given in class 2 weeks before the experimental period. In fact, all selected participants were unable to answer at least 3 out of 5 questions of level 1 of the Van Hiele Geometry Test. We then selected thirty children (age 6.1 ± 0.7 years, physical education experience: 0.6 ± 0.4 years).

2.2 Procedure

A physical education teacher and a maths teacher agreed to collaborate with the research team after receiving a detailed description of the aims and procedures of the study. They agreed to allow access to their classes for the implementation of the 'Exchequer Motor Game' (EMG) intervention during the physical education sessions and the Conventional Geometry Course (CGC) during the Maths sessions.

The research was conducted over the 2022–2023 school year, lasting three weeks. During the first week, designated as the familiarization phase, participants received orientation sessions to familiarize themselves with the intervention (EMG and CGC) and the testing procedures. Moreover, students were randomly assigned to two groups, each comprising 15 pupils, to facilitate a counterbalancing procedure. During the subsequent two weeks, referred to as the experimental phase, both groups engaged in the EMG in the playground and the Conventional Geometry Course (CGC).

Group 1 first participated in the Motor Game of the Exchequer session and then the following week in the Conventional Geometry Course.

Group 2, however, first attended the geometry conventional teaching and then the Motor Game of the Exchequer (Fig. 1A).

2.3 Exchequer motor game

The EMG was designed to engage students in an activity that combines physical movement with geometric reasoning.

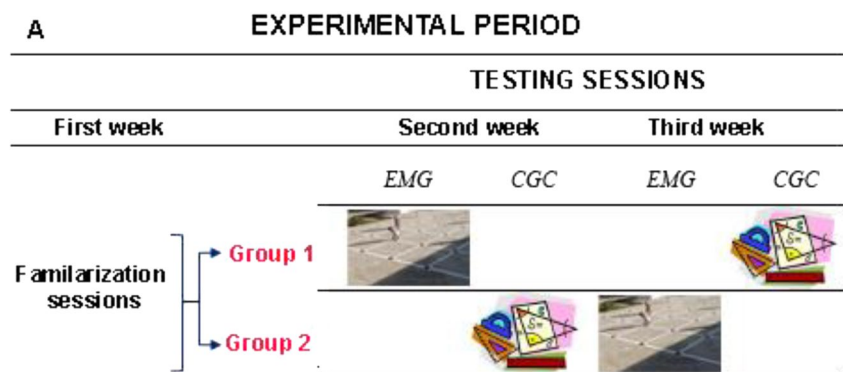
The game is played on a 12 x 12 m grid, divided into 36 squares, each containing a black dot, which serves as a reference point. The objective of the game is for students to correctly position themselves on the grid according to the teacher’s instructions, forming specific geometric shapes such as squares, triangles, or rectangles.

The activities within the EMG support the stages of the van Hiele model as follows:

- **Level 1: Visualization**—At this stage, students visually perceive shapes and properties. EMG offers a tactile, kinesthetic experience where students associate shapes with physical movements, deepening their understanding of shape recognition and spatial relationships.
- **Level 2: Analysis**—Students analyze geometric properties by applying knowledge of sides, angles, and symmetry to position themselves on the grid. This fosters an analytical approach to geometry by exploring relationships between geometric elements.
- **Level 3: Informal Deduction**—Here, students make logical connections between properties. EMG challenges students with spatial problems, encouraging them to deduce how to manipulate shapes on the grid.
- **Level 4: Deduction**—Students engage in more formal deductive reasoning. EMG supports this by offering opportunities to test properties, use logical reasoning, and discuss geometric relationships in a structured way.

