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A new approach for assessing the probability of museum opening choices and its spatial continuity

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ABSTRACT

In the literature, several approaches and methods were applied for studying the visitors' profile or the managerial performance and economic efficiency of the museums; however, none of them investigated the museums opening decisions. To this aim an innovative approach which combines multilevel multinomial ordered models and spatial correlation models, is introduced and some advances in logit data geostatistical modeling is proposed together with an extended form of regression kriging, called multilevel logit kriging. Thus, the variation of the probability of the museums opening decisions both at regional and provincial levels for some peculiar museums characteristics, as well as the effect of some specific regional/provincial key factors which might influence their regular/non regular opening are modeled, also with respect to different types of institution (private/public).

The ISTAT microdata concerning the Italian survey on the museums and cultural institutions, will be considered. The empirical findings will provide worthy advices for the development of suitable management policies.

1. Introduction

The Italian cultural heritage is characterized by a wide range of museums, which differ for institutional features, types of collection, geographical location, exhibition space, number of visitors and other important factors. The management and enhancement of museums, together with their cultural heritage have been receiving growing attention in recent years from the policy makers [1–5]. Logistic approach plays a key role [6], since it may support public and private museums in taking strategic decisions, in order to ensure effectiveness, efficiency and efficacy of functioning. In this context, the opening periods and the corresponding costs represent a trade-off that the manager has to evaluate, also in compliance with the criteria for opening to the public, supervision and safety of museums and cultural sites defined by current legislation.

The focus of most of the works available in the literature was concentrated on empirical evidences of the specific dynamics related to museum visitors [7] or to assess the managerial performance and economic efficiency of the museums [8–10]. In particular, [7] proposed a combination of two machine learning algorithms with the clustering analysis to, on one hand, reconstruct visitor trajectories at room-scale and, on the other hand, analyze the visitors paths to catch behavioral insights. Successively, the authors implemented a stochastic model based on a probability transition matrix among museum rooms for simulating museum visits and forecasting the visitors' dynamic path in crowded museums. [10] employed a generalized conditional efficiency model to evaluate the performance of Italian museums, in terms of museums' service potential. In particular, this study showed that museums' service potential is higher for private museums, with respect to public institutions. In addition, multilevel logit models were considered by [11], for determining the museums inclination to adopt digital innovations. Moreover, [9] applied the Dynamic-Network Data Envelopment Analysis [12,13], which is a non-parametric technique with the dual goal to measure the efficiency and to assess the productivity of a sample of Spanish museums over time. Nevertheless, none of them gives prominence to the specific determinants concerning the factors that might influence the opening periods. Indeed, to the best of our knowledge, no systematic scoping review exists on these aspects. For this reason, the objective of this paper is to evaluate the probability of defining continuous, seasonal or occasional museums opening and provide an assessment of the logit data continuity over the Italian territory by using a dataset collected by the ISTAT (Italian Institute of Statistics) concerning a survey on museums and cultural institutions.

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The novelty is to propose a multilevel multinomial approach [14–20], combined with a geostatistical modeling [21–30], where (1) the spatial components (represented by regions and provinces where the museums are located) of the covariates have been treated as different units of the multilevel multinomial model in order to assess the complex pattern of variability at each level and (2) the spatial correlation of the logit data has been analyzed for a further investigation on their continuity over the domain. To this aim, the logit data (referred to the probability of the Italian museums opening decisions, where an increasing hierarchical criterion is used to define the tendency for the museums to stay open all over the year, seasonally or occasionally) have been interpreted as a finite realization of a non-stationary random field, where the drift component [31] has been described by a multilevel multinomial ordered model. Various contextual factors and relevant determinants (such as "Expenditure for recreation, culture and religion", "Gross domestic product per capita, current prices", "Number of tourist accommodation establishments") which can affect the regular opening have been also explored. Then, the resulting spatially indexed correlated residuals has been considered as a realization of a stationary random field, in order to examine its continuity over the territory. The spatial similarity/dissimilarity associated to the propensity, measured by the logit, of remaining open all over the year or at least seasonally for the Italian museums (located in the 108 Italian provinces nested in the 20 Italian regions), has been estimated and modeled through the variogram; thus, a multilevel logit kriging, has been used for interpolating the data. In other terms, the geostatistical analysis has been developed by focusing on the logit results, in order to corroborate the hypothesis that there might be a spatial relationship in the probability of museums opening choices. Indeed, the use of spatial statistical tools might offer significant elements to evaluate the service efficiency, in terms of spatial continuity, of an existing system and to guide and support planning policy makers [32]. In addition, the impact of the type of institutions (public or private), has been studied in the assessment of the likelihood for museums to opt for occasional, seasonal or year-round opening.

After a brief outline of the combined methodological approaches (i.e. the multilevel ordered modeling, the advances in logit data geostatistical modeling and the new notion of the multilevel logit kriging), the description of the ISTAT data architecture for Italian museum opening choices has been given (Section 2). Then, the trend functions, defined through a multilevel multinomial ordered model for the logit data, have been described, both in the case of all the museums (Section 3) and for the museums classified by type of institution (Section 4). The use of the multinomial ordered models has been justified by taking into account the appropriateness to graduate the opening periods for museums, according to an ordinal scale, ranging from the occasional to the year-round opening. Furthermore, the exploratory spatial data analysis of multilevel logit results has been displayed by focusing on the estimated probabilities of respecting the decision to stay open all over the year, seasonally or occasionally computed separately for the regions and provinces where the museums are placed (Section 5). Then, the structural analysis to assess the spatial continuity of the multilevel logit ordered residuals has been performed for the statistical units under study and their subsets and the results obtained through the multilevel logit kriging have been shown (Section 6). Finally, the results discussion and some concluding remarks on the museums' opening decision are presented (Section 7).

2. Methods and materials

In this section, after a short theoretical review on multilevel ordered modeling and some hints in logit data geostatistical modeling included the new notion of the multilevel logit kriging, the description of the ISTAT data architecture for Italian museum opening choices is presented.

2.1. Short review of the multinomial ordered logistic regression model

The multilevel approach is often recalled in Statistics for the study of hierarchical data structure characterized by complex patterns of variability [17,18,33,34]. This structure organizes the cases into known clusters and a set of explanatory variables (covariates) associated with each group level. In the following, the multilevel multinomial ordered logistic regression models is presented. This model is a cumulative regression model that links an ordinal variable of multiple categories (i.e. polychotomous outcome variable consisting of ordered categories) to a set of covariates [35]. In the literature, the early works regarding multilevel regression models for ordinal data are introduced by [11,36–38]. Furthermore, reviews about ordered logistic regression models are provided by [14,15,19], as well as by [39] among others.

The most common multilevel ordinal logistic regression model is based on the assumption of proportional odds, according to which the effects of the covariates included in the model are the same across all the categories of the ordinal dependent variable and only the intercept changes for each level. As underlined by Peterson and Harrell [20], an extension of this model is represented by the partial proportional odds model, where this proportionality hypothesis can be relaxed by allowing non-proportional odds for a set of regressors (predictors). In other terms, in case of violation of this proportional odds assumption, covariates can produce "differential effects" on the cumulative logits to stay open "seasonally and year-round ". A Wald test can be applied in order to test the proportional odds assumption [19,20,40]. In the case of rejection of the proportional odds hypothesis, the partial proportion odds model is applied, allowing separate effects for some variables without no loss in accuracy of prediction.

Let $Y_{ijk}^{(s)}$ be a multinomial ordered response variable which takes values $s = 1, 2, \dots, t$ (response categories), where t is chosen as the reference category and with the index i ($i = 1, \dots, n_{jk}$) representing the level 1 unit, the index j ($j = 1, \dots, N_k$) corresponding to the level 2 unit and the index k ($k = 1, \dots, K$) indicating the level 3 unit.

Moreover, given the probability $\pi_{ijk}^{(s)}$ that the ith first level unit presents a response variable value equal to s (i.e. the probability of being in category s, with $s = 1, \dots, t$), the cumulative response probabilities is defined as follows:

$$\psi_{ijk}^{(s)} = \sum_{h=1}^{s} \pi_{ijk}^{(h)}, \qquad s = 1, 2, \dots, t-1.$$

The category probabilities can be also expressed in terms of the cumulative probabilities as specified below:

$$\pi_{ijk}^{(h)} = \psi_{ijk}^{(h)} - \psi_{ijk}^{(h-1)}, \qquad 1 < h < t$$

$$\pi_{ijk}^{(1)} = \psi_{ijk}^{(1)}; \qquad \psi_{ijk}^{(t)} = 1.$$

Thus, assigned a set of covariates $\{X_{1\cdot}, X_{2\cdot}, \dots, X_{H\cdot}\}$, which influences the dependent response variable, a multilevel multinomial ordered model can be defined as follows:

$$\psi_{ijk}^{(s)} = \left\{ 1 + \exp\left[-\left(\beta_{0jk}^{(s)} + \sum_{h=1}^{H} \beta_{hjk}^{(s)} X_{hijk} \right) \right] \right\}^{-1}, \quad s = 1, 2, \dots, t-1, \qquad (1)$$

with

- $\beta_{hjk}^{(s)} = \beta_{h}^{(s)} + v_{hk}^{(s)} + u_{hjk}^{(s)}, \qquad h = 0, 1, \dots, H, \qquad s = 1, 2, \dots, t-1,$
- $\begin{bmatrix} v_{0k}^{(1)} \\ v_{0k}^{(2)} \\ \vdots \\ v_{hk}^{(s)} \\ \vdots \\ v_{Hk}^{(t-1)} \end{bmatrix} \sim N\left(\mathbf{0}, \boldsymbol{\Omega}_v\right),$

- $$\begin{bmatrix} u_{0jk}^{(1)} \\ u_{0jk}^{(2)} \\ \vdots \\ u_{hjk}^{(s)} \\ \vdots \\ u_{Hjk}^{(t-1)} \end{bmatrix} \sim N\left(\mathbf{0}, \boldsymbol{\Omega}_u\right),$$

where

- the generic element of the covariance matrix $\boldsymbol{\Omega}_v$ is $cov(v_{hk}^{(s)}, v_{h'k}^{(s')})$, $h, h' = 0, 1, \ldots, H,\ s, s' = 1, 2, \ldots, t-1,\ \ s \leq s'$,
- the generic element of the covariance matrix $\boldsymbol{\Omega}_u$ is $cov(u_{hjk}^{(s)}, u_{h'jk}^{(s')})$, $h, h' = 0, 1, \ldots, H,\ \ s, s' = 1, 2, \ldots, t-1,\ s \leq s'$.

Note that all of the covariate effects $\beta_{hjk}^{(s)}$ vary across categories ($s = 1, 2, \ldots, t-1$); similarly for the random-effect variance term [19].

Alternatively to the formulation given in (1), the multilevel multinomial ordered model can be written as follows:

$$\eta_{ijk}^{(s)} = \log it(\psi_{ijk}^{(s)}) = \beta_{0jk}^{(s)} + \sum_{h=1}^{H} \beta_{hjk}^{(s)} X_{hijk}, \quad s = 1, 2, \ldots, t-1, \tag{2}$$

where it is clear that increasing values of the linear components are associated with increasing probabilities as s increases [41]. The parameters of such a model can be estimated through the marginal maximum likelihood estimation, where the marginal likelihood of the observed data, obtained by integrating out the distribution of the random effects, is maximized.

2.2. Advances in logit data geostatistical modeling

In the literature, the geostatistical methodology has been widely adopted to analyze the spatial evolution of different phenomena in almost all the areas of the applied research, which include hydrogeology, environmental engineering, climatology, but also demography and socio-economic sciences, just to cite some of them. In all cases, the spatial behavior of the processes involved presents a systematic structure at the macroscopic level and a random behavior at the microscopic level. Both components can contribute in the description of the variable of interest, although in some contexts only the trend component is modeled with the consequent loss of information provided by the microscopic component. In particular, for a multinomial ordered response variable, observed over a territory, it is usually valuable to adapt a multilevel multinomial ordered model, which regards the expectation of the variable under study, however the logit residuals and their potential spatial correlation are not studied and modeled. For this reason, the formalism of a random field which can reasonably generate a geo-referenced logit dataset of a multinomial ordered response variable has been introduced and the spatial geostatistical modeling of the same logit data has been proposed.

By recalling the random field theory [29], the logit values are considered as a realization of a non-stationary spatial random field $Z^{(s)}$, $s = 1, 2, \ldots, t-1$, over a spatial domain D (the Italian territory), which can be decomposed as follows:

$$Z^{(s)}(\mathbf{u}) = \eta^{(s)}(\mathbf{u}) + W^{(s)}(\mathbf{u}), \tag{3}$$

where $\eta^{(s)}(\mathbf{u}) = E(Z^{(s)}(\mathbf{u}))$ is the expected value of $Z^{(s)}$, named *drift*, and is modeled through a multilevel ordered logit function, while $W^{(s)}(\mathbf{u})$ is assumed to be a zero-mean second-order stationary random field, with spatial variogram defined as follows:

$$2\gamma^{(s)}(\mathbf{h}) = Var[W^{(s)}(\mathbf{u}+\mathbf{h}) - W^{(s)}(\mathbf{u})],$$

where $\mathbf{h}$ represents the spatial separation vector for any couple of spatial locations $\mathbf{u}$, $\mathbf{u}'$ in the domain.

Note that, differently from the general notation used in (2), the multilevel ordered logit function proposed to model the drift of $Z^{(s)}$ shows explicitly the dependence on the spatial coordinate $\mathbf{u}$.

Nevertheless, the variability analyzed by the multilevel approach does not consider the spatial proximity which can be studied through the residuals $W^{(s)}$.

Thus, after modeling the drifts, the residual components are computed as specified below:

$$\widehat{W}^{(s)}(\mathbf{u}) = Z^{(s)}(\mathbf{u}) - \widehat{\eta}^{(s)}(\mathbf{u}), \quad s = 1, 2, \ldots, t-1,$$

and the spatial correlation of the residuals is analyzed. The first phase of the structural analysis consists in the estimation of the variogram of the residual components $W^{(s)}$, $s = 1, 2, \ldots, t-1$, over the set of data locations $A = \{\mathbf{u}_i, i = 1, 2, \ldots, n\}$. Then, the sample variogram $\widehat{\gamma}^{(s)}$ is evaluated by means of the estimator:

$$\widehat{\gamma}^{(s)}(\mathbf{h}) = \frac{1}{2|N(\mathbf{h})|} \sum_{N(\mathbf{h})} [W^{(s)}(\mathbf{u}+\mathbf{h}) - W^{(s)}(\mathbf{u})]^2,$$

where $|N(\mathbf{h})|$ is the cardinality of the set $N(\mathbf{h}) = \{(\mathbf{u}, \mathbf{u}+\mathbf{h}) \in A | \mathbf{h} \in Tol(r)\}$ and $Tol(r)$ represents the specified tolerance region r around $\mathbf{h}$ [25]. The second phase of the structural analysis is referred to the selection of a theoretical admissible model [22] and fitting it to the sample spatial variogram.

Furthermore, a special focus will be put on the behavior of the variogram near the origin, in order to assess the spatial continuity and catch the regularity of the variable under study [42]. Moreover, the nugget-to-sill ratio will be considered as a measure of the proportion of total observed variation that could not be explained by the spatial dependence of the variables under study.

