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# On the Construction of Multivariate Drought Indices: Theoretical Foundations and Practical Implications

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**Key Points:**

- Standardization of drought indices exploits the Probability Integral Transform (PIT)
- In a multivariate framework the Kendall distribution plays a fundamental role
- The findings are particularly significant since they draw attention to the risk of misinterpretation arising from probabilistic aspects that may be overlooked

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**Abstract** Drought indices represent essential tools for monitoring and evaluating drought conditions and evolution. Univariate indices employ the Probability Integral Transform to map data onto the standard Gaussian domain. Extending such a Gaussian normalization procedure to multivariate settings requires the usage of the Kendall distribution function for preserving the Normality of the indices when aggregating the variables via their joint CDF. This work aims to clarify the issue and underscore the importance of applying proper standardization in multivariate frameworks. We show that neglecting this aspect can lead to biased estimation of drought occurrences, as illustrated through both theoretical and empirical analyses.

## 1. Introduction

Droughts are among the most economically damaging hazards (Cuevas et al., 2024). Drought indices are essential tools developed to characterize and quantify drought conditions (onset, severity, and termination) and to monitor their spatio-temporal evolution (WMO, 2016). For this reason, numerous drought indicators and indices have been developed to quantify deficits across the hydrologic cycle based on precipitation (McKee et al., 1993), runoff (Shukla & Wood, 2008), snow (Huning & AghaKouchak, 2020), groundwater storage (Mishra & Singh, 2010), soil moisture (Narasimhan & Srinivasan, 2005), vegetation condition (Kogan, 1995), evaporation (Hobbins et al., 2016) and surface water supply indicators (Cheng et al., 2025; Kim et al., 2019). These indicators underpin operational drought monitoring, forecasting, and impact assessment from local to global scales (Heim, 2002; Mishra & Singh, 2010; Mo & Lettenmaier, 2014; Sheffield et al., 2004; Shukla & Wood, 2008; Svoboda et al., 2002).

Traditionally, the value $S_x$ of an univariate standardized drought index is computed as follows:

$$S_x = \Phi^{-1}(F_X(x)), \tag{1}$$

where $\Phi^{-1}$ denotes the inverse of the standard Normal CDF, $F_X$ is the CDF of the variable $X$ of interest (e.g., precipitation), and $x$ is the observed value of $X$. $S_x$ is computed either parametrically (McKee et al., 1993) by fitting a theoretical distribution $F_X$ to $X$ (typically, but not necessarily, a Gamma law), or non-parametrically using the empirical CDF of $X$ (Farahmand & AghaKouchak, 2015; Sołáková et al., 2014).

According to convention (McKee et al., 1993), moderate to extreme drought conditions are identified when $S_x \leq -1$ (corresponding to a probability of approximately 15.9%), while wet conditions are defined by $S_x \geq 1$ (also approximately 15.9%), and normal conditions occur when $-1 < S_x < 1$ (about 68.2%).

The construction of a generic univariate standardized drought index $S$ (i.e., a random variable) relies on the *Probability Integral Transform* (PIT, also known as the "universality of the Uniform law"), which ensures that, irrespective of the original distribution $F_X$ of $X$, the transformed random variable

$$U = F_X(X) \tag{2}$$

follows a Uniform distribution in the interval [0,1], provided that $F_X$ is continuous (Angus, 1994)—as is typical in hydrometeorological applications. Hence, the random variable

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$$S = \Phi^{-1}(U) = \Phi^{-1}(F_X(X)) \tag{3}$$

has a standard Gaussian distribution. Since the law of a standardized index $S$ is Normal, the value taken by $S$ provides information about the statistical behavior of the associated variable $X$. In fact, $S \leq 0$ with probability 50% and, consequently, negative (respectively, positive) values of $S$ correspond to observed values $x$'s of $X$ that are smaller (respectively, greater) than the median of the distribution of $X$, while a value $S = 0$ implies that $x$ coincides with the median of $X$—though not necessarily with the mean of the population of $X$. Conceptually, $S$ "standardizes" $X$ by mapping it onto the standard Normal law.

In the Literature, numerous univariate drought indices adopt such a standardized framework. The primary advantage of the Normal standardization lies in enabling consistent comparisons across different geographical regions, indices and hydrometeorological variables. Among the most widely used univariate indices (WMO, 2016), we highlight the Standardized Precipitation Index SPI (McKee et al., 1993), the Standardized Soil Moisture Index SSMI (Carrao et al., 2013), the Standardized Streamflow Index SSI (Tijdeman et al., 2020), the Standardized Precipitation Evapotranspiration Index SPEI (Beguería et al., 2014), the Standardized Runoff Index SRI (Shukla & Wood, 2008), the Standardized Water-level Index SWI (Bhuiyan et al., 2006), and the Standardized Reservoir Supply Index SRSI (Gusyev et al., 2015).

Univariate indices typically reflect a specific dimension of drought and come with inherent strengths and limitations. For example, Lloyd-Hughes and Saunders (2002) raise concerns about the standardization procedure adopted by the SPI, a critique that extends to similar indices. In fact, by the very construction of the index itself, extreme droughts (or any other drought threshold) will occur with the same probability at all locations. Thus, the index may not be capable of identifying regions that may be more drought-prone than others. Moreover, because the index is defined relative to the median of the calibration sample, its interpretation can also be affected by temporal biases: if the reference data set is skewed toward dry or wet conditions, the resulting values may under- or over-estimate drought severity, with the risk of distorting both spatial and temporal comparisons.