Each participant moves within the grid, positioning themselves at the intersection points according to a coordinate system, using movement to create the requested geometric shapes while relying on their ability to determine their position relative to reference points and associate movements with the various topological relations suggested by the different variations (Fig. 1B). The geometric shapes requested vary each week, and students must apply their geometric knowledge (such as properties of shapes) to form them accurately. The game encourages practical understanding of geometric concepts, improving spatial orientation and the ability to apply geometry to real-life scenarios. To earn the

Fig. 1 A Description of the experimental protocol and the counterbalancing procedure. *EMG* Exchequer Motor Game, *CGC* Conventional Geometry Course. **B** Exchequer Motor Game



B



maximum number of points, the team must quickly construct the geometric shapes requested by the teacher while positioning themselves on the chessboard drawn on the floor, with each correctly formed shape earning a point.

Before the intervention, children participated in familiarization sessions with the EMG, to understand the rules and dynamics of the game. During these sessions, an orientation time was dedicated to explaining to the children the objectives of the game. In particular, the research team provided a brief introduction of the rules and expectations. The session lasted approximately 30 min and included a practical demonstration in which a team member guided the children through the main steps of the game.

Afterwards, children were encouraged to play independently with minimal supervision, to consolidate their understanding of the game. The familiarization sessions took place on two consecutive days, to ensure that the participants were fully prepared for the subsequent phases of the intervention.

2.4 Procedure test

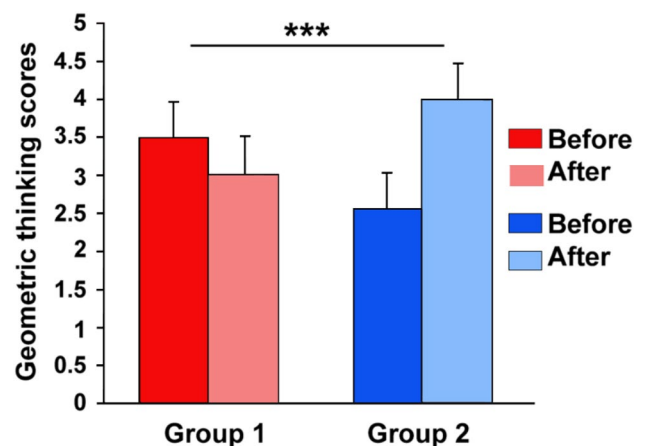
Before and after the intervention, participating pupils were administered the Van Hiele Geometry Test (VHGT) to determine their level of geometric thinking and the Mood Chart for Post-Learning Mood Tracking (PLMT). The VHGT was validated by Usiskin [54], who found that van Hiele's theorem was a good predictor of student success in geometry courses. Indeed, the test items were designed to correspond to the knowledge and skills of pupils at different levels [33]. The VHGT is composed of 25 multiple-choice items and is sequentially organized into blocks of five levels [6, 9, 10, 19]. In this study, given the age and cognitive capacity of our population, we only were interested in level (1) of the VHGT, where pupils can 'identify, name, and understand the properties of geometric shapes (such as squares, rectangles, and triangles) and name the regularities contained in these geometric shapes [45, 46]. Each student is given 3 min to provide their answer. At the end of the tests, the students' teachers will evaluate their geometric thinking (GT) skills, assigning 1 point for the correct answer and 0 points for the incorrect one. As a result, each participant will receive a score between 0 and 5. This methodology aligns with approaches taken in several previous studies [7, 9, 13, 21, 36].

For post-learning mood tracking (PLMT), we used a simple pupils-specific 'mood chart' with a series of written and illustrated moods such as 'happy', 'satisfied', 'bored', 'angry', and 'sad' (Fig. 2). We asked to pupils to select or mark the mood that best represented how they felt at that moment and allowed them to provide comments or additional feedback on why they felt a certain way. The mood scale ranged from sad (− 2) to happy (+ 2) and all odd integers including 'angry' (− 1), 'bored' (0), and 'satisfied' (+ 1).

Before the tests, children participated in familiarization sessions with the Van Hiele Geometry Test (VHGT) and the Mood Chart for Post-Learning Mood Tracking (PLMT), to ensure that they understood the format and the type of questions they would encounter during the measurements. During these sessions, 10 min were dedicated to each test to explain their content and how they would be administered.