As a spatial stochastic interpolation method of the variables $Z^{(s)}$, an extension of the regression kriging is proposed, which will be called multilevel logit kriging. Given the model for the random fields $Z^{(s)}$ in (3), the following linear estimator can be defined:

$$\widehat{Z}^{(s)}(\mathbf{u}) = \widehat{\eta}^{(s)}(\mathbf{u}) + \widehat{W}^{(s)}(\mathbf{u}), \qquad \text{with} \qquad \widehat{W}^{(s)}(\mathbf{u}) = \sum_{i=1}^{n} \lambda_i^{(s)}(\mathbf{u}) W^{(s)}(\mathbf{u}_i),$$
$$s = 1, 2, \ldots, t-1, \tag{4}$$

where $\widehat{\eta}^{(s)}(\mathbf{u})$ are the fitted drifts, $\widehat{W}^{(s)}(\mathbf{u})$ are the interpolated residuals, $\lambda_i^{(s)}$, $i = 1, 2, \ldots, n$, are the kriging weights, which depend on the variogram model and the sampling pattern (sampling geometrical relationship).

Thus, after modeling the drifts, the residual components have to be computed as specified below:

$$W^{(s)}(\mathbf{u}) = Z^{(s)}(\mathbf{u}) - \widehat{\eta}^{(s)}(\mathbf{u}), \quad s = 1, 2, \ldots, t-1;$$

then the following linear estimator for the residuals $W^{(s)}$ can be defined:

$$\widehat{W}^{(s)}(\mathbf{u}) = \sum_{i=1}^{n} \lambda_i^{(s)}(\mathbf{u}) W^{(s)}(\mathbf{u}_i), \qquad s = 1, 2, \ldots, t-1, \tag{5}$$

where the kriging weights $\lambda_i^{(s)}$, $i = 1, 2, \ldots, n$, which satisfy both the unbiasedness and the efficiency properties. More specifically, the first condition is satisfied if the weights $\lambda_i^{(s)}$, $i = 1, 2, \ldots, n$, of the linear combination are such that

$$\sum_{i=1}^{n} \lambda_i^{(s)}(\mathbf{u}) = 1. \tag{6}$$

On the other hand, the kriging estimator in (5) is efficient, if the weights are selected in order to minimize the error variance $\sigma_E^{2(s)} = E[R^{(s)}(\mathbf{u})]^2 = E[\widehat{W}^{(s)}(\mathbf{u}) - W^{(s)}(\mathbf{u})]^2$, $s = 1, 2, \ldots, t-1$, under the unbiasedness condition.

Thus, the following linear system, also named *ordinary kriging system* has to be solved:

$$\begin{cases} \sum_{j=1}^{n} \lambda_j^{(s)}(\mathbf{u})\gamma^{(s)}(\mathbf{u}_i, \mathbf{u}_j) - \omega^{(s)}(\mathbf{u}) = \gamma^{(s)}(\mathbf{u}_i, \mathbf{u}), \\ \sum_{i=1}^{n} \lambda_i^{(s)}(\mathbf{u}) = 1 \qquad i = 1, \dots, n, \ s = 1, 2, \dots, t-1, \end{cases} \tag{7}$$

where $\gamma^{(s)}$ is the spatial variogram of $W^{(s)}$ (identified in the step 3 dedicated to the structural analysis) and $\omega^{(s)}$ stands for a Lagrange multiplier.

2.3. Data architecture for musem opening choices

The dataset used in this study was obtained from the ISTAT official website and is referred to a census questionnaire survey carried out in the year 2018 on the basis of a specific protocol signed in 2017 with the Ministry for Cultural Heritage and Tourism. This survey offers an updated and detailed description regarding management, access and visits, typology, staff, financial resources, structures, support of fruition, activities and services, relationship with the territory and many other aspects of the museums located in Italy. The dataset consists of 3,217 museums, galleries or collection (2,043 public and 1,174 private) [43]. After a meticulous and careful exploratory analysis performed on the ISTAT microdata, the covariates shown in Table 1 have been identified and recoded for computational objectives. Note that the covariate “Type of institution” has been chosen as stratification variable by examining the different impact produced by the selected covariates on the opening decisions of private and public museums. On the other hand, a derived variable has been defined and successively recoded in order to evaluate the degree of digitalization (graduated as low, medium, high level) of Italian museums. More specifically, the encoding of this variable has taken into account the presence of at most 15 significant digital innovations, such as a digital inventory, a digital catalogue, video and audio-guides, applications dedicated for smartphones and tablets, interactive installations and/or virtual reconstructions (touch screen tables, videos), QR Code and/or proximity devices (Bluetooth, Wi-Fi, etc.), tablets available for the visitors, immersion video/multimedia rooms, free Wi-Fi connection, dedicated website, online ticketing service, digital catalogue accessible online, virtual tours online, social media accounts (Facebook, Twitter, Instagram; Pinterest; Foursquare, etc.), link to digital maps and/or geographic coordinates for the location of the museum. In this context, a multinomial multilevel logit model suitable to estimate the probability of the choice of the museums to open the whole year, seasonally or occasionally has been implemented. Such analysis takes into account contextual factors which might influence the opening decisions. The model used has been applied, firstly, on the complete dataset of museums and then on the subsets of public and private museums in order to highlight the potential effect of the type of museum on their opening decisions. In particular, three degrees of aggregation have been proposed: *the first level*, that is the museums (in total 3,217 museums); *the second level* corresponding to the Italian provinces where the museums are placed (108 Italian provinces), *the third level*, concerning the Italian regions where the museums are located (20 Italian regions). This is motivated by the intrinsic hierarchical structure of the data measured with respect to the locations (i.e. provinces aggregated into regions), in which the cultural heritage can offer different opportunities. The right pattern of covariates included into the model has been selected through a backward elimination procedure based on the Wald test at the 95% confidence level. At the end of this deletion procedure, some covariates not statistically significant (such as the total number of visitors, the presence of systematic or occasional satisfaction campaigns in the last five years, the number of bedrooms in the tourist accommodation establishments, the motorway density, the number of airports, the number of ports), as well as the interactions terms, have been neglected.

Remark
Based on the target of this work,

- the multilevel ordered modeling will be considered in order to define the trend of the logit data, that is the probability of museums opening decisions (occasional, seasonal, continuous), classified also by type of institution (public or private);
- the geostatistical analysis will involve the following steps:
    1. exploratory spatial data analysis, where a descriptive investigation of the examined variables is shown,
    2. structural analysis, where a measure of the spatial correlation (usually the variogram function) exhibited by the partial proportional logit residuals is evaluated and then modeled, in order to provide an adequate representation of their spatial behavior, as well as their spatial variability,
    3. interpolation of the selected variables over the spatial domain under study, by means of the model identified in the previous step.

  In particular, the structural analysis (step 2) is oriented to the assessment of the spatial continuity [44] concerning the partial proportional logit residuals for the ordered categorical response variables “continuous museums opening” and “seasonal or year-round museums opening”, over the domain of interest. From a computational point of view, such analysis requires (a) the variogram estimation, as well as (b) the selection of an admissible theoretical model available in the literature to be fitted to the empirical variogram [22]. Then, the selected spatial variogram models for partial proportional logit residuals of the ordered categorical response variables “continuous museums opening” (denoted with $\gamma^{(1)}(\mathbf{h})$) and cumulative odds logit “seasonal or year-round museums opening” (indicated with $\gamma^{(2)}(\mathbf{h})$), identified in the step 2.b, are used for interpolation purposes (step 3).

3. A trend model for the logit data

The Italian cultural heritage includes an extensive range of museums, which are characterized by different institutional features, types of collection, geographical location, exhibition space, number of visitors and other relevant factors.

In this context, a multilevel multinomial ordered model has been applied in order to evaluate the probability of museums opening decisions (occasional, seasonal, continuous) by considering, firstly, all the museums and then by classifying them on the basis of the type of institution (public or private). These models are suitable for analyzing ordinal response variables [20], since the explanatory variables may have either proportional odds (with one parameter for each covariate), unconstrained non-proportional odds (with $t-1$ parameters for each covariate) or constrained non-proportional odds (with a trend in log-odds ratios, where the odds parameter can increase or decrease in a monotonic way through the cut-points of the ordinal values). In particular, as previously mentioned, the multilevel multinomial logit proportional odds model is characterized by the restrictive proportionality assumption which is often released. Indeed, researchers may choose either a *partial* model by releasing the proportionality hypothesis only for a subset of variables or the *generalized* model where the assumption is violated for every independent variable [40].

In this paper, a partial proportional odds model will be fitted, in which the hypothesis of the proportional odds is partially released only for those explanatory variables that violate this hypothesis.

The multilevel logistic regression model has been fitted by using the *MLwiN* software [41].

3.1. Modeling the probability of the opening decisions for the museums

Let $Y_{ijk}^{(s)}$ be a multinomial ordered response variable which takes values $s = 1, 2, 3$ (response categories), where $s = 3$ represents the reference category “occasional museums opening”.

Table 1
Covariates selected for the study.

ISTAT questionnaire variables or derived variables	ISTAT questionnaire modality/ derived modality
• Network of museums	"0" = no network "1" = presence of network
• Access	"0" = absolutely free "1" = with admission fee
• Research activities	"0" = no research activities "1" = presence of research activities
• Partnership	"0" = no partnership "1" = presence of partnership
• Guided tours	"0" = no guided tours "1" = presence of guided tours
• Exhibition space	"0" = not greater than 201 square meters "1" = between 201 and 500 square meters "2" = more than 500 square meters
• Type of institution	"0" = public "1" = private
• Level of digitalization	"0" = low "1" = medium "2" = high
• Total number of visitors	"0" = lower than 700 "1" = between 700 and 3,000 "2" = between 3,002 and 10,000 "3" = more than 10,000
• Presence of systematic or occasional satisfaction campaigns in the last five years	"0" = no satisfaction campaigns "1" = systematic satisfaction campaigns "2" = occasional satisfaction campaigns "3" = systematic and occasional satisfaction campaigns
• Percentage of Italians vs Foreigners	"0" = lower than 50% "1" = greater than or equal to 50%
Provincial-level covariates	
• Number of tourist accommodation establishments	"0" = lower than 1,000 "1" = greater than or equal to 1,000
• Number of bedrooms in the tourist accommodation establishments	"0" = lower than 7,000 "1" = greater than or equal to 7,000
Regional-level covariates	
• Expenditure for recreation, culture and religion	"0" = lower than 470 euro "1" = greater than or equal to 470 euro
• Gross domestic product per capita, current prices	"0" = lower than 0.028 "1" = greater than or equal to 0.028
• Motorway density (km per 100 km^2)	"0" = up to 0.9 "1" = between 1 and 2 "2" = greater than 2
• Number of airports	"0" = no airports "1" = between 1 and 3 "2" = greater than 3
• Number of ports	"0" = no ports "1" = greater than or equal to 1

On the other hand, $s = 2$ corresponds to "seasonal museums opening" and $s = 1$ denotes "continuous museums opening", and let $\pi^{(3)}_{ijk}$ the probability of occasional museums opening, $\pi^{(2)}_{ijk}$ the probability of seasonal museums opening, $\pi^{(1)}_{ijk}$ the probability of continuous (year-round) museums opening, with the index i $(i = 1,\dots,n_{jk})$ representing the Italian museums (level 1 unit), the index j $(j = 1,\dots,N_k)$ corresponding to the provinces (level 2 unit) and the index k $(k = 1,\dots,K)$ indicating the regions (level 3 unit). Moreover, let $\mathbf{X}_{.} = \{X_{1.}, X_{2.},\dots,X_{H.}\}$ be a set of covariates, which influences the dependent response variable (Table 1).

By taking into consideration the ordering, the chosen model is based on the following cumulative response probabilities:

$$\psi^{(s)}_{ijk} = \sum_{h=1}^{s} \pi^{(h)}_{ijk}, \qquad s = 1,2,$$

which can be specified as follows:

$$\psi^{(1)}_{ijk} = \pi^{(1)}_{ijk}, \qquad \psi^{(2)}_{ijk} = \pi^{(1)}_{ijk} + \pi^{(2)}_{ijk}, \qquad \psi^{(3)}_{ijk} = 1.$$

Two sub-equations, i.e. one for each category ("continuous museums opening" and "seasonal or year-round museums opening"), are defined:

$$\begin{aligned}\eta^{(1)}_{ijk} &= \log it(\psi^{(1)}_{ijk})\\ &= \beta^{(1)}_{0jk} + \beta^{(1)}_{1k}\,x_{1ijk} + \beta^{(1)}_{2}\,x_{2ijk} + \beta^{(1)}_{3}\,x_{3ijk} + \beta^{(1)}_{4}\,x_{4ijk} + \beta^{(1)}_{5}\,x_{5ijk} +\\ &\quad \beta^{(1)}_{6}\,x_{6ijk} + \beta^{(1)}_{7}\,x_{7ijk} + \beta^{(1)}_{8}\,x_{8ijk} + h_{ijk},\end{aligned} \tag{8}$$

$$\begin{aligned}\eta^{(2)}_{ijk} &= \log it(\psi^{(2)}_{ijk})\\ &= \beta^{(2)}_{0jk} + \beta^{(2)}_{1k}\,x_{1ijk} + \beta^{(2)}_{2}\,x_{2ijk} + \beta^{(2)}_{3}\,x_{3ijk} + \beta^{(2)}_{4}\,x_{4ijk} + \beta^{(2)}_{5}\,x_{5ijk} +\\ &\quad \beta^{(2)}_{6}\,x_{6ijk} + \beta^{(2)}_{7}\,x_{7ijk} + \beta^{(2)}_{8}\,x_{8ijk} + h_{ijk},\end{aligned} \tag{9}$$

with

- $h_{ijk} = \beta_9 x_{9k} + \beta_{10} x_{10k} + \beta_{11} x_{11jk} + \beta_{12k} x_{12ijk} + \beta_{13} x_{13ijk}$,
- $\beta^{(1)}_{0jk} = \beta^{(1)}_{0} + v^{(1)}_{0k} + u^{(1)}_{0jk}$,
- $\beta^{(2)}_{0jk} = \beta^{(2)}_{0} + v^{(2)}_{0k} + u^{(2)}_{0jk}$,
- $\beta^{(1)}_{1k} = \beta^{(1)}_{1} + u^{(1)}_{1jk}$,
- $\beta^{(2)}_{1k} = \beta^{(2)}_{1} + u^{(2)}_{1jk}$,
- $\beta_{12k} = \beta_{12} + v_{12k}$,
- $\begin{bmatrix} v^{(1)}_{0k} \\ v^{(2)}_{0k} \\ v_{12k} \end{bmatrix} \sim N\left(\mathbf{0}, \mathbf{\Omega}_v\right), \quad \mathbf{\Omega}_v = \begin{bmatrix} \sigma^2_{v0^{(1)}} & & \\ \sigma_{v0^{(1)}\,0^{(2)}} & \sigma^2_{v0^{(2)}} & \\ \sigma_{v0^{(1)}\,12} & \sigma_{v0^{(2)}\,12} & \sigma^2_{v12} \end{bmatrix},$

- $$\begin{bmatrix} u_{0jk}^{(1)} \\ u_{0jk}^{(2)} \\ u_{1jk}^{(1)} \\ u_{1jk}^{(2)} \end{bmatrix} \sim N\left(\mathbf{0}, \mathbf{\Omega}_u\right),$$

$$\mathbf{\Omega}_u = \begin{bmatrix} \sigma^2_{u0^{(1)}} & & & \\ \sigma_{u0^{(1)}\,0^{(2)}} & \sigma^2_{u0^{(2)}} & & \\ \sigma_{u0^{(1)}\,1^{(1)}} & \sigma_{u0^{(2)}\,1^{(1)}} & \sigma^2_{u1^{(1)}} & \\ \sigma_{u0^{(1)}\,1^{(2)}} & \sigma_{u0^{(2)}\,1^{(2)}} & \sigma_{u1^{(1)}\,1^{(2)}} & \sigma^2_{u1^{(2)}} \end{bmatrix}.$$

After evaluating the proportional odds assumption by a Wald test (Section 3.2), a partial proportional odds model has been identified, where:

- the intercept has been hypothesized to change for each level,
- the common coefficient of the covariate "Network of museums" ($X_{12.}$) has been assumed to vary across the 3rd level;
- the separate coefficients of the covariate "Research activities" ($X_{1.}^{(1)}$ and $X_{1.}^{(2)}$, for the two sub-Eqs. (8) and (9), respectively) have been assumed to vary across the 2nd level.

