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The Geography of Knowledge and R&D-led Growth

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Abstract

We analyse how spatial disparities in innovation activities, coupled with migration costs, affect economic geography, market structure, growth and regional inequality. We provide conditions for existence and uniqueness of a spatial equilibrium, and for the endogenous emergence of industry clusters. Spatial variations in knowledge spillovers lead to spatial concentration of more innovative firms. Migration costs, however, limit the concentration of economic activities in the most productive region. Narrowing the gap in knowledge spillovers across regions raises growth, and reduces regional inequality by making firms more sensitive to wage differentials. The associated change in the industry concentration has positive welfare effects.

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1. Introduction

Firm market power has increased across all sectors in the US, as recently documented by De Loecker and Eeckhout (2018), with the rise in mark-ups mainly driven by few firms concentrated in the upper tail, and market shares moving from low to high mark-up firms (except for the retail industry). This trend is most evident in dynamic industries where innovation is increasing.[1] In terms of location, innovative activities in most countries are concentrated in few productive regions. One of the primary reasons why innovation tends to cluster spatially is that knowledge spillovers are stronger in specific places. The link between innovation and the spatial dimension of knowledge spillovers has received significant attention in economics and economic geography (see Feldman and Kogler (2010)). Less attention has been paid to the role played by localised knowledge externalities on the transmission of innovation on market structure, the dynamics of competition and growth.

The research question we consider is: How are regional disparities[2] in innovation activities shaping the geography of the economy, its market structure, the growth rate and inter-regional

1 Most R&D intensive activities, such as ICT, pharmaceuticals and biotechnology, and automobiles are concentrated in few companies. For instance, the 2018 EU Industrial R&D Investment Scoreboard shows that in countries like Germany, France, Japan and the US the top 100 companies account for half of R&D spending.

2 Our use of the term "regional/spatial disparities" differs from the traditional way it is used in the field of economic geography. We refer to spatial disparities as relating to innovation activities and differences in technology. In contrast, in the majority of the economic geography field "spatial disparities" refers to disparities in income differences and inequality refers to (income) differences among households.

inequality? We investigate this question within a setup which encompasses industry location and R&D-led growth with three distinctive features: (i) an endogenous market structure characterised by oligopolistic firms conducting R&D, (ii) spatially constrained knowledge flows and (iii) migration costs. We find that a reduced knowledge spillover deficit across regions leads to higher growth, reduced inter-regional inequality, and overall higher welfare.

We depart from the existing literature in three ways. Firstly, innovation clusters and regional disparities emerge endogenously, hence relating industry concentration and growth. Secondly, we can explain dispersed innovation activity without relying on transport (or congestion) costs, hence emphasising frictions in knowledge flows as the source of agglomeration economies. Thirdly, dispersion is robust to the possibility of migration.

The spatial disparities in technology (knowledge) spillovers encourage the spatial concentration of industries in the most productive region, however, inter-regional migration costs limit the geographical concentration of economic activities. As a result, not all firms necessarily operate in the most productive region. The oligopolistic (Cournot) market structure generates a positive relationship between concentration and market power, with the implication that the spatial concentration of industries is associated with fewer, but more R&D intensive, firms. Individuals endogenously choose their location, implying that the share of population in each region is affected by asymmetries in knowledge externalities across regions.

We find that higher knowledge spillovers increase Lao's demand in R&D, which in turn increases R&D costs (and, thereby, the entry level of R&D), leading to a reduction in the number of operating firms, and a higher mark-up. In equilibrium, the economy has fewer but more productive firms; it has a higher growth rate, and this effect is more pronounced for the lagging region. Intuitively, since labour costs are higher in the advanced region, more firms set up operation in the lagging region when knowledge spillovers increase, than in the advanced region for an equivalent increase in knowledge spillovers there. In this case, regional disparities in innovation clusters (and income) can also be reduced.

Welfare in each region depends on static components related to market imperfection and the relative wage, and on dynamic components associated with the rate of growth. We show that the growth gains associated with fewer and more productive firms outweigh the static losses associated with a lower degree of competitiveness (higher prices). Furthermore, we show that, under mild parameter restrictions, both regions experience welfare gains as spillovers in the lagging region strengthen. In essence, reducing regional disparities and promoting growth can be mutually compatible.

We provide novel insights on the role played by localised knowledge externalities on the emergence of regional disparities in innovation clusters, industry concentration and growth. The ability of advanced regions to better capture knowledge and ideas translates into more labour allocated to R&D, higher wages, higher productivity and higher spatial concentration of industry. The latter, in turn, influences the industry market power and the associated R&D and innovation activities. This prediction is consistent with empirical evidence, and deserves further exploration.[3] Compared to monopolistic competition models

3 Recent work by Rossi-Hansberg et al. (2018) studies whether the positive trend in product market concentration, observed at the national level, persists at the local market level. They document heterogeneous trends across industries, with diverging trends prevailing in the retail industry, while in the manufacturing industry they observe mainly rising concentration both at the national and local level. The key mechanism behind the divergent trends in some industries rests on large enterprises opening up more plants in new local markets, thereby expanding the local number of establishments in the industry. This occurs particularly in sectors where transport costs are relevant (e.g. retail). Their results, therefore, do not contradict ours, which focus on R&D intensive (manufacturing) activities and negligible transportation costs.

with endogenous mark ups due to variable demand elasticities, we focus on models that generate higher market concentration and a rise in mark ups.

Our results also have potentially interesting implications for policy. Particularly, which type of intervention facilitates knowledge spillovers on local economies and a realignment in the spatial disparities in innovation activities across regions. One natural candidate would be to implement place-based university/educational policies. Empirical evidence indicates a casual link between university research activity and productivity in neighbouring firms, as exposure to research and inventors facilitate innovation (Jaffe (1989); Kantor and Whalley (2014)). Going a step further, lagging regions may also need new policies aimed at increasing exposure to innovation during childhood, as new evidence shows that more exposure to inventors and innovation in early life is key to increased innovation Bell et al. (2019).[4]

Throughout the paper we concentrate on spillovers across regions rather than technological gaps (or absorptive capacity). Notwithstanding, our work naturally extends to economies not lying on the technological frontier. [5]Finally, we discuss how our model may be extended to allow for population growth and firm heterogeneity. Specifically, firms belonging to a given sector are identical, while firm productivity differs across sectors. As a result, average knowledge between regions differs, mitigating the scale of firm relocation and inter-regional income and utility changes associated with a reduction in the gap in knowledge spillovers between regions.

Relation to the literature

The literature, pioneered by Krugman (1991), and further developed by the new economic geography discipline (Fujita et al. (1999); Martin and Ottaviano (1999); Ottaviano and Robert-Nicoud (2005)), has traditionally focused on spatial frictions to account for the spatial distribution of industries and the geography of economic activity. Yet, localisation persists despite the rapid decline in the costs of shipping goods and in communication costs (Glaeser and Kohlhase (2004); Head and Mayer (2004)). Alternative explanations, emphasising knowledge diffusion and learning, may be as relevant in accounting for clustering of innovation activities (Audretsch and Feldman (2004)). Regional variations in localised knowledge spillovers have been identified in the empirical literature since the seminal work of Jaffe et al. (1993), lately revitalised by Thompson and Fox-Kean (2005). Notably, it is the non-codified (tacit) type of knowledge that flows more easily locally than over great distances (Gertler (2003)). This is the type of knowledge that is transferred through person-to-person interaction and is clearly facilitated by geographic proximity and hindered by the costs of people moving (Combes and Duranton (2006)). Recent work by Bloom et al. (2013), estimate that the social return to R&D is two to three times higher than the private return. Building on Bloom et al. (2013); Lychagin et al. (2016) find that both intra- and inter-regional spillovers matter. Furthermore, they find empirical support for the hypothesis that reduced face-to-face knowledge flows account for the weakening of cross-regional spillovers in space. This evidence also fits with a vast body of literature

4 Focusing on the geography of innovation, Bell et al. (2019) show that direct exposure to a culture of invention and to role models appears to be playing a key role. Crucially, they show that the geographical aspects hold regardless of where you live as an adult, indicating the importance of early exposure. The authors point to the need to expose young people, especially those who show early-life excellence at math and science, to actual inventors and their workplaces, by promoting schemes ranging from mentoring by current inventors to internship programs at local companies

5 In appendix 9.8 we show that incorporating both spillovers and technological gaps would not alter our qualitative results.

in the urban and regional economic field, which argues that regional (and urban) units are increasingly relevant for the advancement of a country, as innovation leads increasingly rely on knowledge that tends to remain localised (Glaeser and Gottlieb (2009)).