For the VHGT, children were introduced to the basic concepts related to geometric shapes and their properties, with practical examples to help them familiarize themselves with the types of questions they would encounter in the test. A practical example was used for each type of question in the VHGT, so that children could understand how to answer correctly. The duration of this familiarization phase was approximately 10 min, followed by a short discussion to ensure that children understood the task at hand.

Fig. 2 Pre and post intervention difference in group 1 (red) and group 2 (blue) of Geometric Thinking scores. Data are mean \pm S.D. *** $p=0.001$, by independent t-test



For the Mood Chart for Post-Learning Mood Tracking (PLMT), children were explained how to use the mood chart. They were guided through the different categories (e.g., 'happy', 'angry', 'nice') and the meaning of the various options, using visual examples. Each child had time to explore the graph, asking questions and making observations about how to best express their mood after the intervention. This phase also lasted 10 min.

Then, the children had the opportunity to practice both tests with the support of the teachers, so that they could feel more confident during the actual assessment. The overall duration of the familiarization was approximately 20 min for each test.

2.5 Statistical analysis

The data collected from the students were analyzed using the SPSS Statistical Package for the Social Sciences (IBM SPSS Statistics 29), and the results are presented as mean \pm standard deviations (SD).

The normality of the data was assessed using the Shapiro–Wilk test. Based on the normal distribution, parametric tests were utilized. For comparing the results of the same group before and after the intervention regarding geometric thinking and mood, the paired samples t-test was employed. Additionally, Hedges' g coefficient was calculated to ascertain the magnitude of differences between the data collected after the EMG and Concrete Geometry Course (CGC). The independent samples t-test was also utilized to compare the changes in geo-metric thinking and mood (Δ). The significance level was set at $p < 0, 05$. Effect sizes were reported as Cohen's D .

3 Results

3.1 Geometric thinking

In group 1, the statistical analyses showed that there were no significant differences in Geometric Thinking after the Conventional Geometry Course compared to that recorded after the EMG ($p > 0.05$, Hedges' $g = 0.42$, small effect).

The statistical analyses indicated a significant difference in Geometric Thinking after the EMG compared to that recorded after the Conventional Geometry Course ($p < 0.05$, Hedges' $g = - 0.91$, large effect) (Group 2).

Comparing the geometric thinking scores collected from groups 1 and 2, the samples t-test there are statistically significant differences between the groups regarding geometric thinking ($t = - 3.674$ and $p = 0.001$) (Fig. 2).

The statistical analyses indicated a significant increase in scores when transitioning from the Conventional Geometry Course to the EMG ($\Delta = 0.5 \pm 1.08$). Conversely, there was a significant decrease in scores when the pupils switched from EMG to CGC ($\Delta = - 1.42 \pm 1.44$; $p < 0.001$, Cohen's $D = - 1.50$, large effect).

3.2 Post-learning mood tracking (PLMT)

In group 1, the statistical analysis revealed a significant difference in PLMT between Conventional Geometry Courses and EMG ($t = 4.864$; $p < 0.001$ by dependent t-test; Hedges' $g = 1.30$, large effect). For group 2, statistical analyses showed a significant difference between PLMTs obtained by CGCs and those obtained by EMGs ($t = - 7.0$; $p < 0.001$, by paired t-test; Hedges' $g = - 1.87$, large effect).