Furthermore, the coefficients of the contextual factors ("Expenditure for recreation, culture and religion", "Gross domestic product per capita, current prices", "Number of tourist accommodation establishments") included as common coefficients, as well as the slopes of the remaining covariates, implemented into the model as common or separate coefficients, have been assumed to be constant.

The reliability of this model has been confirmed by the residual diagnostics (normal scores of residuals and standardized residuals, ranks of residuals) performed both for the third and second level of aggregation.

3.2. Testing the proportional odds assumption

In order to evaluate the adequacy of the fitted partial proportional odds model, a Wald Chi-square test can be applied [45–47]. In other terms, the Wald Chi-square test is used to verify the partial proportional odds model's overall goodness-of-fit. It checks for the validity of the null hypothesis that the coefficients of some covariates added in the model are not significantly different across the two sub-equations [48, 49]. According to the rejection of the null hypothesis, the coefficients of some independent variables can be assumed different in the two sub-equations. Note that this test is crucial to determine which covariates violate the proportionality assumption in order to avoid the selection of an inadequate model which might drive to misleading inferences.

Table 2 shows the results of the Wald Chi-square test. From the *p-values* in Table 2 it is highlighted that the proportional odds assumption holds only for the covariates "Guided tours" and "Network of museums" at a significance level not exceeding 10%. Thus, the other covariates have been added as separate coefficients to each of the two sub-equations of the model, since the proportional odds hypothesis is rejected. Therefore, it can be assumed that a partial proportional odds model represents a suitable choice for fitting the data, since it allows separate effects for each cumulative logit to stay open "seasonally and year-round" only for some predictors.

Table 2
Test for non-proportionality of the covariates with separate effect, referred to multinomial ordered model for public and private museums.

Covariate's category	Wald statistic	*p-value*
constant	39.349	0.000***
Research activities	11.504	0.003***
Access	23.001	0.000***
Percentage of Italians vs Foreigners	4.918	0.086*
Guided tours	1.037	0.596
Exhibition space between 201 e 500 square meters	14.598	0.001***
Exhibition space of more than 500 square meters	37.502	0.000***
Network of museums	1.484	0.476
Partnership	8.274	0.016**
Medium digitalization	10.115	0.006***
High digitalization	17.084	0.000***

**p-value* < 0.1 ***p-value* < 0.05 ****p-value* < 0.01.

3.3. Estimation results for multilevel multinomial ordered model

Table 3 shows the estimates of the fixed and random parameters of the multinomial ordered logit model for all the Italian museums, obtained by the maximum-likelihood method.

The ORs given in the last column of Table 3 highlight that:

- proposing research activities, compared with do not supporting them, increases the probability of deciding to open all over the year with respect to occasionally, by +51.7%; it also rises the probability for museums to stay open always or seasonally by +39%. Although the effect of this covariate is positive, the OR of the cumulative option "seasonal-continuous opening" (which is statistically significant only at 10%) is lower than the OR associated to the continuous opening: this means that offering research activities produces a dampening impact on the seasonal opening, on the other hand it generates a positive effect on the continuous opening;
- a free access to the museum, with respect to a paid visit, decreases by −10.3% the probability to take the decision of keeping the museum open all over the year with respect to occasionally; it also decreases the probability to guarantee a seasonal or continuous opening by −71.8%. The effect of this covariate is negative on the probability of both seasonal and continuous opening, with a stronger incidence on the probability of seasonal opening than on continuous opening (which provides a further reduction of the OR of −61.5%). This could be explained by considering that museums opened year-round, with respect to the ones with seasonal opening, may receive funding in the form of grants or from other sources, thus recovering the loss of profit from free access;
- regarding the exhibition space, the availability of a medium space (between 201 and 500 square meters) or large space (more than 500 square meters), compared to a small exhibition space (less than 201 square meters) significantly increases the probability to guarantee a continuous opening: +44.2% for a medium exhibition space and +115.1% for a large exhibition space. Also the probability to stay open seasonally or year-round is increased, by +103% for a medium exhibition space and +257.2% for a large exhibition space. The effect of this covariate is positive on the probability of both seasonal and continuous opening, with a stronger incidence on the probability of seasonal opening than on continuous opening, especially for a large exhibition space (which entails a further increase of the OR equal to +58.8% in the case of "medium space" and a significant rise of the OR of 142% in the case of "large space"). This is expected taking into account that a larger space could be more attractive for foreign tourists, who presumably represent the majority of the flow of visitors in the case of seasonal opening;
- the presence of partnership with the territory, with respect to the absence of formal relationships with other public or private cultural institutions located in the territory, increases the probability of respecting the decision of continuous opening by +25.1% or stay open seasonally or year-round by +85%. The effect of this covariate is positive on the probability of both seasonal and continuous opening, with a stronger incidence on the probability of seasonal opening than on continuous opening (which provides a further increase of the OR of 60%); this is presumably because

Table 3
Estimates of fixed and random parameters, together with the standard errors (SE), the Wald statistic, the *p-value* and the ORs of the ordered logit model referred to public and private institutions.

Fixed parameters (Covariate's category)	$\hat{\beta}$	$SE(\hat{\beta})$	Wald statistic	*p-value*	$OR = \exp(\hat{\beta})$
Separate coefficients					
Continuous opening					
Individual-level covariates					
constant	0.319	0.190	1.679	0.093*	1.376
Research activities (x_{1ijk})	0.417	0.127	3.283	0.001***	1.517
Access (x_{2ijk})	−0.109	0.011	−9.909	0.000***	0.897
Medium digitalization (x_{3ijk})	0.344	0.141	2.440	0.015**	1.411
High digitalization (x_{4ijk})	0.652	0.173	3.769	0.000***	1.919
Percentage of Italians vs Foreigners (x_{5ijk})	0.257	0.156	1.647	0.099*	1.293
Partnership (x_{6ijk})	0.224	0.113	1.982	0.047**	1.251
Exhibition space between 201 e 500 square meters (x_{7ijk})	0.366	0.117	3.128	0.002***	1.442
Exhibition space of more than 500 square meters (x_{8ijk})	0.766	0.134	5.716	0.000***	2.151
Seasonal and continuous opening					
Individual-level covariates					
constant	2.760	0.461	5.987	0.000***	15.800
Research activities (x_{1ijk})	0.329	0.171	1.924	0.054*	1.390
Access (x_{2ijk})	−1.265	0.263	−4.810	0.000***	0.282
Medium digitalization (x_{3ijk})	0.623	0.199	3.131	0.002***	1.865
High digitalization (x_{4ijk})	1.015	0.308	3.295	0.001***	2.759
Percentage of Italians vs Foreigners (x_{5ijk})	−0.298	0.331	−0.900	0.368	0.742
Partnership (x_{6ijk})	0.615	0.200	3.075	0.002***	1.850
Exhibition space between 201 e 500 square meters (x_{7ijk})	0.708	0.216	3.278	0.001***	2.030
Exhibition space of more than 500 square meters (x_{8ijk})	1.273	0.319	3.991	0.000***	3.572
Common coefficients					
Regional-level covariates					
Gross domestic product per capita, current prices (x_{9k})	0.379	0.202	1.876	0.061*	1.461
Expenditure for recreation, culture and religion (x_{10k})	0.059	0.035	1.697	0.090*	1.061
Province-level covariate					
Number of tourist accommodation establishments (x_{11jk})	0.220	0.097	2.268	0.023**	1.246
Individual-level covariates					
Network of museums (x_{12ijk})	−0.054	0.033	−1.651	0.099*	0.947
Guided tours (x_{13ijk})	0.142	0.086	1.651	0.099*	1.153

Random parameters

$$\boldsymbol{\Omega}_v = \begin{bmatrix} 0.477(0.184) & & \\ 0.081(0.131) & 0.328(0.167) & \\ 0.169(0.119) & -0.144(0.121) & 0.200(0.145) \end{bmatrix}$$

$$\boldsymbol{\Omega}_u = \begin{bmatrix} 0.312(0.103) & & & \\ 0.189(0.105) & 0.207(0.162) & & \\ -0.056(0.107) & 0.014(0.129) & 0.23(0.183) & \\ -0.663(0.239) & -0.444(0.303) & 0.773(0.337) & 1.534(0.837) \end{bmatrix}$$

**p-value* < 0.1 ***p-value* < 0.05 ****p-value* < 0.01.

the museums with seasonal opening, thanks to the relationship with the territory, can exploit the communication strategies of the territory with targeted promotional campaigns, often seasonal;

- the presence of a percentage of Italians versus non Italian visitors greater than (or equal to) the 50% increases the probability to stay open year-round by +29.3% and decreases the probability to stay open continuously or seasonally by −25.8% (not significant) than a higher percentage of foreign visitors, because a larger and continuous presence of Italian visitors versus seasonal or occasional presence of foreigners could lead to stay open year-round, allowing more revenue thanks to the more regular flow of Italian visitors;
- belonging to a network of museums (individual-level covariate) decreases the probability to guarantee the seasonally or continuously opening by −5.3%, compared with not being part of a network of museums; this could be justified by considering that a network is governed by formal acts, with financial and administrative constraints. As a consequence, membership of a network is based on the compliance of rules defined by the network: this could presumably lead to a slowdown of museums management, including the museums opening decisions, the museum schedules, as well as the activities and services provided;
- concerning the degree of digitalization, a medium level of digitalization (compared to a low level of digitalization) increases the probability to guarantee a continuous opening by +41.1%, on the other hand, a high level of digitalization increases the probability to stay open all over the year by +91.9%. In addition, a medium or high degree of digitalization enhances the probability to stay open seasonally or year-round by +86.5% or +175.9%, respectively. This covariate produces a propulsive effect for both the options "continuous opening" and "continuous or seasonally opening", with a major incidence on the probability of seasonal opening than on continuous opening, especially for a high digitalization (which entails a further increase of the OR equal to +45.4% in the case of "medium digitalization" and a significant rise of the OR of 84% in the case of "high digitalization"). This could be justified by considering that high digitalization could contribute to enhance the attractiveness of seasonally opening museums especially due to foreign tourists, who are often more concentrated in some periods of the year and their turnout is presumably higher than that of Italian tourists in the case of seasonal opening;
- museums offering guided tours (individual-level covariate), compared to those not offering guided visits, show a +15.3% increase

of the probability of deciding for a seasonal and continuous opening. The effect of this covariate is positive on the probability of both seasonal and year-round opening. It can be justified by considering that the guided tours contribute to enrich the museum experience, by improving the effectiveness for the promotion of the culture, which is the fundamental aim of museums.

With reference to the contextual factors it is evident that:

- a high provincial number of tourist accommodation establishments determines a positive effect on the probability to stay open continuously or seasonally, with an increment of the +24.6%;
- an available regional financial support to cover the expenditure for recreation, culture and religion generates a positive effect on the probability to stay open continuously or seasonally of the +6.1%;
- a high regional gross domestic product per capita increases the probability to stay open continuously or seasonally of the +46.1%.

Moreover, by focusing on the random part of the model, it is evident that the variation in the probability to stay open all over the year or seasonally, occurring between the third and second level, concerns the covariates:

- "Research activities" with a low variability at level 2 for continuous opening than in the case of continuous or seasonal opening, as confirmed by the estimated intra-class correlation coefficient corresponding to 7% and 32%, respectively;
- "Network of museums" with a low variability only at level 3 for continuous or seasonal opening, as confirmed by the estimated intra-class correlation coefficient corresponding to 6%.

4. Trend modeling by type of institution

Let $Y^{(s)}_{1ijk}$ and $Y^{(s)}_{2ijk}$ be multinomial ordered response variables which take values $s = 1, 2, 3$ (response categories), where $s = 3$ represents the reference category "occasional opening" for public and private museums, respectively.

On the other hand, $s = 2$ corresponds to "seasonal opening" and $s = 1$ denotes "continuous opening", and let

- $\pi^{(3)}_{1ijk}$ and $\pi^{(3)}_{2ijk}$ the probabilities of occasional museums opening for public and private museums, respectively,
- $\pi^{(2)}_{1ijk}$ and $\pi^{(2)}_{2ijk}$ the probabilities of seasonal museums opening for public and private museums, respectively,
- $\pi^{(1)}_{1ijk}$ and $\pi^{(1)}_{2ijk}$ the probabilities of continuous (year-round) museums opening for public and private museums, respectively,

with the index i representing the Italian public $i = 1, \dots, n_{1jk}$ or private $i = 1, \dots, n_{2jk}$ museums (level 1 unit); the index j ($j = 1, \dots, N_k$) corresponding to the provinces (level 2 unit) and the index k ($k = 1, \dots, K$) indicating the regions (level 3 unit). Moreover, let $\mathbf{X}_{.} = \{X_{1.}, X_{2.}, \dots, X_{H.}\}$ be the set of covariates, which influences the dependent response variable (Table 1).

As previously clarified, by considering the ordering, the selected models for public and private museums are based on the cumulative response probabilities defined as follows:

$$\psi^{(s)}_{1ijk} = \sum_{h=1}^{s} \pi^{(h)}_{1ijk}, \qquad s = 1, 2,$$

$$\psi^{(s)}_{2ijk} = \sum_{h=1}^{s} \pi^{(h)}_{2ijk}, \qquad s = 1, 2.$$

The multilevel ordered logit models for public and private museums are specified in the following paragraphs.