New empirical research on knowledge spillovers, technological diffusion and regional innovation (Cortinovis and van Oort (2019); Miguélez and Moreno (2015); Caragliu and Nijkamp (2016)) also stress the importance of absorptive capacity of regions to address issues related to the impact of knowledge spillovers on innovation activities. Caragliu and Nijkamp (2016) analyse how various aspects of proximity (such as geography, cognitive, relational) impact on regional knowledge spillovers, and find that absorptive capacity is critical for a region to benefit from the knowledge produced in other regions. Miguélez and Moreno (2015) and Cortinovis and van Oort (2019) support these findings and stress the importance of networks and spatial mobility. They both emphasise that it takes a critical mass of existing knowledge to capture R&D spillovers. The formal model we propose borrows insights from this literature by incorporating disparities in the spatial extent of knowledge externalities, and migration costs, so that the region with the larger knowledge spillovers is characterised by higher productivity and a higher population share.

The present work is closely related to the literature on endogenous growth and endogenous industry location pioneered by Grossman and Helpman (1991) and subsequently adopted by the new economic geography literature (Martin and Ottaviano (1999), Baldwin and Forslid (2000); Martin and Ottaviano (2001); Baldwin et al. (2011)). Grossman and Helpman (1991) show that, when knowledge spillovers are global, initial conditions determine the pattern of trade and growth. In their approach, however, there is a single R&D sector that innovates and each innovation is used to produce a new variety. Hence, innovation takes place only in the country with the larger stock of knowledge capital. In our setup, in contrast, dispersion of innovation activities can be compatible with regions of different sizes and of, potentially, different knowledge endowments. Building on Grossman and Helpman (1991), Martin and Ottaviano (1999) consider the role of global and local R&D spillovers; they show that geography influences productivity only if spillovers are local and, then, study the effect of lower transport costs on agglomeration and growth. Rather than assuming one or the other type of spillover, we take into consideration both the strength and spatial extent of knowledge externalities, and show how regional disparities in innovation-enhancing activities, rather than transport frictions, can affect both industry location and growth. Both Baldwin and Forslid (2000); Martin and Ottaviano (2001), develop growth-and-geography models featuring monopolistic competition, endogenous industrial location and inter-regional migration, with the aim to analyse the spatial evolution of economic activities. Namely, the conditions that support multiple equilibria and the role played by impediments to trade and factor mobility.[6] Compared to this literature, an added feature of our approach is that the number of firms is endogenous, allowing us to relate industry concentration and growth.[7] We show that reducing the knowledge spillover deficit across regions may lead to higher growth and more concentrated markets. Although the link between market power concentration, innovation and

6 Baldwin et al. (2011) provide a comprehensive overview of this literature.

7 In its basic structure, our model is closely related to the GOLE framework developed by Neary, in that we develop a two-region model (without any competitive numeraire sector) featuring a large number of variety of goods produced by differentiated sectors with firms competing à la Cournot (see, e.g., Eckel and Neary (2010); Neary (2016)). In our oligopolistic economy, though, there is endogenous growth and the focus is on how regional disparities in innovation activities affect the geography of economic activity and inter-regional inequality, rather than on the implications of international trade.

growth is well established in the literature (Aghion and Howitt (1992); Peretto (1996); Aghion et al. (1997); Etro (2009)), we explicitly consider location as an additional variable, which allows for the endogenous emergence of regional disparities in innovation clusters. In the paper we also touch upon the issue of scale distortions embedded in the NEG approach (Bond-Smith (2019)), and extend the model so that the spatial consequences of innovation are not amplified by scale effects.

Our present contribution also resonates with recent work on spatial economics incorporating trade costs and labour mobility, where part of the spatial variation in income across regions is explained by variations in trade costs (Allen and Arkolakis (2014); Desmet et al. (2018)). Differently from this literature, we focus on the endogenous emergence of industry clusters, growth and inter-regional inequality. Compared to this literature, our contribution is to show that spatial differences in knowledge externalities (coupled with migration costs) not only ensure uniqueness and existence of a spatial equilibrium but are relevant for industry location and growth. Crucially, we are able to explain dispersed innovation activity in the presence of no transportation costs.[8]

The paper is organised as follows. Section 2 describes the model. Section 3 analyses the equilibrium, first conditional on a given population distribution across regions, and then conditional on individuals' location choice. Section 4 analyses the effect of knowledge spillovers on the patterns of agglomeration, or dispersion, of activities and individuals. Section 5 explores the implications for growth and inter-regional income inequality. Section 6 briefly discusses the role of transport costs, while in Section 7 we look at the implications of relaxing firm homogeneity and fixed population. Section 8 concludes. All proofs are presented in the Appendix Table 1 below details the notation (parameters and variables) used throughout the paper.

2. The Model

We consider a two-region setup (North and South) where the same disembodied form of technology (e.g., blueprints, intangibles) may be adopted in the whole economy, but one region is better than the other at capturing outside (non-codified) knowledge. As a result, the region with the larger knowledge spillovers experiences lower innovation costs and, thereby, higher productivity. We assume that knowledge is embedded in labour hired and used in R&D activities. Workers are homogeneous and are used for both R&D as well as the production of goods. They are perfectly mobile within a region but imperfectly mobile between regions. In what follows, we restrict attention to the description of technologies and preferences in the Northern region. Analogous expressions apply to the Southern region. Whenever a distinction is needed variables and/or parameters for the South are denoted with a star, *.

Time t is continuous and goes from zero to infinity. The economy as a whole has a constant, exogenous number of identical, infinitely-lived, skilled workers, L^w, each endowed with one unit of labour-time supplied inelastically.

It is assumed that, at date zero, a share η of individuals are born and reside in the North, while the remaining share $\eta^* = 1 - \eta$ is born and reside in the South.

The labour market is perfectly competitive and workers incur a positive non-pecuniary cost of migration. The latter accounts, among other things, for the cost of adapting to a

8 Tabuchi et al. (2018) also have migration costs acting as the dispersion force. In their paper, however, the distribution of activities is determined by the interplay between labour productivity and migration costs; moreover, in their work, technological progress is exogenous and affects all regions equally. In contrast, we assume that firms' ability to capture knowledge differs across regions, and identify knowledge spillovers as an agglomeration force.

Table 1. Notation

Symbol	Description
N	Number of industrial sectors, $j = 1, \ldots, N$
Q_j	Number of firms in each sector $j = 1, \ldots, N$, each denoted by $q_j = 1, \ldots, Q_j$
α_j	Parameter of taste for each good j, $j = 1, \ldots, N$
ρ	Rate of time preference
η	Share of total population residing in the North
L^w	Total number of individuals/total quantity of labour (North+South)
τ_D	Intra-regional transportation cost
τ_I	Inter-regional transportation cost
μ	Degree of inter-regional knowledge spillover in R&D
E_t	Per-capita level of expenditures
$c_{j,t}$	Consumption of good j, $j = 1, \ldots, N$
$p_{j,t}$	Price of good j, $j = 1, \ldots, N$
ν	Returns to knowledge in the R&D sector
δ	Productivity parameter in the R&D sector
$A_{q_j,t}$	Quantity of knowledge produced by a Northern firm q_j, $q_j = 1, \ldots, Q_j$
$X_{q_j,t}$	Quantity of good produced by firm q_j, $q_j = 1, \ldots, Q_j$
$L^X_{q_j,t}$	Quantity of labour devoted to the production of good q_j, $q_j = 1, \ldots, Q_j$
$L^A_{q_j,t}$	Quantity of labour devoted to R&D in firm q_j, $q_j = 1, \ldots, Q_j$
γ	Endogenous share of sectors located in the North
w_t	Wage rate in the Northern region
$\Pi_{q_j,0}$	Present values of expected profits of firm q_j, $q_j = 1, \ldots, Q_j$
$\pi_{q_j,t}$	Time t profits of firm q_j, $q_j = 1, \ldots, Q_j$
r_t	Real interest rate
m	Individual migration cost
ξ_t	Co-state variable associated with the R&D technology in every firm q_j, $q_j = 1, \ldots, Q_j$, $j = 1, \ldots, N$

new environment, moving away from friends and family, and similar. As made clear later on, such a migration cost allows a steady-state equilibrium in which individuals stay put while firms, responding to shocks, set up operations in one region or the other.

Preferences and technologies are described next. Individuals derive utility from the consumption of diverse goods with preferences given by

$$U = \int_0^\infty \left[\sum_{j=1}^{N} \alpha_j \log\left(c_{j,t}\right) \right] e^{-\rho t} dt \,, \tag{1}$$

where α_j is a parameter of the taste for variety j, with $\sum_{j=1}^{N} \alpha_j = 1$, $N > 2$ represents the exogenous set of commodities produced in the whole economy, $c_{j,t}$ is the consumption of variety j ($j = 1, \ldots, N$) and $\rho > 0$ is the rate of time preference. To simplify we impose $\alpha_j = \alpha = 1/N$. Lifetime utility, expression (1), is slightly different from that used in standard models in the new economic geography literature (NEG). In particular, there is no homogeneous (agricultural) sector good which can be traded at no cost between regions.[9] The budget constraint of an individual residing in the North is given by

9 In NEG models this is the device used to equalise the wage of (unskilled) individuals in both regions. Another point of departure with the standard literature is the use of a Cobb-Douglas felicity function instead of a standard CES form. Such formalisation, however, is made to simplify the analysis. Detailed calculations, available from the authors upon request, show that none of the results we derive hinge on this specification.

$$E_t = \sum_{j=1}^{\gamma N} \tau_D p_{j,t} c_{j,t} + \sum_{j=\gamma N+1}^{N} \tau_I p^*_{j,t} \bar{c}_{j,t} \, , \tag{2}$$

where E_t denotes the per-capita level of expenditure, $p_{j,t}$ ($p^*_{j,t}$) is the price of commodity j produced in the North (South), τ_D and τ_I stand for transport (iceberg) costs, and γ represents the (endogenous) share of industries located in the North. Accordingly, γN is the number of commodities produced in the North. The upper-bar indicates the consumption of a good produced in the foreign region (imported good).