## 2. The Multivariate Case

To better capture the multifaceted nature of droughts, composite and multivariate drought indices have been developed over the past two decades (Hao & AghaKouchak, 2013; Hao & Singh, 2015; Kao & Govindaraju, 2010). Droughts are complex phenomena that can impact multiple components of the hydrological cycle and cause a wide range of effects across various sectors, including agriculture, energy production, navigation, and broader socio-economic systems. A key justifications for multivariate indices is that integrating multiple variables can reveal compound hazardous conditions that may be missed by univariate indicators, thereby improving detection and characterization of events that would otherwise remain undetected (Svoboda et al., 2002).

As pointed out in Salvadori et al. (2013) and later underscored in Hao et al. (2018), the main difficulty of working in a multidimensional framework is the lack of a total order, which in turn may affect the construction of multivariate drought indices (e.g., yielding a standardized multivariate indicator whose distribution is skewed such that negative values occur more frequently than positive ones). In fact, a "natural" threshold that can be used to define extremes in a multivariate setting does not exist. A possible general way to summarize the information provided by different sources (drought indices and/or physical variables) could be as follows:

1. Aggregate the variables of interest (say, $X_1, \ldots, X_d$, with $d \geq 2$) into one single univariate random variable $A$ via a transformation of type $A = \Psi(X_1, \ldots, X_d)$ for a suitable function $\Psi$ (e.g., a linear combination, maximum, minimum, joint CDF of the $X_i$'s, etc.).
2. Transform the aggregated random variable $A$ in such a way that the resulting output has a standard Gaussian distribution as in the univariate case.

Concerning the first step, a natural far-reaching choice is to use as an aggregation function $\Psi$ the joint distribution $\mathbf{F}$ of the $X_i$'s, which fully encompasses all the statistical and probabilistic properties of the variables under consideration, viz.

$$\mathbf{F}(x_1, \ldots, x_d) = \mathbf{P}(X_1 \leq x_1, \ldots, X_d \leq x_d) \tag{4}$$

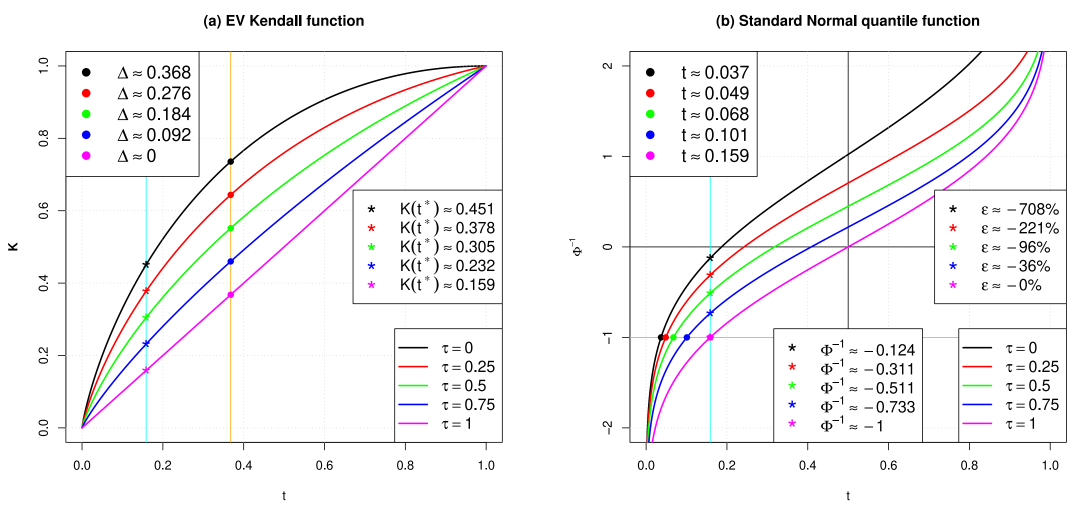


**Figure 1.** (a) Comparison of the Kendall function $\mathbf{K_C}$ for Extreme Value copulas parametrized by different Kendall rank correlation coefficients $\tau$'s. (b) Comparison of the Standard Normal quantile function $\Phi^{-1}$ evaluated in the Kendall functions $\mathbf{K_C}$'s plotted in the left panel, yielding the correct standardized drought index $S$ (see Equation 11). See text for explanation.

for all $(x_1, \dots, x_d) \in \mathbf{R}^d$, with marginal distributions $F_i$'s, and $\Psi = \mathbf{F}$. However, in a multivariate setting the PIT (see Equations 2 and 3) does *not* exhibit the same behavior as in the univariate case. Therefore, appropriate techniques must be employed to achieve the intended Gaussian representation in a multivariate framework, as shown below.

Let $\mathbf{X} = (X_1, \dots, X_d)$ be a $d$-dimensional vector ($d \geq 2$) of physical variables and/or drought indices, and let $\mathbf{x} = (x_1, \dots, x_d)$ be an occurrence of $\mathbf{X}$. According to Sklar's Theorem representation (Sklar, 1959), the corresponding joint CDF $\mathbf{F_X}$ can be written as

$$\mathbf{F_X}(x_1, \dots, x_d) = \mathbf{C}(F_1(x_1), \dots, F_d(x_d)), \tag{5}$$

where $\mathbf{C}$ is the unique Copula (i.e., the dependence structure) associated with $\mathbf{X}$, assuming that the marginals are continuous, as is usually the case in hydrometeorological applications.