4.1. Multilevel multinomial ordered model for public museums

The category probabilities for the public museums can be used to determine the cumulative probabilities, as follows:

$$\psi^{(1)}_{1ijk} = \pi^{(1)}_{1ijk}, \qquad \psi^{(2)}_{1ijk} = \pi^{(1)}_{1ijk} + \pi^{(2)}_{1ijk}, \qquad \psi^{(3)}_{1ijk} = 1.$$

The two logit models, one for each category ("continuous museums opening" and "seasonal or year-round museums opening") are:

$$\eta^{(1)}_{ijk} = \log it(\psi^{(1)}_{1ijk}) = \beta^{(1)}_{0jk} + \beta^{(1)}_{1k}\, x_{1ijk} + \beta^{(1)}_{2}\, x_{2ijk} + \beta^{(1)}_{3}\, x_{3ijk} + \beta^{(1)}_{4}\, x_{4ijk} +$$

$$+ \beta^{(1)}_{5}\, x_{5ijk} + \beta^{(1)}_{6}\, x_{6ijk} + h_{ijk}, \tag{10}$$

$$\eta^{(2)}_{ijk} = \log it(\psi^{(2)}_{1ijk}) = \beta^{(2)}_{0jk} + \beta^{(2)}_{1}\, x_{1ijk} + \beta^{(2)}_{2}\, x_{2ijk} + \beta^{(2)}_{3}\, x_{3ijk} + \beta^{(2)}_{4}\, x_{4ijk} +$$

$$+ \beta^{(2)}_{5}\, x_{5ijk} + \beta^{(2)}_{6}\, x_{6ijk} + h_{ijk}, \tag{11}$$

with

- $h_{ijk} = \beta_7 x_{7k} + \beta_8 x_{8k} + \beta_9 x_{9jk} + \beta_{10} x_{10ijk} + \beta_{11} x_{11ijk} + \beta_{12} x_{12ijk} + \beta_{13} x_{13ijk}$,
- $\beta^{(1)}_{0jk} = \beta^{(1)}_{0} + v^{(1)}_{0k} + u^{(1)}_{0jk}$,
- $\beta^{(2)}_{0jk} = \beta^{(2)}_{0} + v^{(2)}_{0k} + u^{(2)}_{0jk}$,
- $\beta^{(1)}_{1k} = \beta^{(1)}_{1} + u^{(1)}_{1jk}$,
- $\begin{bmatrix} v^{(1)}_{0k} \\ v^{(2)}_{0k} \end{bmatrix} \sim N\left(\mathbf{0}, \boldsymbol{\Omega}_v\right), \quad \boldsymbol{\Omega}_v = \begin{bmatrix} \sigma^2_{v0^{(1)}} & \\ \sigma_{v0^{(1)}\,0^{(2)}} & \sigma^2_{v0^{(2)}} \end{bmatrix},$
- $\begin{bmatrix} u^{(1)}_{0jk} \\ u^{(1)}_{0jk} \\ u^{(1)}_{1jk} \end{bmatrix} \sim N\left(\mathbf{0}, \boldsymbol{\Omega}_u\right), \quad \boldsymbol{\Omega}_u = \begin{bmatrix} \sigma^2_{u0^{(1)}} & & \\ \sigma_{u0^{(1)}\,0^{(2)}} & \sigma^2_{u0^{(2)}} & \\ \sigma_{u0^{(1)}\,1^{(1)}} & \sigma_{u0^{(2)}\,1^{(1)}} & \sigma^2_{u1^{(1)}} \end{bmatrix}.$

Note that the covariates implemented in the Eqs. (10)–(11) are the same included in the fitted models specified in Eqs. (8)–(9). After testing the proportional odds hypothesis by a Wald test (Section 3.2), a partial proportional odds model has been selected. In particular:

- the intercept in Eqs. (10)–(11) has been hypothesized to change for each level,
- the separate coefficient of the covariate "Research activities" ($X_{1.}$) has been assumed to vary across the 2nd level only for Eq. (10), whilst it does not vary for Eq. (11) since in this case the covariate is not statistically significant.

On the other hand, the coefficients of the contextual factors ("Expenditure for recreation, culture and religion", "Gross domestic product per capita, current prices", "Number of tourist accommodation establishments") included as common coefficients, as well as the slopes of the remaining covariates, implemented into the model as separate coefficients, have been assumed to be constant.

The residual diagnostics computed both at the third and second level has tested the normality assumption.

4.2. Multilevel multinomial model for private museums

Given the cumulative probabilities:

$$\psi^{(1)}_{2ijk} = \pi^{(1)}_{2ijk}, \qquad \psi^{(2)}_{2ijk} = \pi^{(1)}_{2ijk} + \pi^{(2)}_{2ijk}, \qquad \psi^{(3)}_{2ijk} = 1.$$

the following two logit models, one for each category ("continuous museums opening" and "seasonal or year-round museums opening") are defined as follows:

$$\eta_{ijk}^{(1)} = \log it(\psi_{2ijk}^{(1)}) = \beta_{0jk}^{(0)} + \beta_1^{(1)} x_{1ijk} + \beta_2^{(1)} x_{2ijk} + \beta_3^{(1)} x_{3ijk} + \beta_4^{(1)} x_{4ijk} + h_{ijk}, \tag{12}$$

$$\eta_{ijk}^{(2)} = \log it(\psi_{2ijk}^{(2)}) = \beta_{0k}^{(2)} + \beta_1^{(2)} x_{1ijk} + \beta_2^{(2)} x_{2ijk} + \beta_3^{(2)} x_{3ijk} + \beta_4^{(2)} x_{4ijk} + h_{ijk}, \tag{13}$$

with

- $h_{ijk} = \beta_5 x_{5k} + \beta_6 x_{6k} + \beta_{7k} x_{7jk} + \beta_8 x_{8ijk} + \beta_9 x_{9ijk} + \beta_{10} x_{10ijk} + \beta_{11} x_{11ijk} + \beta_{12} x_{12ijk} + \beta_{13} x_{13ijk}$,
- $\beta_{0jk}^{(1)} = \beta_0^{(1)} + v_{0k}^{(1)} + u_{0jk}^{(1)}$,
- $\beta_{0k}^{(2)} = \beta_0^{(2)} + v_{0k}^{(2)}$,
- $\beta_{7k} = \beta_7 + v_{7k}$,
- $\begin{bmatrix} v_{0k}^{(1)} \\ v_{0k}^{(2)} \\ v_{7k} \end{bmatrix} \sim N\left(\mathbf{0}, \boldsymbol{\Omega}_v\right), \quad \boldsymbol{\Omega}_v = \begin{bmatrix} \sigma_{v0^{(1)}}^2 & & \\ \sigma_{v0^{(1)}0^{(2)}} & \sigma_{v0^{(2)}}^2 & \\ \sigma_{v0^{(1)}7} & \sigma_{v0^{(2)}7} & \sigma_{v7}^2 \end{bmatrix}$,
- $\begin{bmatrix} u_{0jk}^{(1)} \end{bmatrix} \sim N\left(\mathbf{0}, \boldsymbol{\Omega}_u\right), \quad \boldsymbol{\Omega}_u = \begin{bmatrix} \sigma_{u0^{(1)}}^2 \end{bmatrix}$.

After checking the proportionality assumption (Table 4), it has been underlined that:

- the intercept added in (12) has been hypothesized to change for each level, on the other hand, the intercept included in (13) has been assumed to vary across the 3rd level,
- the common coefficient of the covariate "Number of tourist accommodation establishments" ($X_{7.}$) has been assumed to vary across the 3rd level.

Moreover, the coefficients of the contextual factors ("Expenditure for recreation, culture and religion", "Gross domestic product per capita, current prices"), included as common coefficients, as well as the slopes of the remaining covariates, implemented into the model as common or separate coefficients, have been assumed to be constant.

The efficiency of this model has been proved by the residual diagnostics (standardized residuals against normal scores or rank of residuals) computed both for the third and second level of nesting.

Remark

From a comparison between the models (10) and (12), it has to be highlighted that in the case of public museums, the covariate "Research activities" shows variability at the provincial level, while in the case of private museums, there is no variability at the provincial level. This can be justified by taking into account that private museums are typically founded on economic, management and financial policies which do not depend on the provincial or regional level. Furthermore, for private museums, the covariate "Number of tourist accommodation establishments" explains variation at regional level, since museums react differently on the basis of the regional context and infrastructures. This kind of variability is not reasonably found in the public case, based on specific institutional settings that differ across regions.

4.3. Test for non-proportionality and estimation results for public and private museums

In order to test the null hypothesis that the effects of the covariates are proportional, the Wald Chi-Square test has been applied (Table 4).

From the *p-values* in Table 4 it is evident that:

- for public museums, the proportional odds assumption holds only for the covariate "Percentage of Italians vs Foreigners" at a significance level not exceeding 10%. The other covariates reported in the Table 4 have been implemented as separate coefficients to each of the two sub-equations of the model, since the proportional odds hypothesis is violated,

Table 4
Test for non-proportionality of the covariates with separate effect, referred to multinomial ordered model by type of institutions.

Covariate's category for public museums	Wald statistic	*p-value*
constant	16.35	0.000***
Research activities	9.743	0.008***
Access	20.485	0.000***
Percentage of Italians vs Foreigners	2.893	0.235
Exhibition space between 201 e 500 square meters	17.884	0.000***
Exhibition space of more than 500 square meters	29.148	0.000***
Medium digitalization	9.973	0.007***
High digitalization	8.027	0.018**
Covariate's category for private museums	**Wald statistic**	***p-value***
constant	26.472	0.000***
Research activities	5.895	0.052**
Access	5.566	0.062**
Percentage of Italians vs Foreigners	4.195	0.123
Network of museums	1.279	0.528
Partnership	1.405	0.495
Medium digitalization	6.728	0.035**
High digitalization	15.874	0.000***
Guided tours	0.342	0.843
Exhibition space between 201 e 500 square meters	0.778	0.678
Exhibition space of more than 500 square meters	9.786	0.007***

**p-value* < 0.1 ***p-value* < 0.05 ****p-value* < 0.01.

- for private museums, the proportional odds assumption is confirmed for the covariates "Percentage of Italians vs Foreigners", "Network of museums", "Partnership", "Guided tours" and "Exhibition space between 201 e 500 square meters", at a significance level not exceeding 10%. Thus, the other covariates reported in the Table 4 have been added as separate coefficients to each of the two sub-equations of the model, since the proportional odds hypothesis is rejected.

Tables 5–6 show the estimates of fixed and random parameters of the multinomial ordered logit model for private and public institutions.

The ORs' results suggest that, for public museums, the factors with the largest impact on the probability of guaranteeing continuous and regular opening or seasonal opening are the "Exhibition space", the "Partnership" and the macro-economic covariate "Regional expenditure for recreation, culture and religion". Indeed, it is worth highlighting the following details:

- for what concerns public museums, the availability of a medium (large) space increases the probability to allow a continuous opening by +76.1% (+149.7% for a large exhibition space); moreover, the availability of a medium (large) space raises the probability to stay open seasonally or year-round by +171.8% (+314.1% for large space). In other terms, this covariate produces a propulsive effect on the probability of both the options, with a stronger incidence on the probability of seasonal opening than on year-round ones (which entails a further increase of the OR equal to +95.7% in the case of medium space and a significant rise of the OR of +164.4% in the case of large space);
- the presence of partnership with the territory, with respect to the absence of formal relationships with other public or private cultural institutions located in the territory, increases the probability of respecting the decision of continuous or seasonal opening by +39.2% for public (+19% for private museums); this result is justified by the idea that both the public and private museums can avail the integrated cultural services to relaunch their image and promote their cultural heritage;
- an available regional financial support to cover the expenditure for recreation, culture and religion generates a positive effect on the probability to stay open continuously or seasonally of the +65.4% and +19.7% (no significance) for public and private museums, respectively.

Table 5
Estimates of fixed and random parameters, together with the standard errors (SE), the Wald statistic, the *p-value* and the ORs of the ordered logit model concerning public institutions.

Fixed parameters (Covariate's category)	$\hat{\beta}$	$SE(\hat{\beta})$	Wald statistic	*p-value*	$OR = \exp(\hat{\beta})$
Separate coefficients					
Continuous opening					
Individual-level covariates					
constant	0.134	0.065	2.062	0.039**	1.143
Research activities (x_{1ijk})	0.521	0.168	3.101	0.002***	1.684
Access (x_{2ijk})	−0.107	0.042	−2.548	0.011**	0.899
Medium digitalization (x_{3ijk})	0.195	0.083	2.349	0.019**	1.215
High digitalization (x_{4ijk})	0.497	0.225	2.209	0.027**	1.644
Exhibition space between 201 e 500 square meters (x_{5ijk})	0.566	0.153	3.699	0.000***	1.761
Exhibition space of more than 500 square meters (x_{6ijk})	0.915	0.176	5.199	0.000***	2.497
Seasonal and continuous opening					
Individual-level covariates					
constant	2.048	0.451	4.541	0.000***	7.752
Research activities (x_{1ijk})	0.246	0.274	0.898	0.369	1.279
Access (x_{2ijk})	−1.332	0.307	−4.339	0.000***	0.264
Medium digitalization (x_{3ijk})	0.728	0.244	2.984	0.003***	2.071
High digitalization (x_{4ijk})	0.908	0.346	2.624	0.009***	2.479
Exhibition space between 201 e 500 square meters (x_{5ijk})	1.000	0.277	3.610	0.000***	2.718
Exhibition space of more than 500 square meters (x_{6ijk})	1.421	0.378	3.759	0.000***	4.141
Common coefficients					
Regional-level covariates					
Gross domestic product per capita, current prices (x_{7k})	0.328	0.194	1.692	0.091*	1.389
Expenditure for recreation, culture and religion (x_{8k})	0.503	0.287	1.753	0.080*	1.654
Province-level covariate					
Number of tourist accommodation establishments (x_{9jk})	0.133	0.070	1.900	0.057*	1.142
Individual-level covariates					
Percentage of Italians vs Foreigners (x_{10ijk})	0.261	0.125	2.088	0.037**	1.298
Partnership (x_{11ijk})	0.331	0.144	2.299	0.022**	1.392
Network of museums (x_{12ijk})	0.002	0.144	0.014	0.989	1.002
Guided tours (x_{13ijk})	0.269	0.123	2.187	0.029**	1.309

Random parameters

$$\mathbf{\Omega}_v = \begin{bmatrix} 0.471(0.193) & \\ 0.042(0.118) & 0.133(0.131) \end{bmatrix}$$

$$\mathbf{\Omega}_u = \begin{bmatrix} 0.285(0.129) & & \\ -0.002(0.119) & 0.247(0.197) & \\ -0.075(0.14) & 0.197(0.158) & 0.253(0.243) \end{bmatrix}$$

**p-value* < 0.1 ***p-value* < 0.05 ****p-value* < 0.01.

On the other hand, regarding private museums, the factors which present the most evident incidence on the probability to stay open always or seasonally are "Research activities " and "High digitalization", as specified below:

- offering research activities for private museums increases the probability to stay open all over the year with respect to occasionally opening, by +42.2% or boostes the probability to prefer the continuous or seasonally opening by +109.4%;
- a high level of digitalization for a private museum, compared to a low level of digitalization, increases the probability to guarantee a continuous opening by +135.4% and seasonal or continuous opening by +680.7%. This covariate produces a triggering effect on the probability of both seasonal and continuous opening, with a larger incidence on the probability of seasonal opening than on continuous opening especially in the case of high digitalization (which entails a further increase of the OR equal to +545.3%);
- a high regional gross domestic product per capita increases the probability to stay open continuously or seasonally of the +38.9% and +56% for public and private museums, respectively.

In addition, it is important to clarify that a free access to public museums, with respect to a paid visit, decreases the probability to stay open all over the year with respect to occasionally by −10.1% or reduces the probability to opt for the continuous or seasonally opening by −73.6%; similarly, a free access to private museums decreases the probability of respecting the decision to open all over the year with respect to occasionally, by −0.5% or diminishes the probability to guarantee a seasonal or continuous opening by −72%; thus, in terms of this covariate, there are no significant discrepancies for private and public museums.