The iceberg costs can also be interpreted as capturing the quality of infrastructure within a region, τ_D, or between regions, τ_I (see Martin (1999)). In line with the literature we impose the restriction $1 \leq \tau_D \leq \tau^*_D < \tau_I = \tau^*_I$, that is, transport costs are less costly within a region than between regions, and infrastructure in the North is of better quality than in the South. In each period t, every variety j of commodities ($j = 1, \ldots, N$) is produced by an endogenous number of identical firms $Q_{j,t} > 1$, each designated by q_j ($q_j = 1, \ldots, Q_{j,t}$), competing "à la Cournot".[10] Both $Q_{j,t}$ and γ are crucial variables of the model as they are related to the extent of firms' market power and the extent of firms' agglomeration, respectively. To keep the analysis simple and in line with empirical evidence, we suppose that in each industry j, every firm q_j engages simultaneously in the production of good j and in R&D (see, Dasgupta and Stiglitz (1980)).

Denoting by $X_{q_j,t}$ the quantity of good produced by firm q_j, the total amount of variety j is $X_{j,t} = \sum_{q_j=1}^{Q_{j,t}} X_{q_j,t}$. The technology of production of every firm is given by

$$X_{q_j,t} = (A_{q_j,t})^{\nu} L^X_{q_j,t} \, , \tag{3}$$

where $0 < \nu < 1$ denotes the returns to knowledge, $L^X_{q_j,t}$ is the quantity of labour devoted to the production of a good, and $A_{q_j,t}$ is the stock of (specific) knowledge-capital produced and used by firm q_j. Each firm can improve its productivity over time by engaging in in-house R&D via a process of cost reduction driven by the accumulation of firm-specific knowledge-capital (cf. Peretto (1996)). The production function (3) does not include a fixed or sunk cost, only variable costs. However, notice that in the (infinite horizon) open-loop dynamic model we present, firms' R&D expenditures are effectively sunk at every point in time by all active firms. Hence, they are formally equivalent to fixed production or maintenance costs (Spence (1984)).

The number of units of knowledge-capital produced per unit of time by each firm q_j is given by

$$A^*_{q_j,t} = \delta L^A_{q_j,t} \left[\sum_{j=1}^{\gamma N} \sum_{q_j=1}^{Q_{j,t}} A_{q_jt} + \mu \sum_{j=\gamma N+1}^{N} \sum_{q_j=1}^{Q_{j,t}} A^*_{q_jt} \right] , \tag{4}$$

where $\delta > 0$, is the R&D productivity parameter. The term $L^A_{q_j,t}$ is the amount of labour devoted to R&D, and μ denotes the degree of inter-regional knowledge spillover. Expression (4) represents the flow of knowledge generated by R&D. The R&D technology (4) above exhibits constant returns to scale in the factor that is accumulated, i.e knowledge. Within each region firms are able to take full advantage of each other's knowledge, also helped by intra-regional perfect mobility of workers; however, outside each region

10 Notice that there are no segmented markets in this economy. Specifically, we assume a world market for each commodity $j = 1, \ldots, N$, in which firms cannot price discriminate. Notice also, that we consider a symmetric Cournot equilibrium, which entails industrial specialisation within regions.

knowledge spillovers are not perfect and are region specific. Formally, this amounts to imposing the restriction: $0 \leq \mu^* < \mu \leq 1$.[11]

To close the model, we set the labour constraint in the North as

$$\eta L^w = L_t^X + L_t^A , \tag{5}$$

where $L_t^X = \sum_{j=1}^{\gamma N} \sum_{q_j=1}^{Q_{j,t}} L_{q_j,t}^X$ and $L_t^A = \sum_{j=1}^{\gamma N} \sum_{q_j=1}^{Q_{j,t}} L_{q_j,t}^A$ denote the *aggregate* quantity of labour employed in the production of differentiated goods and employed in R&D, respectively. In the sequel since the total number of workers is fixed, without loss of generality, we set $L^w = 1$.

3. Equilibrium

We proceed in three steps. First, we describe the behaviour of individuals and firms. Second, we derive the equilibrium of the model assuming a given population distribution across regions, $\eta \in [1/2, 1]$, and no migration. Third, we analyse the choice of location of individuals, and study the spatial equilibrium. As regards market structure, we assume Cournot competition with free entry in the goods market (see below), and perfect competition in the labour market. We denote by w_t the price of labour in the North and normalise the price of labour in the South to one, that is $w_t^* = 1$.

3.1 Individuals and Firms

Each individual maximises lifetime utility (1) subject to the budget constraint (2). The solution of this programme is standard. The demand function for consumption good j of an individual living in the North is given by $c_{j,t} = E_t/(N\tau_D p_{j,t})$ if $0 < j \leq \gamma N$ and $\overline{c}_{j,t} = E_t/(N\tau_I p_{j,t}^*)$ if $\gamma N < j \leq N$. The aggregate demand function for variety j, denoted $c_{j,t}^d$, is thus given by,

$$c_{j,t}^d = \frac{1}{N}\left(\frac{\eta E_t}{\tau_D p_{j,t}} + \frac{(1-\eta)E_t^*}{\tau_I p_{j,t}}\right). \tag{6}$$

Firms perform two activities: (i) they produce and sell in an oligopolistic Cournot market with free entry and exit and, (ii) they generate new pieces of knowledge-capital via their in-house R&D using labour.

The market equilibrium we consider is a symmetric Nash equilibrium in open-loop strategies.[12] Denote by $s_{q_j} = [X_{q_{j,t}}, L_{q_{j,t}}^A, A_{q_{j,t}}]$ for $t \geq 0$ firm q_j's strategy vector. To make the

11 It would be possible to also introduce imperfect intra-regional knowledge spillovers. In this case, the technology (4) would read: $\dot{A_{q_j,t}} = \delta L_{q_j,t}^A[(1-\lambda)A_{q_j,t} + \lambda \sum_{j=1}^{\gamma N} \sum_{q_j=1}^{Q_{j,t}} A_{q_j,t} + \mu \sum_{j=\gamma N+1}^{N} \sum_{q_j=1}^{Q_{j,t}} A_{q_j t}^*]$, where λ would stand for the degree of intra-regional knowledge spillover, and verify: $0 \leq \mu^* < \mu < \lambda^* < \lambda \leq 1$. However, imperfect intra-regional knowledge spillovers are not essential for our results and seriously complicate the model. The more general analysis of the model for $0 \leq \mu^* < \mu < \lambda^* < \lambda \leq 1$ is available from the authors upon request.

12 The advantage of focusing on an open-loop equilibrium is that it allows a closed-form solution. The drawback of using an open-loop equilibrium is that, typically, it does not have the property of subgame perfection. Unfortunately, closed loop or feedback equilibria complicate the model substantially, as they do not guarantee a closed form solution and, often, do not allow for a solution at all.