Now, in a multivariate setting (i.e., $d \geq 2$), the PIT does *not* behave as in the univariate case. In fact, the transformed univariate random variable $T = \mathbf{F_X}(\mathbf{X}) \in [0, 1]$ is *not* necessarily Uniformly distributed in [0,1]. As shown in Genest and Rivest (2001); Nelsen et al. (2003), the CDF of $T$ is given by, for $t \in [0, 1]$,

$$\mathbf{K_C}(t) = \mathbf{P}(T \leq t) = \mathbf{P}(\mathbf{F_X}(\mathbf{X}) \leq t) = \mathbf{P}(\mathbf{C}(F_1(X_1), \dots, F_d(X_d)) \leq t), \tag{6}$$

and is known as the *Kendall distribution function*—see Figure 1a below for a graphical illustration. This law depends solely on the copula $\mathbf{C}$ associated with the multivariate vector $\mathbf{X}$, and

$$\mathbf{K_C}(t) \geq t \tag{7}$$

for all $t \in [0, 1]$—i.e., $T$ is *not* Uniform in [0,1]. For some popular copula families, like Archimedean copulas, the corresponding $\mathbf{K_C}$ admits an explicit analytical expression (see, e.g., Equations 15 and 17 later). Otherwise, it can be estimated non-parametrically by using its empirical version—see, for example, Barbe et al. (1996) and Salvadori et al. (2011, Algorithm 1). A note of warning is raised in Erhardt and Czado (2017) when the dimension $d$ of the problem is very large: "As in such high dimensions the Kendall distribution becomes almost degenerate at 0 (see e.g. Brechmann (2014))". In fact, when $d$ goes to $\infty$, the Kendall distribution function tends to the Heaviside step function: however, such problems do not occur when small/moderate dimensions are used (as in the present case).

Here the crucial point is that the random variable $T = \mathbf{F}_{\mathbf{X}}(\mathbf{X})$ is Uniformly distributed in [0,1] (i.e., $\mathbf{K}_{\mathbf{C}}(t) = t$—see the *magenta* line in Figure 1a) *if, and only if*, the copula $\mathbf{C}$ associated with $\mathbf{F}_{\mathbf{X}}$ is the co-monotonic one (Nelsen, 2006; Salvadori et al., 2007), that is

$$\mathbf{C}(u_1, \ldots, u_d) = \mathbf{M}_d(u_1, \ldots, u_d) = \min(u_1, \ldots, u_d), \tag{8}$$

which corresponds to the case where the components $X_i$'s are perfectly co-monotonic. In turn,

$$\Phi^{-1}(T) = \Phi^{-1}(\mathbf{F}_{\mathbf{X}}(\mathbf{X})) \tag{9}$$

cannot be standard Normally distributed in general (since $T$ is generally *not* Uniform), contrary to what occurs in the univariate case (see Equations 2 and 3): for a graphical illustration, compare the *magenta* line in Figure 1b (corresponding to the inverse standard Gaussian distribution) with the other curves. In simple words, $\Phi^{-1}(\mathbf{F}_{\mathbf{X}}(\mathbf{X}))$ is *not* law-invariant, rather it depends upon the distribution $\mathbf{F}_{\mathbf{X}}$ of $\mathbf{X}$, and hence it *cannot* always be Gaussian—unless in the very special case when $\mathbf{F}_{\mathbf{X}}$ is Uniform in [0,1] and $\mathbf{C} = \mathbf{M}_d$.

The procedure for performing a standard Normal index transformation in a multivariate context when the aggregation function $\Psi$ is given by the joint distribution $\mathbf{F} = \mathbf{C}(F_1, \ldots, F_d)$ involves the use of the Kendall function $\mathbf{K}_{\mathbf{C}}$ associated with $\mathbf{C}$, as shown below. Given an observed realization $\mathbf{x}$ of the vector $\mathbf{X}$, let $t = \mathbf{F}_{\mathbf{X}}(\mathbf{x}) \in [0,1]$. Note that (Nelsen, 2006; Salvadori et al., 2007):

1. $t$ is the constant value taken by $\mathbf{F}_{\mathbf{X}}$ on the iso-hypersurface crossing $\mathbf{x}$ (e.g., an isoline in the bivariate case);
2. $\mathbf{K}_{\mathbf{C}}(t)$ is the probability measure induced by the copula $\mathbf{C}$ (and hence by $\mathbf{F}$) on the region $\mathcal{R}_t^< \subset \mathbf{R}^d$ where $\mathbf{F}_{\mathbf{X}} \le t$ (i.e., "below" the iso-hypersurface of level $t$ crossing $\mathbf{x}$).
3. Since $T = \mathbf{F}_{\mathbf{X}}(\mathbf{X})$ has distribution $\mathbf{K}_{\mathbf{C}}$, it *is* the random variable

$$U = \mathbf{K}_{\mathbf{C}}(T) = \mathbf{K}_{\mathbf{C}}\,(\mathbf{F}_{\mathbf{X}}(\mathbf{X})) \tag{10}$$

that has a Uniform distribution in [0,1], *not* $T = \mathbf{F}_{\mathbf{X}}(\mathbf{X})$, by virtue of the univariate PIT—see Equation 2.

4. Since $\mathbf{F}_{\mathbf{X}}(\mathbf{x})$ corresponds to the probability measure of the hyper-rectangle $[\mathbf{0}, \mathbf{x}]$, which is included in $\mathcal{R}_t^<$, then $\mathbf{K}_{\mathbf{C}}(t) > t = \mathbf{F}_{\mathbf{X}}(\mathbf{x})$. In fact, $\mathbf{K}_{\mathbf{C}}(t) > t$ for all $t \in (0,1)$, independently from (a) the copula $\mathbf{C}$ at play (or, equivalently, the corresponding multivariate distribution $\mathbf{F}_{\mathbf{X}}$) and (b) the observation $\mathbf{x}$ on the iso-hypersurface of level $t$, with equality only if $\mathbf{C} = \mathbf{M}_d$—for a graphical illustration see the functions $\mathbf{K}_{\mathbf{C}}$'s plotted in Figure 1a.