The different behavior of private museums with respect to the public ones could be due both to a different policy of incentives, since they do not rely on a public budget [50], as well as to a lower incidence of the cumbersome administrative processes which is binding for the public institutions.

5. Exploratory spatial data analysis of multilevel logit results

Fig. 1 shows the colormap of the logit odds for "continuous museums opening" and "seasonal or year-round museums opening", respectively, together with the corresponding box-plots, classified in Northern, Central and Southern Italy (including the islands).

From Fig. 1-(a), a greater likelihood to stay open all the year is observed for the museums located in the Northern and Central Italy, with respect to those in the South Italy. Moreover, the lowest and the highest logit values of year-round opening for the museums are exceptionally placed in Aosta Valley (Northern Italy) and in the province of Napoli (in Campania region), respectively. On the other hand, by focusing on the cumulative logit odds ("seasonal and year-round museums opening"), the differences between the Central-Northern regions and the Southern regions tend to be smoother (Fig. 1-b). Note that in case of more than one museum at province level, the logit values have been averaged.

Table 6
Estimates of fixed and random parameters, together with the standard errors (SE), the Wald statistic, the *p-value* and the ORs of the ordered logit model concerning private institutions.

Fixed parameters (Covariate's category)	$\hat{\beta}$	$SE(\hat{\beta})$	Wald statistic	*p-value*	$OR = \exp(\hat{\beta})$
Separate coefficients					
Continuous opening					
Individual-level covariates					
constant	0.429	0.223	1.924	0.054*	1.536
Research activities (x_{1ijk})	0.352	0.174	2.023	0.043**	1.422
Access (x_{2ijk})	−0.005	0.003	−1.667	0.096*	0.995
Medium digitalization (x_{3ijk})	0.495	0.226	2.190	0.029**	1.640
High digitalization (x_{4ijk})	0.856	0.270	3.170	0.002***	2.354
Seasonal and continuous opening					
Individual-level covariates					
constant	2.769	0.559	4.953	0.000***	15.943
Research activities (x_{1ijk})	0.739	0.386	1.915	0.056*	2.094
Access (x_{2ijk})	−1.274	0.416	−3.063	0.002***	0.280
Medium digitalization (x_{3ijk})	0.742	0.308	2.409	0.016**	2.100
High digitalization (x_{4ijk})	2.055	0.582	3.531	0.000***	7.807
Common coefficients					
Regional-level covariates					
Gross domestic product per capita, current prices (x_{5k})	0.445	0.317	1.402	0.161	1.560
Expenditure for recreation, culture and religion (x_{6k})	0.180	0.181	0.994	0.320	1.197
Province-level covariate					
Number of tourist accommodation establishments (x_{7jk})	0.008	0.004	2.000	0.046**	1.008
Individual-level covariates					
Percentage of Italians vs Foreigners (x_{8ijk})	0.226	0.136	1.662	0.097*	1.254
Partnership (x_{9ijk})	0.174	0.102	1.706	0.088*	1.190
Exhibition space between 201 e 500 square meters (x_{10ijk})	0.152	0.080	1.900	0.057*	1.164
Exhibition space of more than 500 square meters (x_{11ijk})	0.584	0.203	2.877	0.004***	1.793
Network of museums (x_{12ijk})	−0.230	0.139	−1.655	0.098*	0.795
Guided tours (x_{13ijk})	−0.022	0.167	−0.132	0.895	0.978

Random parameters

$$\boldsymbol{\Omega}_v = \begin{bmatrix} 0.409(0.306) & & \\ 0.239(0.243) & 0.142(0.247) & \\ -0.258(0.258) & -0.054(0.206) & 0.124(0.259) \end{bmatrix}$$

$$\boldsymbol{\Omega}_u = \begin{bmatrix} 0.451(0.148) \end{bmatrix}$$

**p-value* < 0.1 ***p-value* < 0.05 ****p-value* < 0.01.

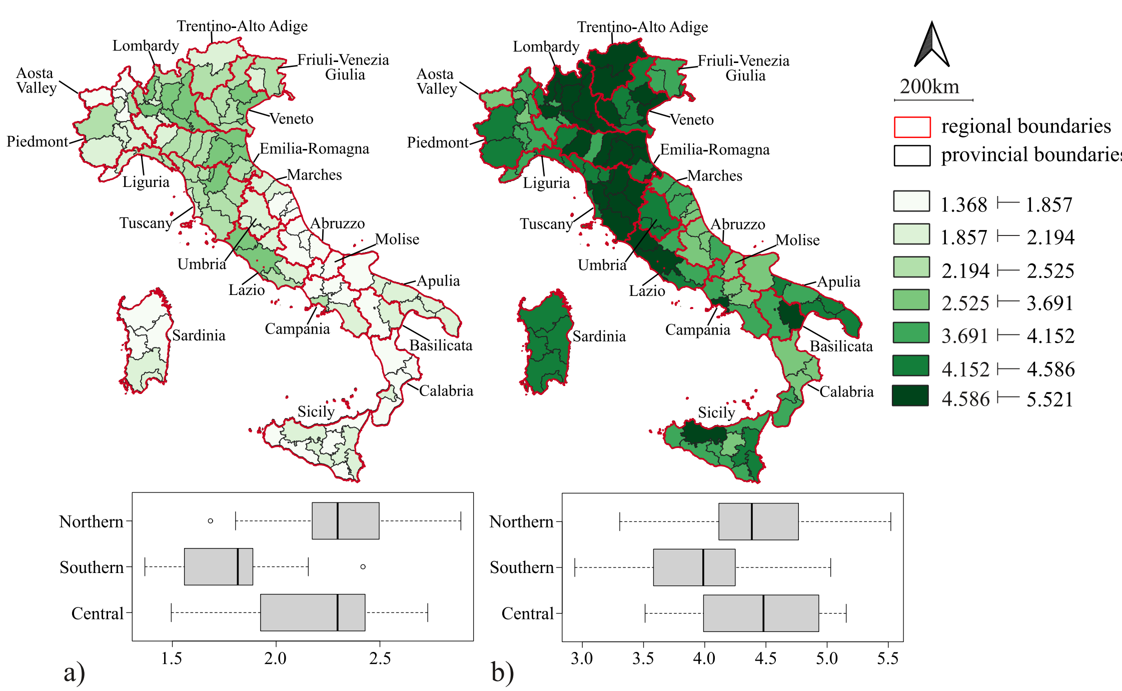

Fig. 1. Colormap and box-plots for logit odds to stay open (a) all year-round (b) at least seasonally or continuously for all the Italian museums.

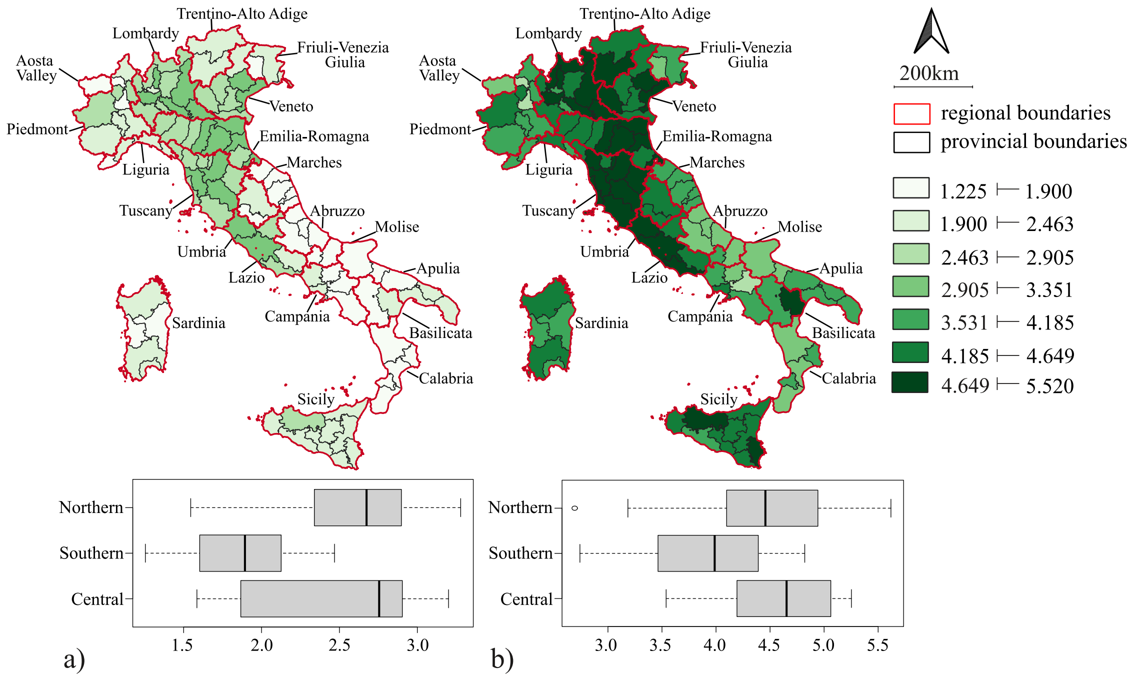

Fig. 2. (a) Colormap for logit odds to stay open all year-round (b) colormap for cumulative logit odds to stay open seasonally and continuously, for public museums.

Furthermore, in Figgs. 2 and 3 are illustrated the colormaps of the logit odds for "continuous museums opening", as well as the cumulative logit case for public and private museums, respectively, jointly with the respective box-plots categorized in the three main territorial divisions North, Centre and South of Italy (including the islands).

For what concerns the public museums, from the inspection of Fig. 2-(a) it is evident that the provinces with the greatest logit odds values for the option "continuous museums opening" are in the Northern and Central Italy, while a lower likelihood for the museums located in the Southern provinces has been detected. By analyzing the cumulative case (Fig. 2-b), it is found that the discrepancies among the cumulative logit odds for the museums in the South and the others in the Northern and Central Italy tend to decrease. In addition, it is worth highlighting that the lowest cumulative logit odd for the public museums is placed in the Northern part of Italy and in particular in the province of Vercelli (Piedmont region), even though it represents an isolated case.

Also from Fig. 3-(a) it is evident that the museums located in the Northern and Central regions show logit odds to stay open year-round greater than those in the South of Italy, with a few exceptions. Indeed, the lowest logit odds to stay open continuously for the private museums are sparsely placed in Aosta Valley, Liguria (provinces of Savona, Imperia and La Spezia), Trentino-Alto Adige (province of Bolzano) and Marches (province of Fermo). By focusing on the cumulative odds logit ("seasonal or year-round museums opening"), the differences between the Central-Northern Italy and the South tend to be toned down, as in the previous cases (Fig. 3-b), with the lowest and the highest values located in Marches (provinces of Fermo) and Tuscany (province of Prato), respectively.

All the colormaps have been computed by the *R* package *gstat* [27, 28].

6. Structural analysis and interpolation by multilevel logit kriging

As previously mentioned, the structural analysis (conducted both by considering all museums and by splitting them according to the type of institution) is characterized by the variogram estimation and the model fitting.

Thus, after completing the first phase of the structural analysis, the following spatial variogram models for the residuals of the partial proportional logit odds of the ordered categorical response variables "continuous museums opening" (all the year), denoted with $\gamma^{(1)}(\mathbf{h})$, and "seasonal or year-round museums opening" (cumulative logit), indicated with $\gamma^{(2)}(\mathbf{h})$, have been identified:

$$\gamma^{(1)}(\mathbf{h}) = \begin{cases} 0 & \|\mathbf{h}\| = 0 \\ 0.005 + 0.165\, Exp(\mathbf{h};\, 615{,}000) & \|\mathbf{h}\| > 0 \end{cases} \tag{14}$$

where

- the nugget is equal to 0.005,
- the sill corresponds to 0.165,
- the range coincides with 615,000 m,
- $Exp(\cdot)$ represents the exponential variogram model [23],

$$\gamma^{(2)}(\mathbf{h}) = \begin{cases} 0 & \|\mathbf{h}\| = 0 \\ 0.055 + 0.31\, Exp(\mathbf{h};\, 450{,}420) & \|\mathbf{h}\| > 0 \end{cases} \tag{15}$$

where

- the nugget is equal to 0.055,
- the sill coincides with 0.31,
- the range corresponds to 450,420 m,
- $Exp(\cdot)$ is the exponential variogram model [23].

The sample variograms for the residuals of the logit values to stay open all the year or at least seasonally, together with the corresponding fitted models (14)–(15) are illustrated in Figgs. 4(a)–(c). The nugget/sill ratio demonstrates that for the residuals of the logits of staying open all the year-round in Fig. 4(a), the spatial variability is largely explained by the model, with a range covering at least half of the maximum observable distance over the Italian peninsula. These aspects reflect the presence of vast areas characterized by strong spatial similarity. In particular, the entire Central-Eastern, as well as Southern and Insular regions, show logit values tendentially below the median value, differently from the rest of Italy. This feature is attenuated for the residuals of the logits of staying open at least seasonally in 4(c), where the range of the variogram model in (15) is smaller than the one in (14) and the corresponding spatial similarity is reduced.

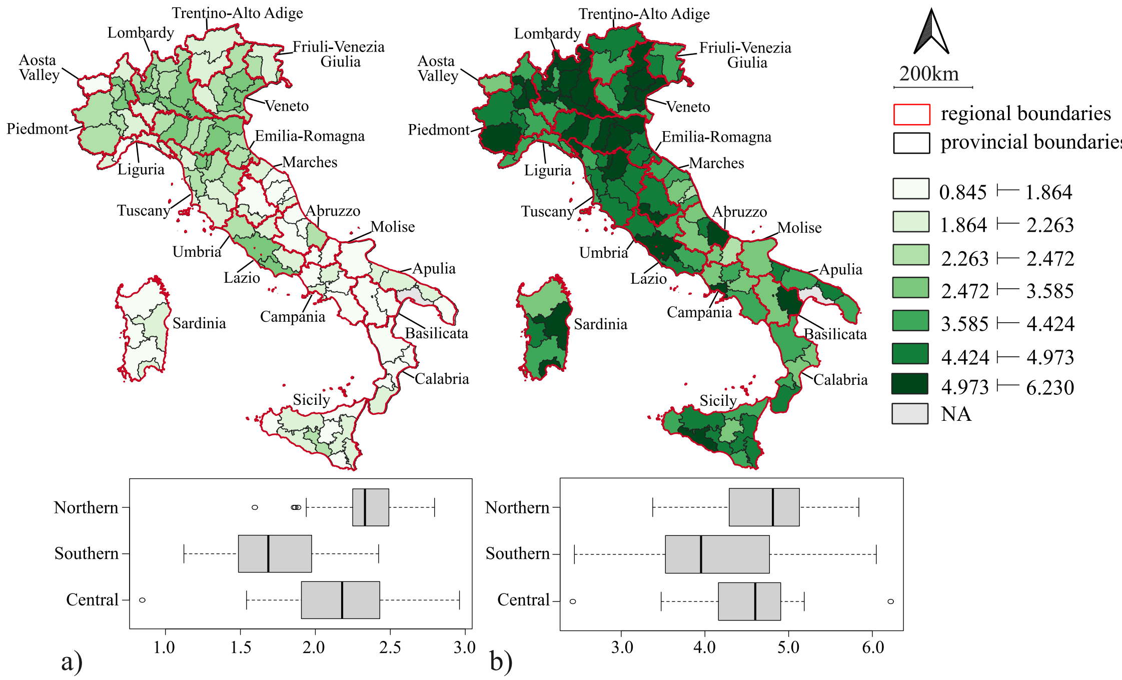

Fig. 3. (a) Colormaps for odds logit to stay open all year-round (b) colormaps for odds cumulative logit to stay open seasonally or continuously for private museums.