analysis simple, we assume that entry and exit involve zero costs,[13] meaning that the number of firms can freely adjust to its equilibrium level. In equilibrium firms commit to time paths for production, R&D, and labour at time t, with entry and exit determining the number of active firms, Q_j. Therefore, at time t the vector $[Q_j, s_1, \ldots, s_j, \ldots s_{Q_j}]$ is an instantaneous equilibrium with free entry and exit if for all firms q_j (and in all sectors, $j = 1, \ldots N$)

$$\Pi_{q_j,t}[Q_j, s_1, \ldots, s_j, \ldots s_{Q_j}] \geq \Pi_{q_j,t}[Q_j, s_1, \ldots, s'_j, \ldots s_{Q_j}] \geq 0\,, \tag{7}$$

and for $Q_j > 1$

$$\Pi_{q_j,t}[Q_j + 1, s_1, \ldots, s_j, \ldots s_{Q_j+1}] \leq 0\,, \tag{8}$$

where $[Q_j, s_1, \ldots, s'_j, \ldots s_{Q_j}]$ is the strategy vector when firm q_j deviates from its optimal time-paths while all other firms do not. Condition (7) requires that a firm maximises the sum of present values of its net profits while taking as given the behaviour of the other firms, and this value be non-negative. Condition (8) is a standard zero-profit condition. Accordingly, each firm q_j (in the North) maximises

$$\Pi_{q_j,t} = \int_0^\infty [p_{j,t}X_{q_j,t} - w_t L^X_{q_j,t} - w_t L^A_{q_j,t}]e^{-\int_0^t r_u du} dt\,, \tag{9}$$

subject to equations (3) and (6), and taking as given the law of motion of knowledge-capital in the R&D sector (4) and the real interest rate, r_t. After substitution, the current value Hamiltonian becomes

$$CVH_{q_j,t} = \left\{ \begin{array}{c} X_{q_j,t}\left[\Omega_t\left(\sum_{q_j=1}^{Q_{j,t}} X_{q_j,t}\right)^{-1} - w_t(A_{q_j,t})^{-\nu}\right] - w_t L^A_{q_j,t} \\ +\xi_t \delta L^A_{q_j,t}\left[\sum_{j=1}^{\gamma N}\sum_{q_j=1}^{Q_{j,t}} A_{q_j,t} + \mu \sum_{j=\gamma N+1}^{N}\sum_{q_j=1}^{Q_{j,t}} A^*_{q_j,t}\right] \end{array} \right\},$$

where

$$\Omega_t \equiv \frac{1}{N}\left[\frac{\eta E_t}{\tau_D} + \frac{(1-\eta)E^*_t}{\tau_I}\right]\,, \tag{10}$$

and $\sum_{j=1}^{\gamma N}\sum_{q_j=1}^{Q_{j,t}} A_{q_j,t} + \mu \sum_{j=\gamma N+1}^{N}\sum_{q_j=1}^{Q_{j,t}} A^*_{q_j,t}$ are taken as given by each firm. The term ξ_t is the co-state variable associated with (4) and $\sum_{q_j=1}^{Q_{j,t}} X_{q_j,t} = c^d_{j,t}$. In this problem, the choice variables are: $X_{q_j,t}$ (production of commodity j), $L^A_{q_j,t}$ (quantity of labour employed in R&D) and $A_{q_j,t}$ (the path of knowledge-capital). The first order conditions are given by: $\partial CVH_{q_j,t}/\partial X_{q_j,t} = 0$, $\partial CVH_{q_j,t}/\partial L^A_{q_j,t} = 0$ and $\partial CVH_{q_j,t}/\partial A_{q_j,t} = -\dot{\xi_t} + r_t\xi_t$. The transversality condition is $lim_{t\to\infty}\, \xi_t A_{q_j,t} e^{-\int_0^t r_u du} = 0$.

13 Obviously this is a strong assumption. Effectively it is implying that R&D knowledge (as embodied in labour hired and used in R&D activities) is substitutable across firms and varieties. It is made in order to keep the model and its dynamics tractable. Notice though that R&D expenditure forms part of firm's total costs and is determined endogenously in market equilibrium, cf. (Peretto, 1996, p.897). Prospective entrants are aware that these costs have to be incurred in the post-entry equilibrium.

Note that, the only plausible Nash-equilibrium of the model is associated with $\partial CVH_{q_j,t}/\partial L^A_{q_j,t} = 0$, at which the marginal revenue of an extra unit of labour devoted to R&D equals its cost (here w_t) and there is no incentive for firms to deviate from the strategy $L^A_{q_j,t} > 0$.[14]

Straightforward computations yield

$$X_{j,t} = \frac{\Omega_t}{w_t(A_{q_j,t})^{-\nu}}\left(1 - \frac{X_{q_j,t}}{X_{j,t}}\right), \tag{11}$$

$$w_t = \xi_t\delta\left(\sum\nolimits_{j=1}^{\gamma N}\sum\nolimits_{q_j=1}^{Q_{j,t}} A_{q_j,t} + \mu\sum\nolimits_{j=\gamma N+1}^{N}\sum\nolimits_{q_j=1}^{Q_{j,t}} A^*_{q_j,t}\right), \tag{12}$$

$$r_t = \nu(1 - X_{q_j,t}/X_{j,t})\left(\frac{w_t L^A_{q_j,t}}{X_{q_j,t}/X_{j,t}}\right)\frac{1}{\xi_t A_{q_j,t}} + \frac{\dot{\xi_t}}{\xi_t}. \tag{13}$$

Equation (11), where $X_{j,t} \equiv \sum_{q_j=1}^{Q_{j,t}} X_{q_j,t}$, is the total production of variety j, implicitly gives the best response of firm q_j (i.e., $X_{q_j,t}$) to the choice of production of good j of the other firms. Note that this condition is used to determine the price level of each variety.

Since $p_{j,t} = \Omega_t(c^d_{j,t})^{-1} = \Omega_t(X_{j,t})^{-1}$ (see 6), we obtain,

$$p_{j,t} = \frac{w_t(A_{q_j,t})^{-\nu}}{(1 - X_{q_j,t}/X_{j,t})}. \tag{14}$$

Equation (14) shows that the price of each variety is determined by the product between its marginal cost of production $\left(w_t(A_{q_j,t})^{-\nu}\right)$ and the markup $1/(1 - X_{q_j,t}/X_{j,t}) > 1$. Equation (12) is a static condition equating the marginal cost (w_t) and benefit $(\xi_t\delta(\sum_{j=1}^{\gamma N}\sum_{q_j=1}^{Q_{j,t}} A_{q_j,t} + \mu\sum_{j=\gamma N+1}^{N}\sum_{q_j=1}^{Q_{j,t}} A^*_{q_j,t}))$ of an additional unit of labour spent in R&D. Finally, equation (13) is a dynamic condition stating that the return (r_t) of a new piece of knowledge-capital depends on three factors: the productivity gains from knowledge accumulation (cost reducing effect of R&D, (first term on the *rhs*)), the future units of knowledge-capital (second term on the *rhs*) and the change in the shadow price of knowledge-capital (third term on the *rhs*).[15]

Using (9), we derive the standard condition $r_t\Pi_{q_j,t} = \dot{\Pi}_{q_j,t} + \pi_{q_j,t}$, where $\pi_{q_j,t} = p_{j,t}X_{q_j,t} - w_tL^X_{q_j,t} - w_tL^A_{q_j,t}$ is the instantaneous profit of a firm. Given that there is free entry and exit, we have $\Pi = \dot{\Pi} \leq 0$. Using (3) and (14), we then obtain,

14 If $\partial CVH_{q_j,t}/\partial L^A_{q_j,t} < 0$, the marginal revenue of an extra unit of labour devoted to R&D is always negative implying no labour allocated to R&D in equilibrium (i.e., $L^X_t = \sum_{j=1}^{\gamma N}\sum_{q_j=1}^{Q_{j,t}} L^X_{q_j,t} = \eta L^w$). Such an outcome can be ruled out, as any firm would have an incentive to deviate from the $L^A_{q_j,t} = 0$ strategy and choose $L^A_{q_j,t} > 0$ to produce new pieces of knowledge-capital, thereby improving their productivity and thus profitability vis-a-vis their rivals. If $\partial CVH_{q_j,t}/\partial L^A_{q_j,t} > 0$, the marginal revenue of an extra unit of labour allocated to R&D is always positive implying all labour allocated to R&D in equilibrium (i.e., $L^A_t = \sum_{j=1}^{\gamma N}\sum_{q_j=1}^{Q_{j,t}} L^A_{q_j,t} = \eta L^w$). The latter can also be ruled out, as it leads to a meaningless solution with no production of (differentiated) goods (cf. Peretto (1996)).

15 Note that, under perfect foresight, a firm can finance its own R&D through debt or equity, as the no arbitrage condition (13) implies that the rate of return from a risk-less loan must be equal to the cost of financing R&D by borrowing (cf. (Peretto, 1996, p. 904)).

$$\frac{\mathrm{pdw}_t L^X_{q_j,t}}{(1 - X_{q_j,t}/X_{j,t})} - w_t L^X_{q_j,t} - w_t L^A_{q_j,t} \leq 0\,. \tag{15}$$

This is the zero profit condition for every firm $q_j = 1, \ldots, Q_j$. Note that the standard asset-pricing equation collapses into a a zero-profit condition because the number of firms is a free to jump variable. [16]

Finally, by combining (13), (3) and the zero profit condition above (15) we obtain the rate of return to R&D in the industry (partial) equilibrium, that is

$$r_t = \nu(1 - X_{q_j,t}/X_{j,t})\left(\frac{w_t L^A_{q_j,t}}{X_{q_j,t}/X_{j,t}}\right)\frac{1}{\xi_t A_{q_j,t}} + \frac{\dot{\xi_t}}{\xi_t}\,. \tag{16}$$