Thus, given any occurrence $\mathbf{x}$, a multivariate Gaussian standardized index $S_{\mathbf{x}}$ (when the aggregation function $\Psi$ equals $\mathbf{F}_{\mathbf{X}}$) should be computed as

$$S_{\mathbf{x}} = \Phi^{-1}(\mathbf{K}_{\mathbf{C}}(t)) = \Phi^{-1}(\mathbf{K}_{\mathbf{C}}\,(\mathbf{F}_{\mathbf{X}}(\mathbf{x}))). \tag{11}$$

Here, the fundamental point is that Equation 11 is the proper one to be used for calculating a standardized drought index $S$ in a multivariate framework when the random variables $X_i$'s of interest are $\Psi$-aggregated via their joint CDF $\mathbf{F}_{\mathbf{X}}$. Note that, due to the monotonicity of $\Phi$ and Equation 7, it holds

$$S_{\mathbf{x}} = \Phi^{-1}(\mathbf{K}_{\mathbf{C}}(t)) = \Phi^{-1}(\mathbf{K}_{\mathbf{C}}\,(\mathbf{F}_{\mathbf{X}}(\mathbf{x}))) > \Phi^{-1}(\mathbf{F}_{\mathbf{X}}(\mathbf{x})) = S_{\mathbf{x}}^{\dagger}, \tag{12}$$

where $S_{\mathbf{x}}^{\dagger}$ represents the estimate of the drought index of interest often present in the Literature (incorrectly treated as Normally distributed). Using $S_{\mathbf{x}}^{\dagger}$ can be a computational shortcut for deriving a multivariate drought index; nevertheless, if the aim is to obtain an index with a Gaussian distribution, this approach is not appropriate. As a practical consequence, $S_{\mathbf{x}}^{\dagger}$ underestimates the factual (Normally standardized) drought index $S_{\mathbf{x}}$, and hence false drought warnings and alarms may be raised, as also noticed in Erhardt and Czado (2017): "This approach does not yield a real standardization. Usually, negative values of the index proposed are more probable…". Furthermore, being standard Gaussian, $S_{\mathbf{x}} \le 0$ (respectively, $S_{\mathbf{x}} \ge 0$) with probability 1/2. In turn, the iso-hypersurface of level $t = \mathbf{F}_{\mathbf{X}}(\mathbf{x})$ crossing $\mathbf{x}$ and such that $\mathbf{K}_{\mathbf{C}}(t) = 1/2$ plays the role of "median" in a multivariate framework.

These issues have been acknowledged in the Literature since the introduction of the Kendall function in hydrology by Salvadori and De Michele (2004). The first application of the multivariate Normal standardization procedure dates back to Kao and Govindaraju (2010, Equation 7), although it did not explicitly reference to the PIT machinery, and was later emphasized by Bateni et al. (2018). Hao et al. (2016, 2020) also pointed out that using $\mathbf{F_X}(\mathbf{X})$ as the argument of $\Phi^{-1}$ leads to a multivariate drought index that is not Normally distributed. Despite these observations, the issue has often been overlooked—whether intentionally or not. Of course, any type of multivariate index can be defined; however, if the goal is to obtain a standardized indicator with a Normal distribution, then the appropriate procedure is the one outlined above exploiting the Kendall function $\mathbf{K_C}$ in case the aggregation function is $\Psi = \mathbf{F_X}$. Note that, over the past decades, several are the studies where the role played by $\mathbf{K_C}$ has been ignored.

However, it is rather easy to normally standardize a non-Gaussian multivariate drought index $S^{\dagger}_{\mathbf{x}}$ into $S_{\mathbf{x}}$ simply by making a proper use of the PIT. In fact,

$$S_{\mathbf{x}} = \Phi^{-1}\left(\mathbf{K_C}\left(\Phi\left(\Phi^{-1}\left(\mathbf{F_X}(\mathbf{x})\right)\right)\right)\right) = \Phi^{-1}\left(\mathbf{K_C}\left(\Phi\left(S^{\dagger}_{\mathbf{x}}\right)\right)\right) \tag{13}$$

turns out to have the desired Normal distribution.

From Equation 13, the main steps required to compute the two indices become apparent. The index $S^{\dagger}_{\mathbf{x}}$ requires the estimation of all the univariate marginals $F_i$'s and the copula $\mathbf{C}$ of $\mathbf{X}$. By contrast, the index $S_{\mathbf{x}}$ requires the computation of the Kendall distribution function $\mathbf{K_C}$, which only depends on the copula $\mathbf{C}$. Interestingly enough, in the Archimedean case, the evaluation of the Kendall distribution may itself serve as a preliminary step for identifying the associated copula, as presented by Genest et al. (2011).

Further considerations can be made, using as an example the Multivariate Standardized Drought Index (MSDI) introduced by Hao and AghaKouchak (2013), Equation 5:

$$\mathrm{MSDI}_{x,y} = \Phi^{-1}(\mathbf{P}(X \le x, Y \le y)) = \Phi^{-1}(\mathbf{C}(F_X(x), F_Y(y))),$$

where $X$ and $Y$ represent, respectively, the precipitation and soil moisture "for overall meteorological and agricultural drought characterization"—the corresponding univariate (Normally standardized) drought indices are, respectively, SPI for $X$ and SSMI for $Y$. Now, by its very construction, it turns out that

$$\mathrm{MSDI}_{x,y} < \min\left(\mathrm{SPI}(x) = \Phi^{-1}(F_X(x)), \mathrm{SSMI}(y) = \Phi^{-1}(F_Y(y))\right) \tag{14}$$

due to the fact that $\Phi^{-1}$ is strictly increasing and that the value attained by the copula $\mathbf{M}_2\,(F_X(x), F_Y(y))$, see Equation 8, represents the Fréchet-Hoeffding upper bound for all bivariate copulas and all $(x, y) \in \mathbf{R}^2$ (Durante & Sempi, 2015; Nelsen, 2006; Salvadori et al., 2007). As a consequence, the drought severity expressed by the MSDI (practically, the probability of values less than $-1$) is always larger than the one indicated by any of its marginal indices SPI and SSMI. As already shown above, the MSDI is not a traditional Normally standardized index, and cannot be directly compared with other Normally standardized indices. However, the inequality in Equation 14 suggests that the severity of a (multivariate) drought, as monitored via the MSDI, could be larger than the one indicated by the corresponding univariate indices. This behavior aligns with a prudential early warning philosophy, in which an alert may be triggered even when neither SPI nor SSMI alone reaches a critical threshold. Consistent with Moradian et al. (2025), the MSDI may therefore diagnose drought more severely than its univariate components, which can be advantageous when early warning procedures are implemented by water managers.