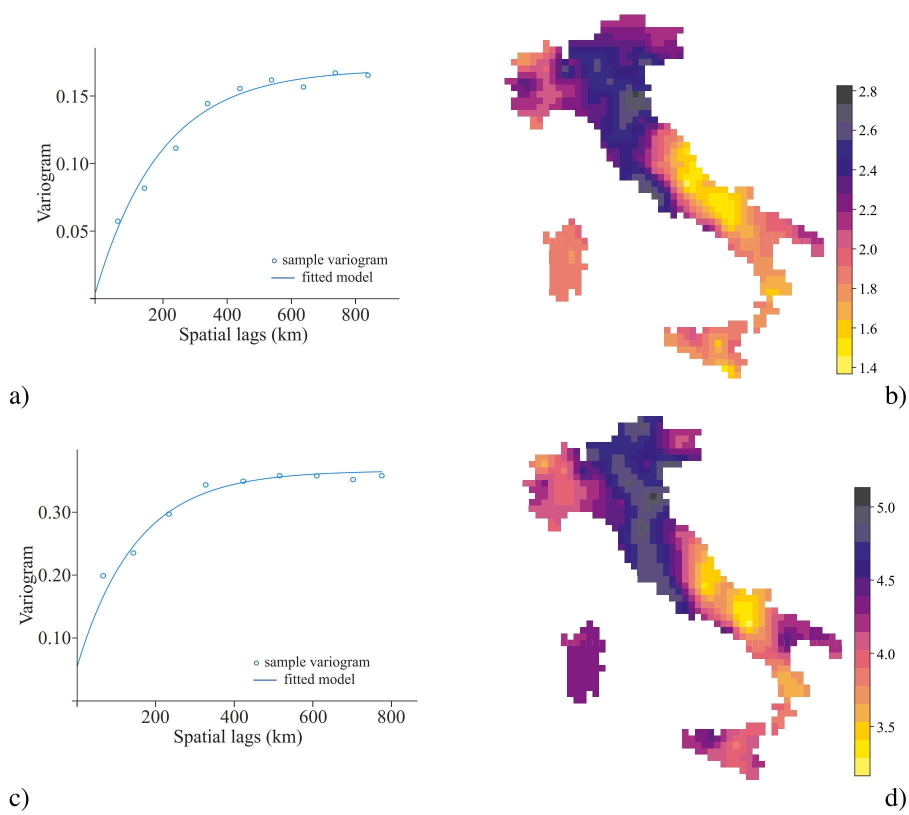

Fig. 4. (a) Sample variogram and fitted model for the residuals of the logit values to stay open all year-round (b) Colormap of the logit values to stay open all year-round, (c) Sample variogram and fitted model for the residuals of the logit cumulative values to stay open seasonally and continuously (d) Colormap of the logit cumulative values to stay open seasonally and continuously.

The variogram models have been used in order to interpolate the selected variables over the domain. In particular, through the multilevel logit kriging interpolations, computed by using the available data, the variogram models (14)–(15) and the *R* package *gstat* [27,28], the predicted logit values have been produced, as illustrated in Figgs. 4(b)–(d). Although the smoothing does not account for the covariates at the predictive locations, the interpolated maps can be at least useful to visually appreciate the high spatial relationship (detected

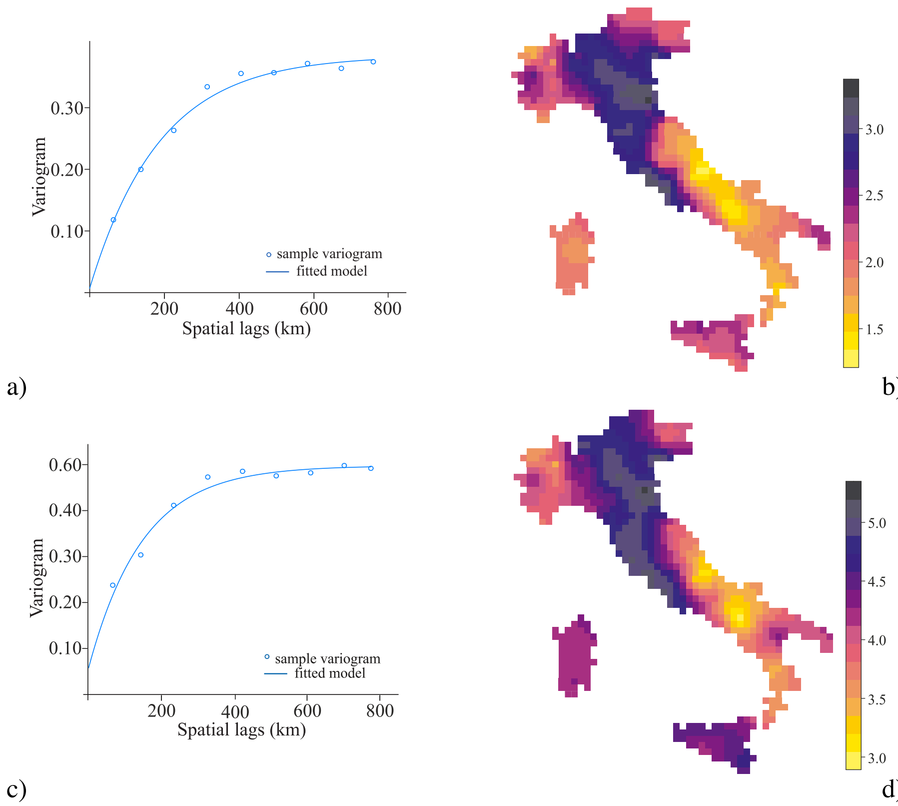

Fig. 5. (a) Sample variograms and fitted models for the residuals of the logit values to stay open all year-round for public museums (b) Colormap of the logit values to stay open all year-round, (c) Sample variograms and fitted models for the residuals of the logit cumulative values to stay open seasonally and continuously (d) Colormap of the logit cumulative values to stay open seasonally and continuously.

through the variograms) according to which the opening decisions of museums located within a certain neighborhood might be reciprocally influenced.

Analogously to the case of the all museums, the same variographic analysis has been carried out for public and private museums.

6.1. Structural analysis and interpolation using multilevel logit kriging, by type of institutions

After the variogram estimation, the following spatial variogram models for the residuals of the logit partial proportional odds of the ordered categorical response variables “continuous museums opening” and “seasonal and year-round museums opening”, respectively, regarding public museums under study, have been chosen:

$$\gamma_1^{(1)}(\mathbf{h}) = \begin{cases} 0 & \| \mathbf{h} \| = 0 \\ 0.008 + 0.380\, Exp(\mathbf{h};\ 585{,}000) & \| \mathbf{h} \| > 0 \end{cases} \qquad (16)$$

where

- the nugget is equal to 0.008,
- the sill corresponds to 0.380,
- the range coincides with 585,000 m,
- $Exp(\cdot)$ represents the exponential variogram model [23],

$$\gamma_2^{(2)}(\mathbf{h}) = \begin{cases} 0 & \| \mathbf{h} \| = 0 \\ 0.06 + 0.44\, Exp(\mathbf{h};\ 446{,}868) & \| \mathbf{h} \| > 0 \end{cases} \qquad (17)$$

where

- the nugget is equal to 0.06,
- the sill corresponds to 0.44,
- the range coincides with 446,868 m,
- $Exp(\cdot)$ represents the exponential variogram model [23].

The sample variogram for the residuals of the logit values to stay open all the year or both continuous and seasonally for public museums, together with the corresponding fitted models (16)–(17) are illustrated in Fig. 5. On the basis of the nugget/sill ratio, it is highlighted that for the residuals of the logits of staying open continuously in (a) the spatial variability is broadly explained by the model, with a range spanning at least half of the maximum detectable distance across the Italian peninsula. These characteristics, similarly to the case of the total museums, reveal the existence of wide domains with high spatial similarity. More specifically, it is worth recalling that the macro-areas with the lowest logit values is the Centre-East, as well as South Italy (including islands), unlike the other parts of Italy. This is toned down for the residuals of the logits of staying open at least seasonally in (c), where the range of the variogram model in (17) is smaller than the one in (16) and the spatial similarity in the latter case is less extensive than the former case. The variogram models have been applied for interpolation purposes by using the multilevel logit kriging, which allow to obtain the maps shown in Figgs. 5-(b)–(d). It is clear that the opening decisions of museums located within a wide neighborhood are reciprocally influenced and this effect is much more evident for the probability of continuous opening.

For what concerns the private museums, the following spatial variogram models for the residuals of the logit partial proportional odds of the ordered categorical response variables “continuous museums opening” and “seasonal or year-round museums opening”, respectively, have been selected:

$$\gamma_1^{(1)}(\mathbf{h}) = \begin{cases} 0 & \|\mathbf{h}\| = 0 \\ 0.075 + 0.16\, Gau(\mathbf{h};\ 640{,}859) & \|\mathbf{h}\| > 0 \end{cases} \qquad (18)$$

where

- the nugget is equal to 0.075,

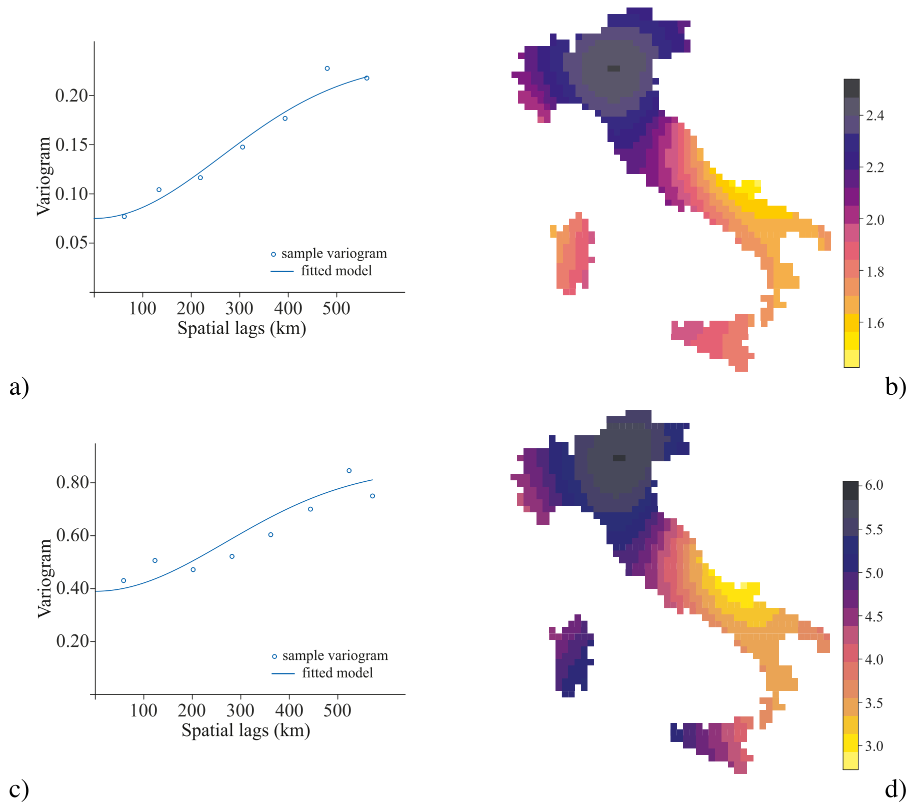

Fig. 6. (a) Sample variograms and fitted models for the residuals of the logit values to stay open all year-round for private museums (b) Colormap of the logit values to stay open all year-round, (c) Sample variograms and fitted models for the residuals of the logit cumulative values to stay open seasonally and continuously (d) Colormap of the logit cumulative values to stay open seasonally and continuously.

- the sill coincides with 0.16,
- the effective range corresponds to 640,859 m,
- $Gau(\cdot)$ is the Gaussian variogram model [23].

$$\gamma_2^{(2)}(\mathbf{h}) = \begin{cases} 0 & \|\mathbf{h}\| = 0 \\ 0.39 + 0.47\, Gau(\mathbf{h};\ 658{,}179) & \|\mathbf{h}\| > 0 \end{cases} \tag{19}$$

where

- the nugget is equal to 0.39,
- the sill coincides with 0.47,
- the effective range corresponds to 658,179 m,
- $Gau(\cdot)$ is the Gaussian variogram model [23].

The sample variograms for the residuals of the logit values to stay open year-round and for the cumulative logit values, together with the corresponding fitted models (18)–(19) are illustrated in Figgs. 6(a)-(c).

In this case, the parabolic behavior of the variograms near its origin (fitted by a Gaussian model) implies a very regular spatial variability of the investigated variables. Hence, this aspects is indicative of a high spatial continuity, although the nugget/sill ratio for the residuals of the logits in (a) and (c) is larger than that of the other two scenarios (total and private museums). As a consequence, one can conclude that the private museums are more affected by the spatial continuity, especially in the scenario of the probability to stay open all year-round. Indeed, the public sector is often linked to political decisions in the region, moreover, the spatial correlation declines faster than the spatial correlation related to the private sector, where the geographical component might have greater impact on the opening decisions.

The variogram models (18)–(19) have been used for interpolating the selected variables over the whole spatial domain. In particular, the multilevel logit kriging interpolations have been calculated by producing the kriged maps as shown in Figgs. 6-(b)–(d), which confirm the high spatial continuity characterized by the data. Thus, especially in this case, the benefit from the possible economies of scale may affect the opening choices of various museums in the hinterland.

6.2. Estimated probabilities for province and region levels

In order to clarify the modeling findings, the estimated probabilities for the museums to choose the occasional, seasonal, continuous opening have been determined at the regional and provincial level as reported in Table 7.

By concentrating on the regional level, it is much more likely that museums in Italy are willing to stay open all year (with predicted values ranging from 0.801 to 0.910) than seasonally (with estimated values ranging from 0.069 to 0.149) or occasionally (with estimated values ranging from 0.015 to 0.059).

Moreover, it is pointed out that the Italian regions with the highest estimated probability to stay open “continuously”, correspond to the ones with more than three hundred structures, i.e Emilia-Romagna, Tuscany and Veneto followed by Lombardy, Lazio, Trentino Alto-Adige and Friuli Venezia-Giulia. Apart from the Lazio and the Tuscany regions located in the Center, the other regions are positioned in the Northern part of the Italian peninsula. This could be explained by considering that these regions represent the cradle of the most important museums in the world, which acts as a driving force to stimulate the small museums located around these famed institutions to remain open all the year. In addition, Molise and Abruzzo are the regions with the biggest cumulative estimated probability of respecting the decision to stay open seasonally or occasionally.

This is confirmed, at the provincial level, from the results shown in Table 7, which reveal a greater likelihood for Italian museums to stay open all the year (with predicted values spanning from 0.787 to 0.941) than seasonally (with estimated values ranging from 0.048 to 0.168) or occasionally (with predicted values spanning from 0.009 to 0.071).