Noticeably, the cost reducing effect of R&D can be decomposed into the return to R&D earned by the increase in the market share $\nu(1 - X_{q_j,t}/X_{j,t})$, and the returns to R&D earned for a given market share $\frac{w_t L^A_{q_j,t}}{X_{q_j,t}/X_{j,t}}$. The latter implies that the incentive to innovate is the incremental profit (*gross-profit effect*), while the former captures the potential gains from an increase in rivals' market shares (*business-stealing effect*). This is in keeping with many IO models of R&D and is in line with Peretto (1996).[17]

3.2 Intra-regional equilibrium

Intra-regional equilibrium determines the number of firms in each sector, quantities and prices for given shares of population (η and $1 - \eta$). We focus on an *intra*-regional symmetric equilibrium, whereby prices and quantities of goods are identical within a region but, as it will be the case, different between regions. Formally,

Definition 1. *For all sectors j in a given region, an intra-regional symmetric equilibrium is characterised by the number of firms in each sector Q_{jt}, quantities $X_{q_{jt}}$ and prices p_{jt} that are identical for all firms q_j. For the North, we have*

1. $Q_{j,t} = Q_t$ for all $j \leq \gamma N$,
2. $X_{q_j,t} = X_{j,t}/Q_t = X_t/Q_t$, $L^X_{q_j,t} = L^X_{j,t}/Q_t = L^X_t/(\gamma N Q_t)$ and $L^A_{q_j,t} = L^A_{j,t}/Q_t = L^A_t/(\gamma N Q_t)$ for all $\mathrm{q_j}$ and all $j \leq \gamma N$, and
3. $p_{j,t} = p_t$ for all $j \leq \gamma N$.

And similarly for the South.

Using the definition above, and combining the labour constraint (5) and the zero-profit condition (15), we obtain aggregate employment in the production of goods and R&D

16 This is a feature specific to the setting developed here and can be related to Peretto and Connolly (2007), who show that both horizontal and vertical dimension of technology are complementary. In their model, the quantity of resources required to perform horizontal innovation is decreasing due to some external effect that makes entry costs fall, and leads to closed-form solutions. In the limit case of zero entry cost, assumed here, it also implies that the transitional dynamics compresses to a jump to the new steady state (see (Peretto and Connolly, 2007, p. 339)).

17 In Peretto (1996) the return to R&D is non-monotonic in the number of firms as a result of the tension between the gross profit and the business stealing effects. Consequently, spillovers exert different influences on incentives to undertake R&D depending on which effect dominates. In our model, in contrast, the gross-profit effect always dominates the business stealing effect in (general) equilibrium (see Appendix 9.2). Recent empirical work by Bloom et al. (2013) supports our result.

$$L_t^X = \eta\left(1 - \frac{1}{Q_t}\right), \tag{17}$$

$$L_t^A = \eta\frac{1}{Q_t}. \tag{18}$$

That is, the share $1/Q$ of labour force is allocated to the production of R&D and, since $\eta \geq 1/2$ and $\mu > \mu^*$, the share of workers employed in each sector will be larger in the North (See proposition 1 below). Interpreting the number of firms in each sector as a proxy for the degree of markets' competitiveness ($Q = X_j/X_{q_j}$) it follows that: as firms' market share in a given sector increases ($Q \downarrow$), firms allocate more labour to R&D and less to the production of commodities.

To ensure existence, and other results to follow, from now on we impose the following restriction on the returns to knowledge parameter, ν.

Assumption 1. *Let* $\nu > \frac{\rho}{\eta\delta}$.

As shown in Appendix 9.1, Assumption 1 is a necessary and sufficient condition to ensure a strictly positive long-term growth rate and is easily satisfied for plausible parameter values.

In Appendix 9.1 we also formally demonstrate that a steady-state equilibrium exists, that the adjustment to steady state is instantaneous, and that growth of knowledge-capital is the same in the North and in the South. To simplify we assume that every firm starts with the same endowment of knowledge-capital ($A_{q_j,0} = A^*_{q_j,0}$), though this is not essential for the results.[18] The following proposition summarises the main properties of the steady state.

Proposition 1. *(Steady state) For any given population distribution* $\eta \in [1/2, 1]$

a. *There exists a unique steady-state equilibrium solution to which the economy jumps immediately,*
b. *The steady state is characterised by a constant (and common to all firms) level of growth of capital-knowledge,* $g = g^*$*; and a constant and equal number of firms in each sector,* $Q = Q^*$*, where* $Q > 1 + \frac{1}{\nu}$*,*
c. $1/2 \leq \eta < \gamma$.

Proof. *See Appendix 9.1* ■

Regions grow at the same rate in steady state, but crucially relative productivity levels differ according to a region's ability to make the most of innovation. Also, the number of firms in each sector is the same for both regions in symmetric equilibrium (part b), and the more productive region has a higher share of sectors with $\gamma > \eta$ (part c). The latter is supported by ample empirical evidence (Carlino and Kerr (2014)) documenting that R&D activities are more concentrated than employment (the measure of agglomeration economies).

The equilibrium system expressed in terms of γ and Q (derived in Appendix 9.1) reads as

18 If $A_{q_j,0} \neq A^*_{q_j,0}$, the knowledge gap between firms of different regions would remain constant over time as the economy jumps immediately to the steady state (see Appendix 9.1).

$$(1-\eta)\left[1+\mu^*\frac{\gamma}{(1-\gamma)}\right]=\eta\left[1+\mu\frac{(1-\gamma)}{\gamma}\right], \tag{19}$$

$$Q=\frac{(1+\nu)[\gamma+\mu(1-\gamma)]\frac{\delta\eta}{\gamma}}{\nu[\gamma+\mu(1-\gamma)]\frac{\delta\eta}{\gamma}-\rho}>1\,. \tag{20}$$

From equation (19) we can express γ as a function of exogenous parameters, which then, through (20), gives Q as function of exogenous parameters.

Starting from the equilibrium share of sectors, γ, few comments are in order. First, there is a differentiated spillover effect according to which the share of industries located in each region is positively related to the region specific spillover (for details see Appendix 9.3). This effect is reinforced by a factor endowment effect (captured by the terms in η in (19)) according to which the share of sectors located in a given region is positively related to its population share: $d\gamma/d\eta>0$ (readily follows from (19)). Intuitively, a larger population share creates a larger market for commodities and allows firms to conduct R&D at a greater scale. This is also an agglomerating force, and reminiscent of a home market effect whereby the larger region hosts a more than proportionate share of industries (see Krugman (1980)).

Turning to the equilibrium number of firms, Q, we can establish that $dQ/d\mu<0$, $dQ/d\mu^*<0$ and $dQ/d\eta<0$.[19] Intuitively, when μ^* and/or μ take greater values, the productivity of labour in R&D improves. As a result, the demand of labour in R&D increases thereby rising R&D costs. This is equivalent to a rise in the entry level of R&D, leading to a reduction in the number of operating firms. Lastly, a higher concentration of people in the North, ceteris paribus, reduces competitiveness ($dQ/d\eta<0$). In fact, a larger pool of workers allows firms to conduct R&D at a larger scale, which pushes up L^A (relative to L^X) and increases the mark up over marginal costs. These steady state properties are reminiscent of Smulders and Van de Klundert (1995) and imply, as we shall see shortly, that market concentration triggered by inter-regional knowledge spillovers is conducive of growth.

The equilibrium wage and general price level are analysed next.

Total production of commodity j must be equal to the number of firms, Q, times quantity produced by each firm, (3); then, using the definition of Ω_t (see (10)) and combining it with (5), (15) and (17), we obtain the wage equation for the North,

$$w=\gamma\left[\frac{E}{\tau_D}+\frac{(1-\eta)}{\eta}\frac{E^*}{\tau_I}\right].$$

Using the same method, we derive the wage for the South,

$$w^*=(1-\gamma)\left[\frac{\eta}{(1-\eta)}\frac{E}{\tau_I}+\frac{E^*}{\tau_D^*}\right].$$

Taking into account that the level of expenditure per capita is determined by the wage ($E=w$ and $E^*=w^*$) and $w^*=1$ (by normalisation), the level of wage in the North, w, must adjust so that both conditions are simultaneously satisfied. After straightforward computations, we obtain the following expression

19 See Appendix 9.3 for analytical derivations.

$$\frac{\eta}{(1-\eta)}\left[\frac{\eta}{(1-\eta)}w+\frac{\tau_I}{\tau_D^*}\right]=\frac{\gamma}{1-\gamma}\left[\frac{\eta}{(1-\eta)}\frac{\tau_I}{\tau_D}+\frac{1}{w}\right], \tag{21}$$

implicitly defining w as function of all exogenous variables. Therefore, we establish the following proposition.