## 3. Illustration

In this section, we illustrate possible consequences of the lack of use of the Kendall function (reads, the PIT) in a specific multivariate aggregation context, both considering a theoretical setting (Section 3.1) and a real-world case study (Section 3.2).

### 3.1. Theoretical Setting

Here we consider firstly the case of bivariate Extreme Value Copulas (EVC). It is important to emphasize that the methodological conclusions drawn from this example are entirely independent of the Copula family employed. What can vary is only the magnitude of the resulting error, which depends upon the specific case study. The key takeaway is that, when the goal is to construct a multivariate drought index that follows a standard Normal distribution, neglecting the role of the Kendall function in the aggregation of variables through their joint CDF $\mathbf{F}_{\mathbf{X}}$ may yield non-Gaussian indices.

This specific example is chosen because the analytical expression of $\mathbf{K}_{\mathbf{C}}$ is known for this class of copulas, and is given by

$$\mathbf{K}_{\mathbf{C}}(t) = t - (1-\tau_{\mathbf{C}})\, t \log(t), \quad t \in [0,1], \tag{15}$$

where $\tau_{\mathbf{C}} \in [0,1]$ is the Kendall rank correlation coefficient associated with the Extreme Value (EV) copula $\mathbf{C}$ (Nelsen, 2006; Salvadori et al., 2007), and can be used here since the framework is two-dimensional. Note that $\tau_{\mathbf{C}} = 1$ for the co-monotone copula $\mathbf{M}_2$, and $\tau_{\mathbf{C}} = 0$ for the Product copula $\mathbf{\Pi}_2(u,v) = uv$, corresponding to the case of independent variables. The EVC's are of utmost importance when investigating extreme catastrophic phenomena (such as droughts). For instance, an EV copula belonging to the Gumbel-Hougaard family was used in the following papers regarding droughts: Hao and AghaKouchak (2013); Hao et al. (2018, 2020); Li et al. (2020); Suo et al. (2024); Yu et al. (2025).

Now, considering the bivariate EV framework of interest here, remember that $\mathbf{K}_{\mathbf{C}}$ is the distribution function of the univariate random variable $T = \mathbf{F}_{\mathbf{X}}(\mathbf{X}) = \mathbf{C}(F_1(X_1), F_2(X_2))$—see Equations 5 and 6—where $\mathbf{C}$ is a bivariate EVC.

Figure 1 clarifies the implications of failing to account for the role played by the Kendall function. In particular, Figure 1a shows that $T$ has a Uniform law (viz., $\mathbf{K}_{\mathbf{C}}(t) = t$) if, and only if, $\tau_{\mathbf{C}} = 1$ (the *magenta* line) as a consequence of Equation 15—i.e. when $\mathbf{C}$ equals $\mathbf{M}_2$, whereas $\mathbf{K}_{\mathbf{C}}(t) > t$ for all the other values of $\tau_{\mathbf{C}} \in (0,1)$—the only obvious equalities take on at $t = 0$ and $t = 1$.It can be shown analytically that the maximum difference $\Delta(\tau_{\mathbf{C}}) = \mathbf{K}_{\mathbf{C}}(t;\tau_{\mathbf{C}}) - t$ always occurs at $t = 1/e \approx 0.37$ (the vertical *orange* line in Figure 1a), and equals $(1-\tau_{\mathbf{C}})/e$: numerical estimates of $\Delta(\tau_{\mathbf{C}})$ are presented in the top-left legend of Figure 1a, showing a monotonic increase from 0 to $1/e$ as $\tau_{\mathbf{C}} \to 0$. It is then worth investigating how large can be the error committed by computing the naive drought index $S^{\dagger}_{\mathbf{x}}$ instead of the correct one $S_{\mathbf{x}}$—see Equation 12. Consider that, conventionally, a drought is at play when the corresponding index is less than $-1$: then, by virtue of Equation 3, this happens when the argument of $\Phi^{-1}$ takes on a value smaller than $t^* \approx 0.159$. The *magenta* line in Figure 1b shows the graph of the standard Normal quantile function $\Phi^{-1}(t)$ for $t \in (0,1)$: as expected it intersects the critical threshold $-1$ (the horizontal *orange* line) at $t^*$ indicated by the vertical *cyan* line.

Figure 1a shows the graphs of the functions $\mathbf{K}_{\mathbf{C}}(t;\tau_{\mathbf{C}})$ for $t \in [0,1]$ and various values of $\tau_{\mathbf{C}}$. At $t^*$ (the vertical *cyan* line) all the $\mathbf{K}_{\mathbf{C}}$'s curves corresponding to $\tau_{\mathbf{C}} < 1$ take on values ("*" markers) larger than $t^*$, as summarized in the right-center legend. As a consequence, by virtue of Equation 12, $\Phi^{-1}(\mathbf{K}_{\mathbf{C}}(t^*)) > \Phi^{-1}(t^*) = -1$, as shown in Figure 1b by the "*" markers on the vertical *cyan* line at $t^*$, and reported in the bottom-center legend. In simple words, should it be $\mathbf{F}_{\mathbf{X}}(\mathbf{x}) = t^*$, then $S^{\dagger}_{\mathbf{x}} = -1$ but $S_{\mathbf{x}} > -1$. In turn, while the correct index $S_{\mathbf{x}}$ is larger than $-1$ (indicating a "regular" situation), the naive index $S^{\dagger}_{\mathbf{x}}$ states a drought condition, and false alerts could be raised.