Table 7
Estimate probabilities of occasional opening ($\hat{\pi}^{(3)}_{ijk}$), of seasonal opening ($\hat{\pi}^{(2)}_{ijk}$) or year-round opening ($\hat{\pi}^{(1)}_{ijk}$) for multilevel ordered logit model, classified by regions and provinces.

Macro-area	Region/Province	$\hat{\pi}^{(3)}_{ijk}$	$\hat{\pi}^{(2)}_{ijk}$	$\hat{\pi}^{(1)}_{ijk}$
Northern Italy	**Liguria**	**0.039**	**0.090**	**0.871**
	Genova	0.039	0.075	0.886
	Imperia	0.044	0.116	0.840
	La Spezia	0.039	0.080	0.881
	Savona	0.033	0.089	0.878
	Lombardy	**0.025**	**0.072**	**0.904**
	Bergamo	0.023	0.063	0.914
	Brescia	0.016	0.067	0.917
	Como	0.011	0.067	0.922
	Cremona	0.021	0.063	0.916
	Lecco	0.011	0.073	0.916
	Lodi	0.037	0.083	0.880
	Mantova	0.013	0.068	0.919
	Milano	0.011	0.048	0.941
	Monza-Brianza	0.057	0.086	0.857
	Pavia	0.035	0.083	0.882
	Sondrio	0.037	0.078	0.885
	Varese	0.027	0.082	0.891
	Piedmont	**0.040**	**0.089**	**0.871**
	Alessandria	0.039	0.092	0.869
	Asti	0.053	0.089	0.858
	Biella	0.030	0.087	0.883
	Cuneo	0.031	0.085	0.884
	Novara	0.035	0.078	0.887
	Torino	0.027	0.069	0.904
	Verbano-Cusio-Ossola	0.036	0.116	0.848
	Vercelli	0.064	0.099	0.837
	Emilia-Romagna	**0.021**	**0.069**	**0.910**
	Bologna	0.014	0.060	0.926
	Ferrara	0.009	0.054	0.937
	Forlì-Cesena	0.031	0.075	0.894
	Modena	0.017	0.069	0.914
	Parma	0.020	0.070	0.910
	Piacenza	0.030	0.086	0.884
	Ravenna	0.018	0.058	0.924
	Reggio nell'Emilia	0.029	0.083	0.888
	Rimini	0.016	0.069	0.915
	Friuli-Venezia Giulia	**0.029**	**0.083**	**0.888**
	Gorizia	0.027	0.084	0.889
	Pordenone	0.039	0.091	0.870
	Trieste	0.021	0.077	0.902
	Udine	0.028	0.080	0.892
	Aosta Valley	**0.059**	**0.112**	**0.829**
	Trentino-Alto Adige	**0.015**	**0.095**	**0.890**
	Bolzano	0.009	0.113	0.878
	Trento	0.021	0.077	0.902
	Veneto	**0.024**	**0.072**	**0.904**
	Belluno	0.031	0.075	0.894
	Padova	0.023	0.075	0.902
	Rovigo	0.032	0.075	0.893
	Treviso	0.020	0.061	0.919
	Venezia	0.010	0.063	0.927
	Verona	0.022	0.079	0.899
	Vicenza	0.027	0.076	0.897
Central Italy	**Lazio**	**0.025**	**0.079**	**0.896**
	Frosinone	0.044	0.097	0.859
	Latina	0.019	0.080	0.901
	Rieti	0.037	0.103	0.860
	Roma	0.013	0.056	0.931
	Viterbo	0.014	0.059	0.927
	Marches	**0.045**	**0.123**	**0.832**
	Ancona	0.033	0.114	0.853
	Ascoli Piceno	0.053	0.132	0.815
	Fermo	0.058	0.139	0.803
	Macerata	0.043	0.123	0.834
	Pesaro-Urbino	0.038	0.108	0.854
	Tuscany	**0.020**	**0.076**	**0.905**
	Arezzo	0.021	0.074	0.905
	Firenze	0.013	0.071	0.916
	Grosseto	0.016	0.078	0.906
	Livorno	0.031	0.096	0.873
	Lucca	0.021	0.074	0.905
	Massa-Carrara	0.017	0.081	0.902
	Pisa	0.014	0.068	0.918
	Pistoia	0.024	0.076	0.900
	Prato	0.020	0.059	0.921
	Siena	0.015	0.077	0.908
	Umbria	**0.022**	**0.114**	**0.864**
	Perugia	0.020	0.109	0.871
	Terni	0.024	0.118	0.858
Southern Italy	**Abruzzo**	**0.056**	**0.135**	**0.809**
	Chieti	0.036	0.127	0.837
	L'Aquila	0.071	0.140	0.789
	Pescara	0.058	0.142	0.800
	Teramo	0.059	0.131	0.810
	Basilicata	**0.027**	**0.126**	**0.847**
	Matera	0.011	0.126	0.863
	Potenza	0.043	0.126	0.831
	Molise	**0.050**	**0.149**	**0.801**
	Campobasso	0.069	0.144	0.787
	Isernia	0.031	0.154	0.815
	Calabria	**0.051**	**0.118**	**0.831**
	Catanzaro	0.067	0.134	0.799
	Cosenza	0.056	0.108	0.836
	Crotone	0.068	0.112	0.820
	Reggio Calabria	0.032	0.124	0.844
	Vibo Valentia	0.034	0.112	0.854
	Campania	**0.046**	**0.110**	**0.844**
	Avellino	0.068	0.132	0.800
	Benevento	0.069	0.123	0.808
	Caserta	0.041	0.114	0.846
	Napoli	0.014	0.077	0.909
	Salerno	0.039	0.106	0.855
	Apulia	**0.030**	**0.107**	**0.862**
	Bari	0.032	0.104	0.864
	Barletta-Andria-Trani	0.025	0.117	0.858
	Brindisi	0.015	0.092	0.893
	Foggia	0.057	0.119	0.824
	Lecce	0.026	0.094	0.880
	Taranto	0.024	0.119	0.857
Islands	**Sardinia**	**0.027**	**0.123**	**0.850**
	Cagliari	0.028	0.114	0.858
	Nuoro	0.024	0.130	0.846
	Oristano	0.036	0.119	0.845
	Sassari	0.023	0.125	0.852
	Sud Sardinia	0.024	0.126	0.850
	Sicily	**0.032**	**0.124**	**0.844**
	Agrigento	0.032	0.138	0.830
	Caltanissetta	0.038	0.106	0.856
	Catania	0.024	0.103	0.873
	Enna	0.042	0.145	0.813
	Messina	0.039	0.114	0.847
	Palermo	0.026	0.096	0.878
	Ragusa	0.030	0.168	0.802
	Siracusa	0.031	0.116	0.853
	Trapani	0.029	0.133	0.838

Bold indicate the mean value of the probability for each region.

In particular, by analyzing the Table 7, it is evident that the provinces with the largest estimated probability of year-round opening are the following:

- Milan (with a predicted probability of 0.941) in the Lombardy region;
- Ferrara (with an estimated probability of 0.937) and Bologna (with a predicted probability of 0.926) in the Emilia-Romagna region;
- Rome (with an expected probability of 0.931) and Viterbo (with a predicted probability of 0.927) in the Lazio region;
- Venice (with an estimated probability of 0.927) in the Veneto region.

On the other hand, the provinces with the greatest cumulative estimated probability of respecting the decision to stay open seasonally or occasionally are the following:

- Campobasso in the Molise region, with a cumulative estimated probability to stay open seasonally or occasionally equal to 0.213 (out of which 0.069 concerning occasional opening and 0.144 for seasonal opening);
- L'Aquila in the Abruzzo region, with a cumulative estimated probability to stay open seasonally or occasionally equal to 0.211 (out of which 0.071 regarding occasional opening and 0.140 for seasonal opening).

In order to support the above mentioned remarks according to which the opening decisions of museums placed within a certain neighborhood might be reciprocally influenced, the geostatistical analysis has been developed by focusing on the logit results.

6.3. Estimated probabilities for province and region levels, by type of institution

In order to explain the results referred to the models reported in the Sections 4.1–4.2, the estimated probabilities have been calculated with respect to the province and region levels, classified according to the type of institution (Tables 8–9).

With reference to the regional level, it is most likely to occur that:

- public museums in Italy are willing to stay open all year (with predicted values ranging from 0.791 to 0.939) than seasonally (with estimated values ranging from 0.045 to 0.157) or occasionally (with estimated values ranging from 0.014 to 0.067);
- private museums in Italy are willing to stay open all year (with predicted values ranging from 0.773 to 0.911) than seasonally (with estimated values ranging from 0.071 to 0.169) or occasionally (with estimated values ranging from 0.014 to 0.058). Note that the minimum and maximum estimated probabilities of continuous opening for private museums are lower than the ones for public museums, meaning that it is less likely to guarantee all year opening in the private sector.

Moreover, from Tables 8–9, it is evident that among the Italian regions with the greatest estimated probabilities to stay open continuously, there are the following:

- Emilia-Romagna (with an estimated probability of 0.939 for public museums and 0.907 for private museums),
- Tuscany (with an estimated probability of 0.933 for public museums and 0.901 for private museums),
- Lombardy (with an estimated probability of 0.929 for public museums and 0.911 for private museums),
- Lazio (with an estimated probability of 0.926 for public museums and 0.903 for private museums),
- Veneto (with an estimated probability of 0.924 for public museums and 0.908 for private museums).

Similar results are also estimated at the provincial level.

By considering the type of institution, for public museums, the largest estimated probabilities of year-round opening are obtained for the following provinces:

- Ferrara (with an estimated probability equal to 0.958), Ravenna (with and estimated probability of 0.957); Bologna and Modena (with an estimated probability of 0.952 and 0.951, respectively), in the Emilia-Romagna region;
- Milan (in the Lombardy region) and Venice (in the Veneto region), both with an estimated probability of 0.955;
- Rome and Viterbo (in the Lazio region) with an estimated probability equal to 0.952 and 0.953, respectively.

On the other hand, for private museums, the highest predicted probabilities of year-round opening are found for the following provinces, such as:

- Prato (with a predicted probability of 0.949) in the Tuscany region,
- Milan (with an estimated probability equal to 0.939) and Como (with an estimated probability equal to 0.935) in the Lombardy region,

Finally, the highest cumulative estimated probabilities to stay open seasonally or occasionally

- for public museums are referred to:
    - Pescara (in the Abruzzo region), with a predicted cumulative of 0.249,
    - Benevento (in the Campania region) with a predicted cumulative of 0.239,
    - Catanzaro (in the Calabria region) with a predicted cumulative of 0.237,
- for private museums concern:
    - Fermo (in the Marche region) with a predicted cumulative of 0.304,
    - Campobasso (in the Molise region) with a predicted cumulative of 0.245,
    - Foggia (in the Apulia region) with a predicted cumulative of 0.245.

7. Results discussion and conclusions

Museums contribute to enrich the Italian social and cultural background, since they are places in which education, research, entertainment and exhibition harmonize in order to create and develop the visitor experience. According to some works [51], public museums are usually more focused on the protection and conservation of their assets (historically prominent in their mission), on the contrary, the private museums are more performing in those areas devoted to visitor orientation and connections with the territory. In this context, it was challenging to study the opening choices of museums by estimating the probability of respecting the decision to stay open all over the year (2,514 museums with permanently open), seasonally or occasionally (703 museums with seasonal or occasional opening), firstly, with reference to all the Italian museums and then by focusing on the public (corresponding to the 63.51% of the total number of museums) and private ones (referred to the 36.49% of the total number of museums).

This was realized by the introduction of an innovative approach based on the combination of multilevel multinomial models and spatial continuity models. In particular, a three-level ordered multinomial logistic model, able to grasp the different opening options for museums, according to an ordinal scale, ranging from the occasional to year-round opening, was applied at first. Successively, the spatial continuity

Table 8
Estimated probabilities of occasional opening ($\hat{\pi}_{ijk}^{(3)}$), of seasonal opening ($\hat{\pi}_{ijk}^{(2)}$) or year-round opening ($\hat{\pi}_{ijk}^{(1)}$) for multilevel ordered logit model, classified by regions and provinces (public museums).