Proposition 2. *(Wage in the North). For any given $\eta \in [1/2, 1]$, an equilibrium level* w *exists and is unique; moreover $w > w^* = 1$.*

Proof. *See Appendix 9.4.* ■

In line with empirical evidence (Head and Mayer (2010); Redding (2011)) we find that the wage differential across regions is positively correlated with the population share in the larger region ($dw/d\eta > 0$).[20] Intuitively, a larger share of workers implies a larger labour supply of labour that, in turn, prompts the location of a greater share of sectors ($d\gamma/d\eta > 0$) and drives up the wage ($dw/d\gamma > 0$). Note, also, that from (14) and $w^* = 1$ it immediately follows that $p/p^* = w$. Finally, transport costs are not crucial to explain the wage/price differential across regions. Indeed, $w > w^*$ for any $1/2 \leq \eta < \gamma$ and no transport costs. [21]

Turning to the general price index, hereafter denoted by P, we obtain [22]

$$P = (\tau_D p)^{\gamma}(\tau_I p^*)^{1-\gamma} < P^* = (\tau_I p)^{\gamma}(\tau_D^* p^*)^{1-\gamma}, \text{ for } \eta \geq 1/2\,. \tag{22}$$

In the region that hosts a larger share of industries, a greater proportion of goods are purchased without incurring the high inter-regional transport cost, τ_I. Therefore, individuals residing in the North benefit not only from a higher wage, but also from a lower price index than individuals located in the South; a set of circumstances that should make the Northern region more attractive. As we shall see shortly, this will play a role in the analysis of the migration decision of individuals.

Finally, we close this subsection by writing the common level of growth of knowledge-capital in the North and South, that is,

$$g \equiv \frac{\dot{A_{q_j,t}}}{A_{q_j,t}} = \delta L^A_{q_j,t}[\gamma NQ + \mu(1-\gamma)NQ]\,. \tag{23}$$

Accordingly, growth depends on labour employed in R&D and on the relative production of knowledge (term in squared brackets). In symmetric equilibrium $L^A_{q_j} = L^A/(\gamma NQ)$, and using (18) into the above we obtain,

$$g = \delta\eta\frac{1}{Q}\left[\frac{\gamma + \mu(1-\gamma)}{\gamma}\right]. \tag{24}$$

Growth is determined by two endogenous variables: the number of firms per industry, Q, and the share of industries located in a given region, γ. Both Q and γ are variables affected by η, μ and μ^*, among other parameters. Later on in the paper, we carry out the equilibrium growth analysis and related comparative statics (see Section 5).

20 Specifically, we obtain $dw/d\eta > 0$, with $\lim_{\eta\to 0} w = \mu\tau_D^*/\tau_I < 1$ and $\lim_{\eta\to 1} w = \tau_I/(\mu^*\tau_D) > 1$. See Appendix 9.4.

21 The role of transport costs is analysed in depth in Section 6.

22 The choice of a Fisher price index is made for convenience. Since it is computed as a geometric mean, it will be easier for us to interpret some of the results we derive later, particularly, regarding the migration condition.

3.3 Spatial equilibrium

Having set out the behaviour of agents, conditional on given shares of individuals in each region (η and $1-\eta$), we now explore the conditions required for a spatial equilibrium. We begin by defining a spatial equilibrium.

Definition 2. *A population share $\hat{\eta} \in (0, 1]$ is a spatial equilibrium, if individuals have no incentives to move away from the region in which they are originally located.*

As long as the differential in utility between North and South (i.e. $\Delta U = U - U^*$) is lower, in absolute value, than the cost of migration, then, individuals born and residing in a given region stay put. Accordingly, using lifetime utility (1), we have,

$$\left| \begin{array}{c} \int_0^\infty [\sum_{j=1}^{\gamma N} \log(c_{j,t}) + \sum_{j=\gamma N+1}^{N} \log(\overline{c}_{j,t})] e^{-\rho t} dt \\ -\int_0^\infty [\sum_{j=1}^{\gamma N} \log(\overline{c}^*_{j,t}) + \sum_{j=\gamma N+1}^{N} \log(c^*_{j,t})] e^{-\rho t} dt \end{array} \right| \leq m,$$

where m denotes the migration cost of an individual.

Using the property that the economy jumps immediately to the steady state, together with the individual demand functions, and expressions (14) and (22), we can simplify the above to obtain,

$$|\Delta U| = \frac{1}{\rho} |\log w \frac{P^*}{P}| \leq m \,. \tag{25}$$

Notice that, as $w > 1$ and $P < P^*$, it follows that $\Delta U > 0$ implying that migration never occurs from North to South. Indeed, in terms of utility, workers residing in the North fare better than those residing in the South, since they benefit from a higher wage and a lower price index. For later purposes, using (25) above and (22), the difference in the present value of utility reads as

$$\rho \Delta U = \log w + \log \left(\frac{\tau_I}{\tau_D}\right)^\gamma \left(\frac{\tau_D^*}{\tau_I}\right)^{1-\gamma}, \tag{26}$$

and, differentiating (26) with respect to η gives,

$$\rho \frac{d\Delta U}{d\eta} = \frac{1}{w}\frac{dw}{d\eta} + \frac{d\gamma}{d\eta} \log\left(\frac{\tau_I}{\tau_D}\frac{\tau_I}{\tau_D^*}\right) > 0 \,. \tag{27}$$

Since $1 \leq \tau_D \leq \tau_D^* < \tau_I$, $d\gamma/d\eta > 0$ and $dw/d\eta > 0$, equation (27) implies that the utility differential is positively correlated with the population share in the North. In other words, a larger population in the North creates more incentives for individuals to migrate from the South to the North.

Keeping the above in mind, we can now turn to the characterisation of the spatial equilibrium, then to the analysis of migration and location of individuals and firms. In the spirit of Murata (2003), location preferences are idiosyncratic, implying a smooth migration function. Specifically, disregarding individuals in the North who always stay put, we assume that the migration costs of individuals in the South, m, are drawn from a uniform distribution with support $[m^{low}, m^{high}]$. Depending on the size of m different scenarios emerge for individuals in the South. To analyse the possible outcomes, let us define the cut-off value $\underline{m}$ such that $\Delta U - \underline{m} = 0$, that is the level of the migration cost at which individuals are indifferent between staying put or migrating

Solving $\Delta U = \underline{m}$ for the population share η, potentially, gives two possible solutions for the population distribution: one for the case of migration from North to South and the other for migration from South to North. As mentioned above, migration from the North to the South cannot occur, hence this solution can be ruled out. We thus denote by $\underline{\eta}$ the unique possible solution for the population distribution for which $\Delta U - \underline{m} = 0$. Notably, there is a one for one correspondence between $\underline{m}$ and $\underline{\eta}$. This is because migration costs are drawn from the set of individuals in the South. Therefore, determining $\underline{m}$ automatically sets $\underline{\eta}$. Moreover, since $\Delta U(\underline{\eta}) > 0$ and $d\Delta U/d\eta > 0$, it follows that $\underline{\eta} > 0$.

If migration costs for individuals in the South verifies $m^{high} < \underline{m}$, and (25) always holds with a strict inequality (i.e., $\Delta U < m$ when $\Delta U - \underline{m} = 0$), then any initial population distribution is a dispersed spatial equilibrium. If, however, migration costs verify $0 < \underline{m} < m^{high}$, then any initial population distribution is an agglomerated spatial equilibrium. In these two extreme cases, migration costs are either low enough so that all individuals migrate to the North, or too large implying that individuals in the South stay put. Lastly, there is the intermediate case in which individuals in the South face migration costs verifying $0 < m^{low} < \underline{m} < m^{high}$. In this scenario, solving $\Delta U = \underline{m}$ determines the population distribution $\underline{\eta}$ that sustains a dispersed spatial equilibrium, with $0 < \underline{\eta} < 1$. The following proposition summarises.

Proposition 3. (SPATIAL EQUILIBRIA). *Define $\underline{\eta}$ as the population distribution satisfying $\Delta U - \underline{m} = 0$. Then there exists a critical cut-off value for the migration cost,$\underline{m}$, such that*

a. *For $0 < m^{low} \leq \underline{m} \leq m^{high}$: Any initial population distribution $\eta_0 \in [0, 1]$ is a dispersed spatial equilibrium, $\hat{\eta} = \underline{\eta}$;*
b. *For $0 < \underline{m} \leq m^{low} < m^{high}$: Any initial population distribution $\eta_0 \in [0, 1]$ is a dispersed spatial equilibrium, $\hat{\eta} = \eta_0$;*
c. *For $0 < m^{low} < m^{high} \leq \underline{m}$: Any initial population distribution $\eta_0 \in [0, 1]$ is an agglomerated spatial equilibrium, $\hat{\eta} = 1$.*

Proposition 3 establishes that, with migration, different types of spatial equilibria may arise. One is an agglomerated equilibrium: in part (c) individuals and firms locate in the North. In the remaining cases, we obtain a dispersed equilibrium where individuals and firms are located in both regions. Notably, while in part (b) some individuals initially born in the South have a migration cost which is too high to induce migration to the North, in the intermediate case (part a) any change in ΔU will affect the corresponding value of the migration cost $\underline{m}$ (solution of $\Delta U = \underline{m}$) and, thereby, the corresponding population distribution, $\underline{\eta}$. In the latter scenario, individuals can potentially migrate to the North, provided that the utility differential between North and South diminishes, as will be discussed further in the following sections.