In addition, in Figure 1b all the curves corresponding to $\tau_{\mathbf{C}} < 1$ intersect the critical threshold $-1$ at abscissas $t$'s smaller than $t^*$, as summarized in the top-left legend. In turn, depending upon the degree of association $\tau_{\mathbf{C}}$ among the variables, the correct probability level of a drought hazard scenario is smaller than 15.9%, and can be as small as 3.7% in case of independence. Hence, ignoring the Kendall device may yield an overestimation of drought occurrence (up to about 12% in this particular example) and, again, incorrect warnings may be issued.

Furthermore, it is of interest to quantify the *relative percentage error* $\epsilon(\tau_{\mathbf{C}})$ incurring when using $S^{\dagger}$ instead of $S$ at the critical value $t^*$, that is at the upper bound of the drought hazard scenario where the drought index equals $-1$:

$$\epsilon(\tau_{\mathbf{C}}) = \frac{S_{t^*} - S^{\dagger}_{t^*}}{S_{t^*}}. \tag{16}$$

This quantity can be readily evaluated by examining the values of the "*" markers along the vertical *cyan* line at $t^*$ in Figure 1b, as reported in the bottom-center legend. The corresponding relative percentage errors $\epsilon$'s are shown in the right-center legend and reveal insightful results. For instance, in the case of independent variables (indicated by the *black* marker), $S^{\dagger}$ may *underestimate* $S$ by as much as 700%. Even under moderate dependence (e.g., $\tau_{\mathbf{C}} \approx 0.5$, a common situation in real-world applications—see the *green* marker), the underestimation remains significant at approximately 100%. In conclusion, neglecting the use of the Kendall framework can lead to a biased assessment of the actual drought condition, further reinforcing the need for a rigorous multivariate standardization.

As a further illustration, consider the case of Archimedean copulas (Nelsen, 2006; Salvadori et al., 2007), quite often used in hydrology and available in all software packages. According to Nelsen (2006, Corollary 5.1.4), the Kendall function of a bivariate Archimedean copula $\mathbf{C}$ can be expressed in terms of its (additive) generator $\gamma$ as

$$\mathbf{K}_{\mathbf{C}}(t) = t - \frac{\gamma(t)}{\gamma'(t)}, \quad t \in (0,1), \tag{17}$$

where $\gamma'$ is the derivative of the generator. The case of the bivariate Gumbel-Hougaard family of EV copulas previously mentioned is just a special case of Equation 17, with generator $\gamma(t) = (-\log(t))^{\theta}$, $\theta \geq 1$, and Kendall correlation coefficient $\tau = 1 - 1/\theta$. For dimensions $d > 2$ the corresponding formula is available in McNeil and Nešlehová (2009); Kasper et al. (2024). Overall, conclusions similar to the ones already expressed above considering bivariate EV copulas can also be drawn in the case of Archimedean copulas in any dimension $d$. Here the point is that the generator $\gamma$ is specific for each family, and hence also the corresponding Kendall function $\mathbf{K}_{\mathbf{C}}$: in turn, as anticipated above, the magnitude of the error depends upon the Archimedean copula family at play.

### 3.2. Real-World Case Study

To complement the theoretical illustration shown above, below we present an applied example documenting how the proposed Kendall-based standardization can be used in practice, and how it differs from the original formulation. For a consistent and fair comparison, we use the same data and study regions as in Hao and AghaKouchak (2013). Specifically, we compute the MSDI drought index using the original approach (hereafter, MSDI-2013) and the Kendall-based approach proposed here (hereafter, MSDI-Kendall). The analysis is conducted using precipitation and soil moisture for California Climate Divisions 3 and 5 over the common period 1932–2010. Monthly precipitation and soil moisture data were obtained from the Climate Prediction Center (CPC). For more information about the data and study area, the interested reader is referred to Hao and AghaKouchak (2013).

Using identical inputs ensures that any differences between MSDI-2013 and MSDI-Kendall arise from the standardization framework rather than data selection or preprocessing. In MSDI-2013, the index is obtained by applying the standard Normal inverse to the bivariate joint CDF, whereas MSDI-Kendall applies the Kendall distribution correction prior to the Gaussian transform as described in this paper. In both cases, the copula, the univariate marginal distributions, and the Kendall distribution function are estimated non-parametrically using their empirical counterparts (although a fully parametric approach could also be adopted).

Figures 2 and 3 summarize results for California Climate Divisions 3 and 5, respectively. In both figures, the top panel shows the MSDI time series, and the horizontal reference lines at $-1$ and $-1.5$ denote moderate and severe drought thresholds. The two pairs of percentages in the main title (one pair for each index) denote the time spent by the corresponding index below the drought threshold $-1$ (left) and $-1.5$ (right). For a properly standardized $N(0,1)$ index, one would expect approximately 15.9% of values below $-1$ and about 6.7% below $-1.5$. In Figure 2, MSDI-2013 yields about 34% of months below $-1$ and about 18% below $-1.5$, indicating substantial probability mis-calibration and an inflated frequency of threshold exceedances. In contrast, MSDI-Kendall shows markedly improved agreement, with about 13% and 5% of months below the $-1$ and $-1.5$ thresholds,