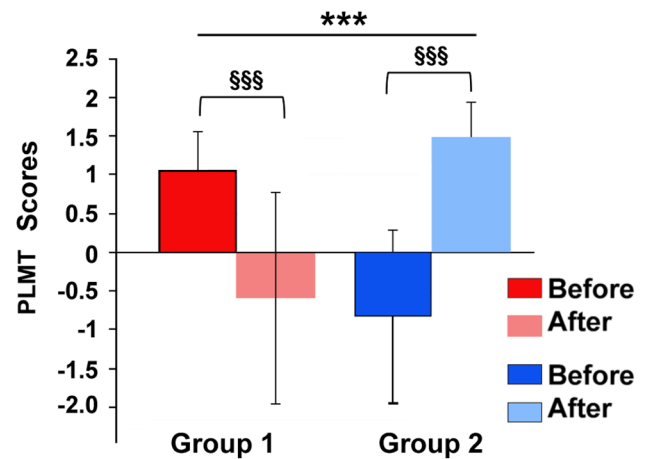
There were statistically significant differences between groups 1 and 2 in PLMT scores ($t = - 8.549$; $p = 0.001$, by independent t test; Hedges' $g = - 3,49$, large effect) (Fig. 3). Exactly, there was a significant increase in PLMT scores during the transition from the Conventional Geometry Course to the EMG (Group 2) ($p < 0.001$). On the contrary, there was a significant decrease in scores when pupils moved from EMG to CGC (Group 1; Fig. 3).

4 Discussion

Research on geometric reasoning utilizing Van Hiele's theory encompasses two primary components: assessing levels of geometric thinking and analyzing errors in solving geometric problems [52]. In this study, our objective is to establish a connection between Van Hiele's theory of geometric thinking and motor activities, such as the EMG, in children.

The principal findings of the study demonstrated that the Exchequer Motor had a significant positive impact on both Geometric Thinking scores and Post-Learning Mood Tracking (PLMT) in first-grade children.

Fig. 3 Pre and post intervention difference in group 1 (red) and group 2 (blue) of post-learning mood tracking (PLMT) scores. Data are mean \pm S.D. *** $p=0.001$, between group 1 and group 2, by independent t-test; \$\$\$ $p=0.001$, among pre and post-intervention in the same group, by paired t-test



Specifically, (i) Geometric Thinking scores increased when pupils switched from the CGC session to the EMG session and decreased when pupils switched from EMG to CGC session; (ii) PLMTs increased more after the EMG session than after the CGC session in pupils; (iii) PLMT scores increased when pupils switched from CGC to EMG and decreased when pupils switched from EMG to CGC session.

Several studies have pointed out the limitations of traditional geometry instruction, which often emphasizes memorization over reasoning [11], leading to dissatisfaction among pupils [13]. Geometry provides a unique context to develop reasoning and justification skills [40].

To foster geometric thinking from an early age, incorporating physical activity interventions like EMG could significantly enhance learning outcomes.

Previous research has highlighted the benefits of physical activity to improve pre-reading and word recognition skills [24], pupils' mathematical skills, pupils' learning time in the first year [26] and have also an effect on basic activities, academic achievement [15], i.e. academic motivation [17], social well-being [16] and learning [42]. Several studies suggest that physical activity interventions can improve pupils' geometric thinking.

Incorporating EMG emphasizes embodied learning, supporting dynamic geometry understanding.

We recognize that physical education classes and embodiment play a crucial role in enhancing students' understanding of dynamic geometric figures. As highlighted in recent studies, the integration of physical activity in educational settings can significantly support cognitive development, especially in mathematics and geometry. For instance, Mallén-Lacambra et al. [57] emphasize the benefits of using non-competitive cooperative motor games in promoting gender equity and transforming socio-affective dynamics. These games not only encourage physical activity but also foster a deeper connection between body movement and cognitive skills, such as geometric reasoning. Similarly, Luchoro-Parrilla et al. [56] argue that traditional sports and games can be powerful tools for teaching sustainability, which aligns with the concept of using embodied activities to support diverse learning objectives. In the case of geometric reasoning, the EMG exemplifies an approach where physical activity is intertwined with the learning of geometric principles, helping children enhance their spatial orientation and apply geometric concepts in a practical context. This approach can be particularly beneficial for young learners, offering them a more engaging and interactive way to grasp abstract mathematical ideas. We believe that emphasizing the connection between physical activity and geometric reasoning in primary education has the potential to significantly enhance students' mathematical skills and cognitive development.