Finally, we link Proposition 3 with Proposition 1 that established conditions of existence and uniqueness for a potential dispersed equilibrium, requiring $\eta \in (1/2, 1)$. Recall that in the context of Proposition 1, individuals were assumed to stay put and migration costs had not been introduced. To proceed, we need to compare $\underline{\eta}$ and 1/2. In principle, we can have $0 < \underline{\eta} < 1/2$ or $1/2 \leq \underline{\eta} < 1$. Indeed, if $\underline{\eta}$ is close to 1/2 we can have

$\Delta U(\underline{\eta}) > 0$, with $\underline{\eta} < 1/2$. However, this case can be ruled out as $\underline{\eta} < 1/2$ would contradict the condition of existence and uniqueness (Proposition 1). Hence, under Proposition 1, the solution to (25) necessarily implies $1/2 \leq \underline{\eta}$. Accordingly, we establish the following.

Corollary 1. *For $0 < m^{low} \leq \underline{m} < m^{high}$ and $\eta_0 \in (1/2, 1)$, there exists a unique spatial equilibrium $\hat{\eta} = \underline{\eta}$ where individuals and firms are dispersed.*

Corollary 1 is instrumental for the analysis carried out in the next section. Before proceeding further, it is instructive to study the parameter conditions under which the corollary applies. As shown in Appendix 9.4, $\lim_{\eta \to 1} w = \frac{1}{\mu^*} \frac{\tau_I}{\tau_D}$; thus, in the limit $\eta = 1$ the utility differential between North and South (25) reads as $|\Delta U| = \frac{1}{\rho} | \log \left[\frac{1}{\mu^*} \left(\frac{\tau_I}{\tau_D} \right)^2 \right] |$. Since the term in square brackets is greater than one, the equation $\Delta U = m$ has a solution for m given by $m = \frac{1}{\rho} \log \left[\frac{1}{\mu^*} \left(\frac{\tau_I}{\tau_D} \right)^2 \right]$. Accordingly, as long as $m^{low} \leq \frac{1}{\rho} \log \left[\frac{1}{\mu^*} \left(\frac{\tau_I}{\tau_D} \right)^2 \right] < m^{high}$, Corollary 1 applies. Closer examination of this condition shows that a lower interregional transportation cost, τ_I, or a greater rate of time preference, ρ, or a larger knowledge spillovers in the South, μ^*, or a higher intra-regional transportation cost in the North, τ_D, reinforce the conditions for a dispersed equilibrium. Intuitively, the parameters τ_I, τ_D and μ^* act through the relative wage and the relative price index (see 25) making the North less desirable. As for ρ, the potential future utility gains of migrating to the North are reduced as the rate of time preference increases.

4. Dispersion and Agglomeration

The previous sub-section has highlighted the role played by the non—pecuniary migration cost in establishing the kind of steady state we obtain. Here, we assess whether the interregional knowledge spillovers either reinforce the dispersed equilibrium or trigger a switch to an agglomerated equilibrium where individuals and firms agglomerate in a single (core) region. We carry out the analysis assuming that, initially, the economy is at a unique dispersed equilibrium (Corollary 1) and look at a decrease in μ^* for a given μ. As seen earlier, a greater gap in knowledge spillovers leads to a greater productivity in the R&D sectors of firms located in the North, thereby the share of industries operating in the North increases. This has also a positive impact on the wage, making the Northern region more attractive. Indeed, by differentiating (26) with respect to μ^* it can be easily checked that the utility differential between North and South expands as μ^*decreases, that is

$$\frac{d\Delta U}{d\mu^*} = \frac{1}{\rho} \left[\frac{1}{w} \frac{dw}{d\mu^*} + \frac{d\gamma}{d\mu^*} \log \left(\frac{\tau_I}{\tau_D} \frac{\tau_I}{\tau_D^*} \right) \right] < 0, \tag{28}$$

making it more desirable for individuals to migrate. Thus, higher regional dispersion in knowledge spillovers may lead to agglomeration. However, it may be also compatible with a dispersed equilibrium, if differences in spillovers between regions are initially not too large, which is the most empirically plausible scenario.[23] In this case, the productivity

23 The most recent empirical work, using U.S. firm level accounting data matched into U.S. Patent and Trademark Office data, documents that cross-regional spillovers are reduced in strength and correlated to geographical distance (Lychagin et al. (2016)).

advantage held by the most advanced region is not high enough to compensate for its higher wage triggered by higher demand of labour in the R&D sector. Notably, firms' dispersion is also robust to the possibility of migration. The following proposition summarises.

Proposition 4. *(SPILLOVERS, DISPERSION AND AGGLOMERATION). Given the migration cost $0 < m^{low} < \underline{m} < m^{high}$ and the corresponding initial population distribution $\underline{\eta}$ that supports the dispersed spatial equilibrium, there exists a cut-off value $\overline{\mu}^*$ such that*

a. $\hat{\eta} = \underline{\eta}$ is a dispersed spatial equilibrium for all $\overline{\mu}^* < \mu^* < \mu$, and
b. $\hat{\eta} = 1$ is an agglomerated spatial equilibrium for all $\mu^* < \overline{\mu}^* < \mu$.

Proof. *See Appendix 9.5.* ■

This Proposition is important as it highlights the role played by the degree of spillover in the South on individuals' decision to migrate and on firms' choice to relocate. To gain intuition, consider the two polar cases of μ^* decreasing and increasing, respectively. First, as μ^* decreases, individuals in the South will migrate to the North by increasing order migration costs. This will also induce firms to relocate to the North. In this case, we have both migration of individuals and relocation of firms. If, on the other hand, we consider an increase in μ^*, the North becomes less attractive. So individuals in the South stay put, meaning that the population distribution $\underline{\eta}$ that supports the initial equilibrium remains unchanged: there is no migration of individuals. However, we obtain a relocation of some sector to the South.

5. Growth and inter-regional inequality

To study the implications for growth and inter-regional income inequality, we focus on the case where the economy is at an equilibrium in which not all R&D activities and not all workers are concentrated in the most productive region (Corollary 1). Evidence suggests that this is indeed the most plausible real world scenario.

Using $L^A_{q_j} = L^A_j/Q = L^A/(\gamma N Q_t)$, (18) and plugging in equation (23) the value of Q given in Appendix 9.1 (see equation 20), we obtain the equilibrium growth rate,

$$g = \frac{1}{1+\nu}\left\{\nu\frac{\delta\eta}{\gamma}[\gamma + \mu(1-\gamma)] - \rho\right\}. \tag{29}$$

Recall that in equilibrium the share of sectors γ depends on η, μ and μ^* (see (19)). Therefore, the equilibrium growth rate depends on the share of population located in the North and on knowledge spillovers. Tedious computations, relegated in appendix, lead to the following.

Proposition 5. *(GROWTH). Along the equilibrium path defined by Corollary 1, and for any $\overline{\mu}^* < \mu^* < \mu$*

a. $\frac{dg}{d\mu^*} > 0$ and $\frac{dg}{d\mu} > 0$;
b. $\frac{dg}{d\mu^*} > \frac{dg}{d\mu}$.

Proof. *See Appendix 9.3.4.* ■

Intuitively, productivity of R&D increases as knowledge spillovers increase (part a). Notably, this effect is stronger in the lagging region (part b). Recall that, as the R&D sector becomes more productive two things happen: (i) firms allocate a greater amount of labour to R&D and, (ii) more industries choose to operate in the region experiencing productivity gains. Since labour cost is higher in the North ($w > w^*$), more industries set up in the South when μ^* increases than in the North for an equivalent increase in μ. As a result, μ^* has a stronger effect on growth than μ.

Turning to inter-regional income inequality, in the context of the present model, this is given by the wage gap between North and South. Recall that $w^* = 1$ and w comes from expression (21). The latter, is indirectly affected by μ and μ^* through γ.

Proposition 6. *(SPILLOVERS AND INTER-REGIONAL INEQUALITY). Along the equilibrium path defined by Corollary 1, and for any $\overline{\mu}^* < \mu^* < \mu$, inter-regional inequality increases (decreases) with μ (μ^*).*

Proof. *See Appendix 9.6.* ■

These findings suggest that strengthening the ability of the lagging region to capture knowledge leads to higher overall growth and a reduction of regional disparities. This is consistent, for example with empirical research documenting local spatial externalities between university research and high technology innovative activity, and the idea that promoting institutions that facilitate knowledge flows are important in supporting regional development (Jaffe (1989); Acs et al. (1992); Anselin et al. (1997); Kantor and Whalley (2014)).