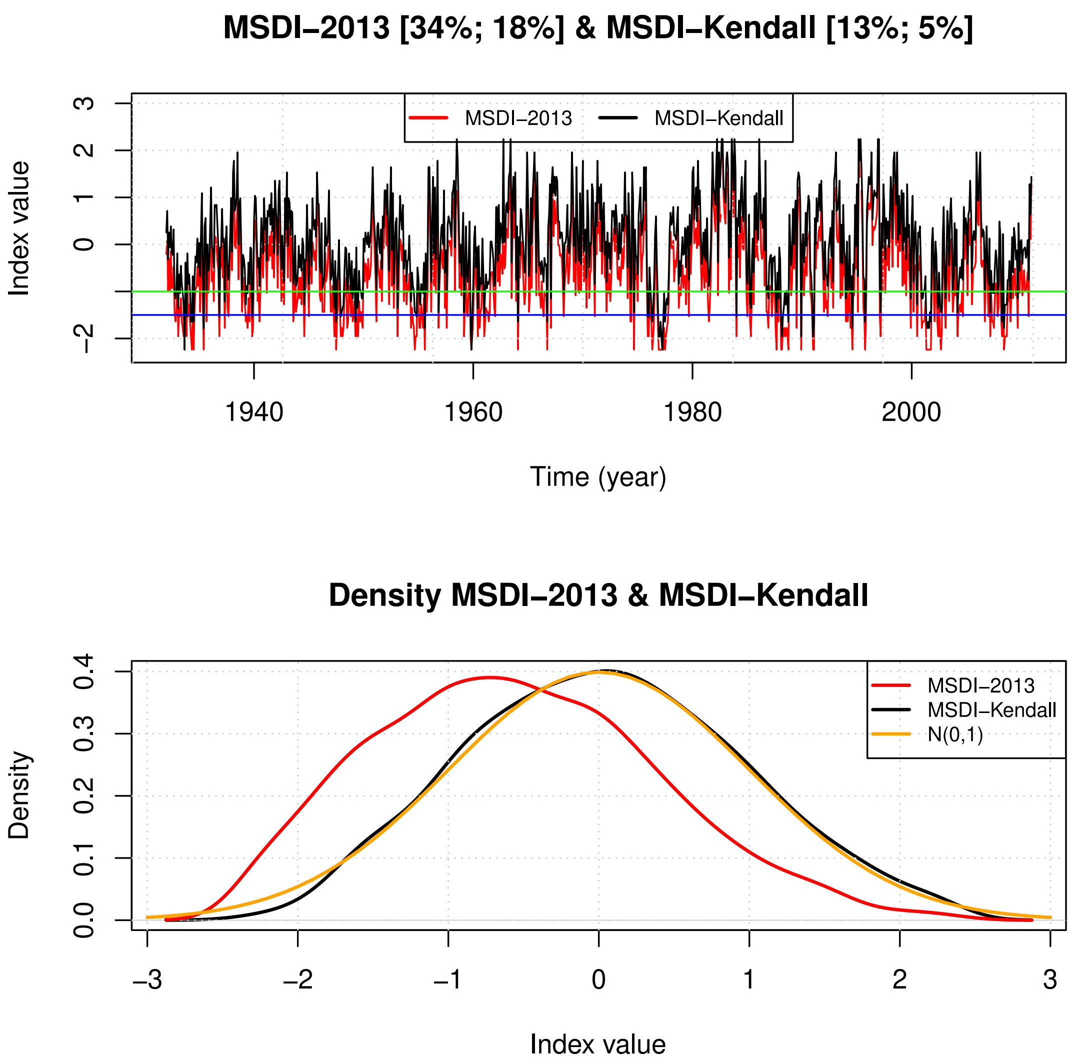


**Figure 2.** Drought assessment in California Climate Division 3 based on monthly precipitation and soil moisture. (top) Time series of Multivariate Standardized Drought Index computed using the original approach (MSDI-2013) and the Kendall-based approach (MSDI-Kendall); horizontal reference lines at $-1$ and $-1.5$ denote, respectively, moderate and severe drought thresholds—see text for explanation. (bottom) Empirical density of MSDI-2013 and MSDI-Kendall compared with the standard Normal density, $N(0,1)$.

respectively, which is substantially closer to the intended standard Normal interpretation. Similar conclusions can be drawn for Figure 3.

The lower panels of Figures 2 and 3 further diagnose these differences by comparing the empirical densities of MSDI-2013 and MSDI-Kendall indices against the reference $N(0,1)$ curve. In both climate divisions, MSDI-2013 exhibits a pronounced negative bias, visible as a leftward shift relative to the standard Normal distribution, which mechanically increases the frequency of drought declarations at fixed thresholds. MSDI-Kendall remains much more consistent with the $N(0,1)$ benchmark, indicating that the Kendall correction mitigates the bias introduced when $\Phi^{-1}$ is applied directly to $\mathbf{F}_{XY}$ without accounting for the multi-dimensional nature of the data. Overall, this example illustrates that the Kendall-based procedure better preserves the probabilistic meaning of the standardized thresholds, improving interpretability and comparability across regions and applications.

## 4. Conclusions

While traditional univariate drought indices are widely adopted and theoretically well founded, largely because they apply the PIT properly, their direct extension to multivariate settings can be problematic. In particular, when