Our results suggest that the use of EMG in primary school provides a promising framework for improving children's geometric thinking in the first year of primary school by providing appropriate learning opportunities based on this educational motor game. However, the rules of EMG include high tactical demands that can stimulate geometric problem solving [37], translation of the geometric problem into a practical model and construction of geometric shapes on the terrain during play. As a result, the inability to translate the problem into a mathematical model [55], is gradually reduced, as pupils are only able to identify shapes because they are similar in shape to those previously encountered, during the practice of EMG [5, 25].

The positive effects of EMG on pupils' geometric thinking highlighted in the present study can be explained by pupils' visual and verbal reasoning, which, according to Hoffer's theory, are essential for learning geometry [28]. More specifically, the visual flair of the EMG game allows pupils to infer advanced information based on visual observations [38]. Furthermore, during the EMG game, pupils' verbal skills allow them to name different geometric constructs and

visualize geometric shapes on the ground according to the verbal description. Verbal ability is the ability to name different geometric constructs, visualize geometric shapes according to their verbal description, identify specific geometric shapes and their properties, formulate definitions correctly and accurately, express relationships, recognize logical structures of oral problems, and formulate generalizations and abstractions [32, 50].

As for the practice of EMG, this game ensures that the pupils not experiencing difficulties because what is presented follows the pupils' ability to understand and absorb the material provided [20, 47].

Thus pupils in grade 1 taught based on the van Hiele theory, together with the use of EMG achieve better results in geometry. The result aligns with a previous study that showed that students who were taught van Hiele theory, with the integration of Google SketchUp, performed better in geometry [12, 34].

In this study, the small number of participants limits the generalisability of the results. Therefore, future research should consider recruiting children with a larger sample size. Secondly, we did not investigate the effect of the EMG on the gender of the participants, as several researchers have stated that this parameter is important in determining the level of children's geometric thinking.

An additional limitations of the present study is the absence of an observational methodology, which could have provided more in-depth insights into the dynamics of the EMG and the cognitive processes involved. While we relied on quantitative measures such as the Van Hiele Geometry Test and the Mood Chart for Post-Learning Mood Tracking (PLMT), incorporating a mixed-methods approach with systematic observational protocols would have allowed for a more comprehensive understanding of how students engage with the activity. As demonstrated in previous research [59, 58], T-pattern analysis and other observational techniques can reveal complex patterns in decision-making, motor behavior, and social interactions during play. Future studies should consider integrating observational methodologies, such as video recordings and behavioral coding, to capture detailed patterns of movement, interaction, and geometric reasoning during motor games, which could enrich the interpretation of the learning outcomes.

5 Conclusions

Our results confirm that first-grade children taught with the EMG at the visualization level (level 1) of the Van Hiele theory of geometric thinking achieve better results in geometry. EMG may be an add-in approach where movement is implicitly integrated with primary pedagogical and didactic purposes, which may be more acceptable to primary teachers and therefore increase the potential for implementation.

From a practical point of view, primary school educators should consider incorporating appropriate motor games that are aligned with the school curriculum for learning geometry.

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Author contributions Aymen Hawani: Conceptualization, methodology, investigation, writing—original draft, visualization, project administration. Anis ben Chikha: methodology, formal analysis, visualization, data curation Wael Zoghalmi: methodology Santo Marsigliante: writing—review and editing, supervision. Antonella Muscella: Conceptualization, writing—review and editing, visualization, supervision.

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Data availability Data is provided within the manuscript or supplementary information files.

Declarations

Ethics approval and consent to participate All procedures performed in this study were conducted in accordance with the relevant guidelines and regulations. The study was approved by the by the local research ethics committee of University of Manouba (CPP N° 0117/2023), and written informed consent was obtained from the parents of the students.

Informed consent Informed consent was obtained from all individual participants (both parents and children) included in the study in written form.

Competing interests The authors declare no competing interests.

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