Macro-area	Region/Province	$\hat{\pi}_{ijk}^{(3)}$	$\hat{\pi}_{ijk}^{(2)}$	$\hat{\pi}_{ijk}^{(1)}$
Northern Italy	**Liguria**	**0.048**	**0.093**	**0.859**
	Genova	0.046	0.077	0.877
	Imperia	0.054	0.116	0.830
	La Spezia	0.052	0.087	0.861
	Savona	0.038	0.092	0.870
	Lombardy	**0.021**	**0.050**	**0.929**
	Bergamo	0.020	0.047	0.933
	Brescia	0.012	0.056	0.932
	Como	0.011	0.055	0.934
	Cremona	0.016	0.034	0.950
	Lecco	0.009	0.044	0.947
	Lodi	0.030	0.061	0.909
	Mantova	0.009	0.042	0.949
	Milan	0.014	0.031	0.955
	Monza-Brianza	0.052	0.064	0.884
	Pavia	0.028	0.054	0.918
	Sondrio	0.028	0.048	0.924
	Varese	0.024	0.061	0.915
	Piedmont	**0.043**	**0.074**	**0.883**
	Alessandria	0.037	0.067	0.896
	Asti	0.059	0.084	0.857
	Biella	0.040	0.066	0.894
	Cuneo	0.032	0.070	0.898
	Novara	0.036	0.062	0.902
	Torino	0.027	0.052	0.921
	Verbano-Cusio-Ossola	0.045	0.089	0.866
	Vercelli	0.070	0.099	0.831
	Emilia-Romagna	**0.016**	**0.045**	**0.939**
	Bologna	0.010	0.038	0.952
	Ferrara	0.008	0.034	0.958
	Forlì-Cesena	0.019	0.060	0.921
	Modena	0.012	0.037	0.951
	Parma	0.020	0.051	0.929
	Piacenza	0.035	0.051	0.914
	Ravenna	0.009	0.034	0.957
	Reggio nell'Emilia	0.021	0.051	0.928
	Rimini	0.013	0.046	0.941
	Friuli-Venezia Giulia	**0.034**	**0.086**	**0.880**
	Gorizia	0.029	0.092	0.879
	Pordenone	0.053	0.099	0.848
	Trieste	0.023	0.067	0.910
	Udine	0.033	0.084	0.883
	Aosta Valley	**0.069**	**0.129**	**0.802**
	Trentino-Alto Adige	**0.015**	**0.099**	**0.887**
	Bolzano	0.015	0.122	0.863
	Trento	0.014	0.076	0.910
	Veneto	**0.023**	**0.053**	**0.924**
	Belluno	0.030	0.068	0.902
	Padova	0.026	0.056	0.918
	Rovigo	0.032	0.054	0.914
	Treviso	0.018	0.048	0.934
	Venezia	0.008	0.037	0.955
	Verona	0.018	0.059	0.923
	Vicenza	0.026	0.054	0.920
Central Italy	**Lazio**	**0.018**	**0.056**	**0.926**
	Frosinone	0.032	0.080	0.888
	Latina	0.013	0.055	0.932
	Rieti	0.022	0.071	0.907
	Roma	0.012	0.036	0.952
	Viterbo	0.010	0.037	0.953
	Marches	**0.049**	**0.129**	**0.822**
	Ancona	0.038	0.117	0.845
	Ascoli Piceno	0.063	0.134	0.803
	Fermo	0.043	0.131	0.826
	Macerata	0.050	0.133	0.817
	Pesaro-Urbino	0.051	0.129	0.820
	Tuscany	**0.018**	**0.049**	**0.933**
	Arezzo	0.019	0.048	0.933
	Firenze	0.010	0.046	0.944
	Grosseto	0.014	0.052	0.934
	Livorno	0.021	0.065	0.914
	Lucca	0.016	0.045	0.939
	Massa-Carrara	0.019	0.052	0.929
	Pisa	0.011	0.044	0.945
	Pistoia	0.025	0.049	0.926
	Prato	0.028	0.047	0.925
	Siena	0.013	0.049	0.938
	Umbria	**0.030**	**0.121**	**0.849**
	Perugia	0.029	0.114	0.857
	Terni	0.031	0.128	0.841
Southern Italy	**Abruzzo**	**0.066**	**0.143**	**0.791**
	Chieti	0.053	0.131	0.816
	L'Aquila	0.071	0.151	0.778
	Pescara	0.091	0.158	0.751
	Teramo	0.049	0.134	0.817
	Basilicata	**0.027**	**0.125**	**0.848**
	Matera	0.012	0.131	0.857
	Potenza	0.041	0.121	0.838
	Molise	**0.043**	**0.157**	**0.800**
	Campobasso	0.063	0.147	0.790
	Isernia	0.024	0.167	0.809
	Calabria	**0.067**	**0.118**	**0.815**
	Catanzaro	0.100	0.137	0.763
	Cosenza	0.067	0.113	0.820
	Crotone	0.077	0.093	0.830
	Reggio Calabria	0.040	0.140	0.820
	Vibo Valentia	0.051	0.109	0.840
	Campania	**0.058**	**0.118**	**0.824**
	Avellino	0.080	0.141	0.779
	Benevento	0.096	0.143	0.761
	Caserta	0.040	0.097	0.863
	Napoli	0.024	0.083	0.893
	Salerno	0.047	0.128	0.825
	Apulia	**0.033**	**0.099**	**0.868**
	Bari	0.036	0.115	0.849
	Barletta-Andria-Trani	0.044	0.113	0.843
	Brindisi	0.020	0.073	0.907
	Foggia	0.045	0.104	0.851
	Lecce	0.022	0.076	0.902
	Taranto	0.027	0.116	0.857
Islands	**Sardinia**	**0.033**	**0.116**	**0.851**
	Cagliari	0.034	0.099	0.867
	Nuoro	0.033	0.133	0.834
	Oristano	0.050	0.115	0.835
	Sassari	0.023	0.113	0.864
	Sud Sardinia	0.024	0.120	0.856
	Sicily	**0.022**	**0.091**	**0.887**
	Agrigento	0.032	0.103	0.865
	Caltanissetta	0.032	0.087	0.881
	Catania	0.018	0.077	0.905
	Enna	0.014	0.100	0.886
	Messina	0.025	0.083	0.892
	Palermo	0.026	0.072	0.902
	Ragusa	0.013	0.132	0.855
	Siracusa	0.014	0.082	0.904
	Trapani	0.022	0.082	0.896

Bold indicate the mean value of the probability for each region.

of the propensity, measured by the logit, of remaining open all over the year or at least seasonally was evaluated and modeled through the variogram; thus, a multilevel logit kriging, was used for interpolating the data over a regular grid and visualizing the effect of the corresponding range. As previously underlined, the integration of both approaches provides the possibility, never explored before to the best of our knowledge, to include the analysis of the spatial component of the logit data.

Table 9
Estimate probabilities of occasional opening ($\hat{\pi}_{ijk}^{(3)}$), of seasonal opening ($\hat{\pi}_{ijk}^{(2)}$) or year-round opening ($\hat{\pi}_{ijk}^{(1)}$) for multilevel ordered logit model, classified by regions and provinces (private museums).

Macro-area	Region/Province	$\hat{\pi}_{ijk}^{(3)}$	$\hat{\pi}_{ijk}^{(2)}$	$\hat{\pi}_{ijk}^{(1)}$
Northern Italy	**Liguria**	**0.036**	**0.105**	**0.859**
	Genova	0.039	0.097	0.864
	Imperia	0.041	0.107	0.852
	La Spezia	0.032	0.107	0.861
	Savona	0.033	0.110	0.857
	Lombardy	**0.018**	**0.071**	**0.911**
	Bergamo	0.018	0.072	0.910
	Brescia	0.017	0.068	0.915
	Como	0.006	0.059	0.935
	Cremona	0.010	0.064	0.926
	Lecco	0.026	0.065	0.909
	Lodi	0.028	0.076	0.896
	Mantova	0.008	0.067	0.925
	Milan	0.006	0.055	0.939
	Monza-Brianza	0.021	0.075	0.904
	Pavia	0.032	0.094	0.874
	Sondrio	0.020	0.082	0.898
	Varese	0.020	0.074	0.906
	Piedmont	**0.019**	**0.077**	**0.904**
	Alessandria	0.024	0.081	0.895
	Asti	0.018	0.072	0.910
	Biella	0.014	0.077	0.909
	Cuneo	0.013	0.091	0.896
	Novara	0.009	0.064	0.927
	Torino	0.022	0.071	0.907
	Verbano-Cusio-Ossola	0.026	0.088	0.886
	Vercelli	0.025	0.071	0.904
	Emilia-Romagna	**0.019**	**0.074**	**0.907**
	Bologna	0.016	0.068	0.916
	Ferrara	0.015	0.058	0.927
	Forlì-Cesena	0.031	0.082	0.887
	Modena	0.014	0.071	0.915
	Parma	0.012	0.073	0.915
	Piacenza	0.017	0.081	0.902
	Ravenna	0.024	0.076	0.900
	Reggio nell'Emilia	0.025	0.076	0.899
	Rimini	0.016	0.082	0.902
	Friuli-Venezia Giulia	**0.020**	**0.083**	**0.898**
	Gorizia	0.007	0.086	0.907
	Pordenone	0.015	0.070	0.915
	Trieste	0.026	0.083	0.891
	Udine	0.031	0.092	0.877
	Aosta Valley	**0.053**	**0.125**	**0.822**
	Trentino-Alto Adige	**0.024**	**0.115**	**0.861**
	Bolzano	0.013	0.136	0.851
	Trento	0.034	0.094	0.872
	Veneto	**0.019**	**0.073**	**0.908**
	Belluno	0.017	0.076	0.907
	Padova	0.012	0.070	0.918
	Rovigo	0.031	0.067	0.902
	Treviso	0.010	0.062	0.928
	Venezia	0.013	0.072	0.915
	Verona	0.032	0.082	0.886
	Vicenza	0.021	0.082	0.897
Central Italy	**Lazio**	**0.020**	**0.077**	**0.903**
	Frosinone	0.027	0.082	0.891
	Latina	0.013	0.067	0.920
	Rieti	0.029	0.095	0.876
	Roma	0.011	0.070	0.919
	Viterbo	0.020	0.069	0.911
	Marches	**0.049**	**0.140**	**0.811**
	Ancona	0.031	0.141	0.828
	Ascoli Piceno	0.030	0.107	0.863
	Fermo	0.109	0.195	0.696
	Macerata	0.046	0.142	0.812
	Pesaro-Urbino	0.029	0.115	0.856
	Tuscany	**0.018**	**0.081**	**0.901**
	Arezzo	0.022	0.078	0.900
	Firenze	0.016	0.081	0.903
	Grosseto	0.021	0.097	0.882
	Livorno	0.034	0.107	0.859
	Lucca	0.025	0.083	0.892
	Massa-Carrara	0.006	0.077	0.917
	Pisa	0.018	0.076	0.906
	Pistoia	0.017	0.065	0.918
	Prato	0.003	0.048	0.949
	Siena	0.019	0.095	0.886
	Umbria	**0.014**	**0.120**	**0.866**
	Perugia	0.017	0.124	0.859
	Terni	0.011	0.116	0.873
Southern Italy	**Abruzzo**	**0.035**	**0.131**	**0.834**
	Chieti	0.005	0.087	0.908
	L'Aquila	0.064	0.144	0.792
	Pescara	0.013	0.149	0.838
	Teramo	0.059	0.144	0.797
	Basilicata	**0.028**	**0.145**	**0.827**
	Matera	0.005	0.135	0.860
	Potenza	0.051	0.155	0.794
	Molise	**0.058**	**0.169**	**0.773**
	Campobasso	0.080	0.165	0.755
	Isernia	0.036	0.173	0.791
	Calabria	**0.039**	**0.128**	**0.833**
	Catanzaro	0.036	0.130	0.834
	Cosenza	0.052	0.132	0.816
	Crotone	0.068	0.147	0.785
	Reggio Calabria	0.014	0.109	0.877
	Vibo Valentia	0.027	0.121	0.852
	Campania	**0.031**	**0.119**	**0.850**
	Avellino	0.033	0.131	0.836
	Benevento	0.030	0.103	0.867
	Caserta	0.049	0.131	0.820
	Napoli	0.009	0.096	0.895
	Salerno	0.034	0.136	0.830
	Apulia	**0.033**	**0.133**	**0.834**
	Bari	0.032	0.116	0.852
	Barletta-Andria-Trani	0.011	0.117	0.872
	Brindisi	0.013	0.120	0.867
	Foggia	0.078	0.167	0.755
	Lecce	0.032	0.145	0.823
Islands	**Sardinia**	**0.021**	**0.129**	**0.850**
	Cagliari	0.003	0.093	0.904
	Nuoro	0.008	0.116	0.876
	Oristano	0.017	0.130	0.853
	Sassari	0.037	0.181	0.782
	Sud Sardinia	0.039	0.128	0.833
	Sicily	**0.028**	**0.122**	**0.850**
	Agrigento	0.002	0.105	0.893
	Caltanissetta	0.009	0.073	0.918
	Catania	0.020	0.097	0.883
	Enna	0.055	0.148	0.797
	Messina	0.056	0.141	0.803
	Palermo	0.018	0.103	0.879
	Ragusa	0.047	0.147	0.806
	Siracusa	0.021	0.116	0.863
	Trapani	0.024	0.166	0.810

Bold indicate the mean value of the probability for each region.

For what concerns the multilevel ordered logit modeling findings, from the inspection of the odd-ratios it was highlighted that the prominent factors which might stimulate the opening choices of the Italian museums depend mainly on the research activities, the exhibition space, the degree of digitalization as well as the presence of relationship over the territory. Then, the predicted probabilities of respecting the decision to stay open occasionally, seasonally or year-round, were evaluated, with respect to the regions and provinces where the museums are placed.

By considering the modeling outcomes concerning all the museums, it was pointed out that, at the regional level, it is much more likely that Italian museums are willing to stay open all year (with predicted values ranging from 0.801 to 0.910) than seasonally (with estimated values ranging from 0.069 to 0.149) or occasionally (with estimated values ranging from 0.015 to 0.059). The Italian regions with the highest estimated probability to stay open "continuously", correspond to the ones with more than three hundred structures, i.e. Emilia-Romagna, Tuscany and Veneto followed by Lombardy, Lazio, Trentino Alto-Adige and Friuli Venezia-Giulia. The motivation could be found by considering that these regions represent the cradle of the most important museums in the world, which is a key driver to encourage the small museums located in their neighborhood to apply the same opening policy. Molise and Abruzzo were the regions with the biggest cumulative estimated probability of respecting the decision to stay open seasonally or occasionally. These empirical evidences were confirmed also at the provincial level.

Moreover, the structural analysis was carried out to assess the spatial similarity of the partial proportional logit odds for the options "continuous museums opening" and "seasonal or year-round museums opening", over the domain under study. According to the spatial modeling results, kriging interpolation was computed without taking into consideration additional information at the predictive points of the fixed grid and the existence of huge areas characterized by strong spatial similarity were identified. More specifically, the entire Central-Eastern, as well as Southern and Insular regions, showed logit values tendentially below the median value, differently from the rest of Italy. This feature was toned down for the logits of staying open at least seasonally, for which wider areas of spatial dissimilarity, with respect to the ones concerning logits of staying open all the year-round, were detected. Thus, from the interpolation results it was highlighted how extensive the areas characterized by similar opening decisions were.

Finally, the variography analysis was also executed by using the data classified with respect to the type of institutions. Hence, the comparison with the public versus private museums highlighted the greater spatial continuity of the private museums than the public ones, especially in the case of the probability to stay open all year-round. This is justified by considering that the public sector is often linked to political decisions in the region, moreover, the spatial correlation declines faster than the spatial correlation related to the private sector, where the geographical component might have greater impact on the opening decisions. Also in this case, it held the assumption according to which there is an evident spatial relationship in the probability of museums opening choices.

A final analysis was performed to obtain the predicted probabilities by type of institutions and with respect to the province and region levels. By looking at the regional level, it is much more likely that: (a) public museums are more ready to stay open all year (with predicted values ranging from 0.791 to 0.939) than seasonally (with estimated values ranging from 0.045 to 0.157) or occasionally (with estimated values ranging from 0.014 to 0.067); (b) private museums prefer to stay open year-round (with predicted values ranging from 0.773 to 0.911) than seasonally (with estimated values ranging from 0.071 to 0.169) or occasionally (with estimated values ranging from 0.014 to 0.058).

Moreover, the Italian regions with the greatest estimated probabilities of continuous opening both for public and private museums were Lombardy, Lazio, Veneto, Emilia-Romagna and Tuscany. Similar results were also observed at the provincial level. More specifically, the highest predicted probabilities of year-round opening for private museums are found in the provinces of Prato (in the Tuscany region), Milan and Como (in the Lombardy region); on the other hand, for public museums, the largest estimated probabilities of year-round opening is observed for the provinces of Ferrara, Ravenna, Bologna and Modena (in the Emilia-Romagna region), followed by Milan (in the Lombardy region) and Venice (in the Veneto region).

In perspective, a hybridization between multilevel and machine learning methods will be proposed in order to evaluate the effects of the permanent presence of museums and their spatial contiguity. In addition, an extension to the spatio-temporal context might be also implemented [24,52], taking into account of the possible effects on the social and economic growth associated to the enhancement of cultural and natural heritage. Furthermore, it might also be stimulating to propose a cross-national comparative study among the Italian museum system and other European museums, in order to evaluate the role of the permanent museum with respect to the geographical area in which they are located. Indeed, as underlined by [53] the cultural policies should strengthen the connection between a cultural cluster and its local context, to ensure an equal distribution of tourists and increase the attractiveness of a particular geographic area to development the territory in terms of authenticity, uniqueness and distinctiveness.

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CRediT authorship contribution statement

Claudia Cappello: Writing – review & editing, Writing – original draft, Methodology, Investigation, Formal analysis, Data curation. **Sandra De Iaco:** Writing – review & editing, Writing – original draft, Supervision, Funding acquisition, Conceptualization. **Sabrina Maggio:** Writing – review & editing, Writing – original draft, Validation, Methodology, Investigation, Formal analysis, Data curation, Conceptualization.

Data availability

The data are open and available at https://www.istat.it/en/archivio/167568. The download requires the free registration on the ISTAT website.

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