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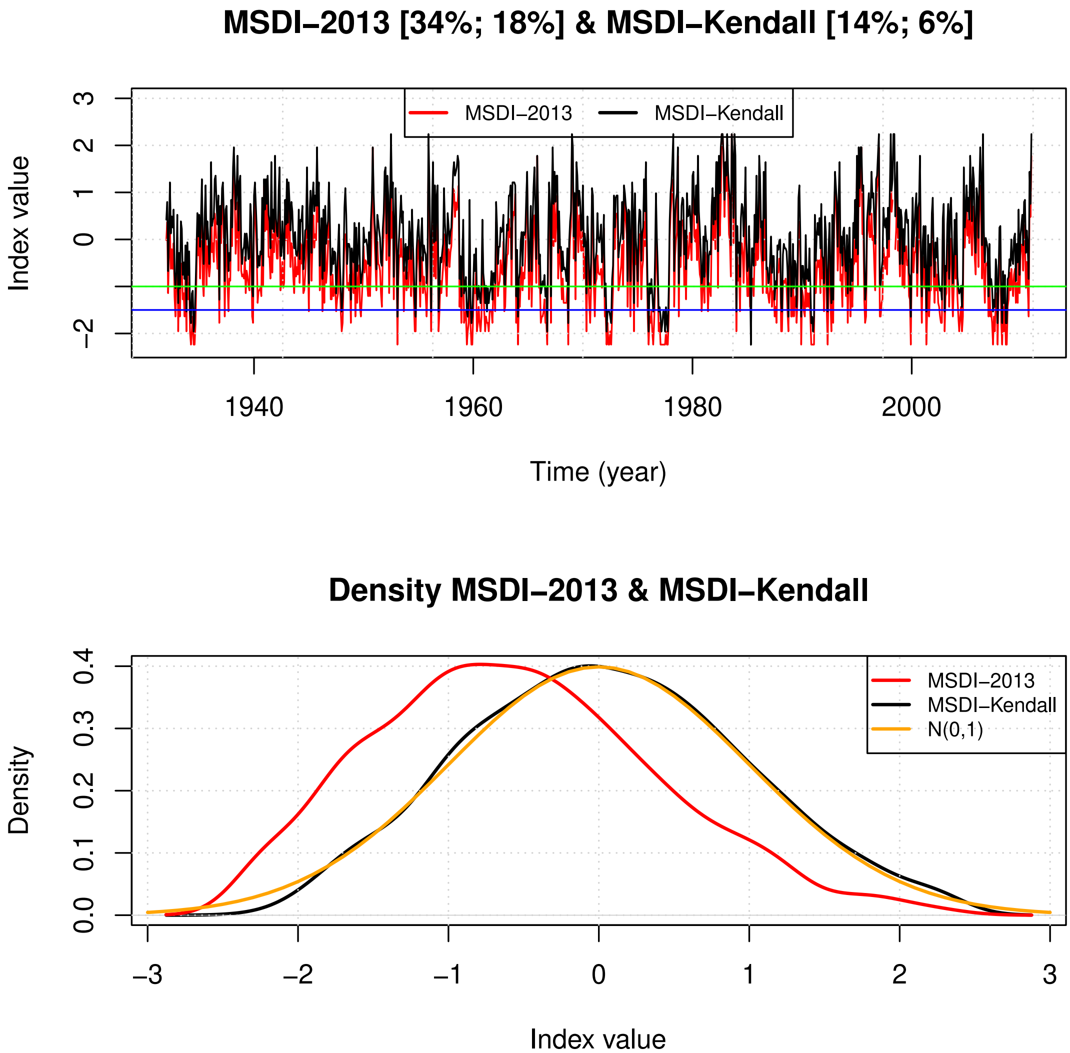


**Figure 3.** Drought assessment in California Climate Division 5 based on monthly precipitation and soil moisture. (top) Time series of Multivariate Standardized Drought Index computed using the original approach (MSDI-2013) and the Kendall-based approach (MSDI-Kendall); horizontal reference lines at $-1$ and $-1.5$ denote, respectively, moderate and severe drought thresholds—see text for explanation. (bottom) Empirical density of MSDI-2013 and MSDI-Kendall compared with the standard Normal density, $N(0,1)$.

multivariate aggregation is defined through certain common operators based on the joint distribution and the Kendall distribution device is ignored, the resulting composite index may no longer satisfy the intended standard Normal calibration. This theoretical inconsistency can translate into substantial numerical discrepancies relative to a correctly standardized multivariate index. In practice, the mismatch may manifest as an overestimation of drought frequency and apparent severity inflating the probability of crossing commonly used thresholds and thereby increasing the likelihood of unjustified drought alarms and misleading warning signals, including false positives. Our results indicate that, within these standard aggregation frameworks, rigorous standardization of multivariate or composite drought indices requires explicit incorporation of the Kendall function. Doing so restores the probabilistic meaning of the transformed index and ensures that threshold exceedances correspond to the intended return frequencies. Beyond drought, the issue is broadly relevant to the construction of indicators for compound extremes, where multiple drivers or state variables must be combined into a single risk-relevant metric without distorting the implied probabilities. In an example using precipitation and soil moisture over California climate divisions, the Kendall-consistent formulation yields an index that remains closely aligned with the intended standard Normal behavior, whereas the original formulation exhibits noticeable departures from the reference distribution. This empirical comparison reinforces the practical value of Kendall-aware standardization in real monitoring settings, where distributional distortions directly affect how thresholds are interpreted and how frequently drought conditions are declared.

The proposed Kendall-aware standardization provides a principled way to build multivariate drought indicators whose values and thresholds remain probabilistically interpretable across regions, seasons, and data sources. This has immediate implications for operational monitoring, where decision protocols often rely on fixed thresholds (e.g., values below $-1$ or $-1.5$) to trigger advisories, allocate water, or initiate contingency actions. By preventing systematic skewness and probability mis-calibration, the approach can reduce false alarms while preserving sensitivity to genuinely compound drought states, such as concurrent precipitation deficits and depleted soil moisture. For prediction, a correctly standardized multivariate index improves the consistency of target variables used in statistical or machine-learning forecasting models and facilitates cleaner evaluation of skill, because forecast probabilities and observed frequencies can be compared on a common, well-calibrated scale. More broadly, Kendall-consistent multivariate indices can support the development of integrated early warning systems that fuse meteorological, hydrologic, and land-surface information while maintaining interpretability, enabling more reliable comparisons across basins and enhancing confidence in both near-real-time monitoring and subseasonal-to-seasonal drought outlooks.

## Conflict of Interest

The authors declare no conflicts of interest relevant to this study.

## Availability Statement

We used monthly precipitation and soil moisture data from the CPC from California Climate Divisions 3 and 5 over the period 1932–2010.

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