From a welfare stand point, along the dispersed equilibrium path (and for any $\overline{\mu}^* < \mu^* < \mu$), we have seen that improving knowledge diffusion in the South reduces the welfare gap between North and South (see expression (28) establishing that $\frac{d\Delta U}{d\mu^*} < 0$). The latter, however, may come at the cost of creating regional winners and losers. Interestingly, we find that individuals in both regions are likely to gain. Computations (relegated in Appendix 9.7), show that the welfare effects of μ^* in the North and in the South amount, respectively, to

$$\frac{dU}{d\mu^*} = \frac{1}{\rho}\left[\frac{1/Q^2}{1-1/Q}\frac{dQ}{d\mu^*} + \frac{1-\gamma}{w}\frac{dw}{d\mu^*} - \frac{d\gamma}{d\mu^*}\log w + \frac{d\gamma}{d\mu^*}\left(\log\frac{1}{\tau_D} - \log\frac{1}{\tau_I}\right) + \frac{\nu}{\rho}\frac{dg}{d\mu^*}\right] \quad (30)$$

and

$$\frac{dU^*}{d\mu^*} = \frac{1}{\rho}\left[\frac{1/Q^2}{1-1/Q}\frac{dQ}{d\mu^*} - \frac{\gamma}{w}\frac{dw}{d\mu^*} - \frac{d\gamma}{d\mu^*}\log w + \frac{d\gamma}{d\mu^*}\left(\log\frac{1}{\tau_I} - \log\frac{1}{\tau_D^*}\right) + \frac{\nu}{\rho}\frac{dg}{d\mu^*}\right]. \quad (31)$$

In the expressions above we can identify several welfare effects. The first term in squared brackets captures the degree of competition (static) effect, which is common to both regions and negative in sign. As seen earlier (Section 3.2), when μ^* increases, the number of firms in each sector decreases ($\frac{dQ}{d\mu^*} < 0$), reducing competitiveness and increasing the price of each good produced. The second term in squared brackets captures the change in the relative wage, and is negative for the North and positive for the South ($\frac{dw}{d\mu^*} < 0$); while the third and fourth terms capture the impact of the change in the share of industries operating in the North ($\frac{d\gamma}{d\mu^*} < 0$). The latter, noticeably, affects the relative price index between regions in opposite directions, positively for the South and negatively for the North. Finally the fifth term, common to both regions, captures the long-term growth effect induced by greater knowledge spillovers in R&D. By differentiating (24) we obtain $\frac{dg}{d\mu^*} = -\frac{g}{Q}\frac{dQ}{d\mu^*} - \delta\eta\frac{1}{Q}\frac{\mu}{\gamma^2}\frac{d\gamma}{d\mu^*} > 0$, suggesting that the dynamic welfare gains are two-fold: (i) associated to the trade off between firms' market power and growth ($-\frac{dQ}{d\mu^*} > 0$), and (ii) related to the effect of knowledge spillovers on the share of industries operating in the North ($-\frac{d\gamma}{d\mu^*} > 0$). Notice, in particular, that the effect of knowledge spillovers on firms' mark up arises because the number of operating firms decreases as R&D becomes more productive; we term this the dynamic competition effect.

Tedious computations (relegated in Appendix 9.7) show that the dynamic competition effect dominates the static competition effect, i.e.$-\frac{g}{Q}\frac{dQ}{d\mu^*} > \frac{1/Q^2}{1-1/Q}\frac{dQ}{d\mu^*}$. In other words, the growth gains associated with fewer and more productive firms outweigh the static losses associated with a lower degree of competitiveness (higher prices). As a result, individuals in the South unambiguously gain from higher knowledge spillover in their region ($\frac{dU^*}{d\mu^*} > 0$). If, in addition, $\frac{dU}{d\mu^*} > 0$, then individuals in the North also gain from narrowing the gap in knowledge spillovers between the advanced and the lagging region. By direct inspection of (30) and (31), and disregarding the term with transport costs, it is apparent that the change in the relative wage separates the two expressions. Hence for $\frac{dU}{d\mu^*} > 0$ to hold it suffices to show that $\frac{1-\gamma}{w}\frac{dw}{d\mu^*} \leq \log w\frac{d\gamma}{d\mu^*} + \frac{1/Q^2}{1-1/Q}\frac{dQ}{d\mu^*}$, where $\frac{dQ}{d\mu^*} = \frac{\partial Q}{\partial\gamma}\frac{d\gamma}{d\mu^*}$. The following proposition restates the result.

Proposition 7. *(WELFARE). Assume transport costs are negligible ($\tau_D^* = \tau_D = \tau_I \to 1$). Then along the equilibrium path defined by Corollary 1, and for any $\overline{\mu}^* < \mu^* < \mu$, narrowing the gap in knowledge spillovers result in welfare gains for both regions, if $\nu \leq \hat{\nu}$ where $\hat{\nu}$ is implicitly given by the solution of*

$$\frac{\gamma\rho(1-\gamma)(1+\hat{\nu})[\gamma+\mu(1-\gamma)]}{\hat{\nu}(1-\eta)\mu\{\hat{\nu}[\gamma+\mu(1-\gamma)]\delta\eta-\gamma\rho\}} = 1$$

and γ is at its equilibrium level, given by equation (A.5) in the appendix.

Proof. *See Appendix 9.7* ■

Since γ is independent of the parameters ν, δ and ρ the conditions ensuring $\nu \leq \hat{\nu}$ are not particularly restrictive; for instance, δ large enough relative to ρ would suffice. Proposition 7 implies that, transport costs aside, the welfare gains associated with the increase in the share of industries operating in the South are likely to outweigh the welfare losses associated with the lower relative wage in the North, implying a net welfare gain for the North too. Allowing for inter-regional transport costs weakens the potential welfare gains in the North, since it generates a larger wage gap across regions. This is explained in more detail in the following section.

6. Transport costs

In this section we briefly mention what role transport costs play in our model. In the economic geography literature it is usually argued that transport costs are crucial in shaping the distribution of activities (Krugman (1991); Fujita et al. (1999)) and in explaining the dynamics between growth and agglomeration (see, e.g, Minerva and Ottaviano (2009)). In these models, which typically assume monopolistic competition and increasing returns to scale, the innovation sector requires goods which incur transport costs, so that industrial concentration, by reducing the input cost of innovation, increases the growth rate.

From direct inspection of (29) one can immediately check that, in our setup, the equilibrium growth rate does not depend on transport costs. This is because inter- and intra-regional transport costs $(\tau_I, \tau_D, \tau_D^*)$ do not directly influence the input costs of innovation and, therefore, play no role in the equilibrium location of industries (nor on their degree of competitiveness).

Transport costs, though, do influence the income differential between regions. Consider, for instance, the effect of a higher inter-regional transport cost. Simple algebra shows that w (see (21)) is increasing in τ_I if the number of industries located in the North is greater than in the South (i.e. $dw/d\tau_I > 0$ as $1/2 \leq \eta < \gamma$). The reason is that an increase in τ_I induces a decrease in individual demands for foreign varieties and, as the Northern region is more populated in terms of firms ($\gamma > 1/2$), such an effect is more pronounced in the South than in the North. As a result, the relative price of varieties increases, which, ultimately, generates a larger wage gap across regions.[24] This also suggests that, ceteris paribus, the lower the transport cost the more likely are dispersed equilibria. Intuitively, when the transport cost τ_I decreases, on the one hand foreign varieties are cheaper and, on the other hand, the wage in the North decreases ($dw/d\tau_I > 0$). Both effects make the Northern region less attractive for individuals in the South.[25]

Finally, due to the price index effect highlighted above, transport costs affect the change in regional welfare associated with higher μ^*. Namely, transport costs strengthen the positive welfare effect in the South but weaken the potential welfare gain in the North.

7. Extensions

In what follows, we discuss how our model may be extended to allow for population growth and firm heterogeneity. We draw from Peretto (1998) to explore the role of population growth, and from Impullitti and Licandro (2018) to introduce firm heterogeneity in our set-up. These extensions require some modifications and additional assumptions which are explained next. The more technical and lengthy computations are relegated in Appendix 9.9.

In line with Peretto (1998), we assume that population is growing exogenously at rate ω; hence, $\frac{L_t^{w\bullet}}{L_t^w} = \frac{L_t^\bullet}{L_t} = \frac{L_t^{*\bullet}}{L_t^*} = \omega$, with $0 < \omega < \rho$. For simplicity, the population growth rate is set identical across regions. The allocation of the world population remains: $L_t = \eta L_t^w$ and $L_t^* = (1-\eta)L_t^w$ at any time. To account for population growth, the utility function now reads as $U = \int_0^\infty L_0 [\sum_{j=1}^{N_t} \log(c_{j,t})/N_t] e^{-(\rho-\omega)t} dt$, where L_0 is the total number of individuals residing in the North at time zero, and $\rho - \omega$ can be interpreted as the net rate of time preference. Finally, to ensure constant growth rates of variables in steady state, the number of varieties grows exogenously at the same rate as population: $N_t = N_0 e^{\omega t}$, with N_0 representing the number of sectors at date zero. Hence, $N_t^\bullet = \omega N_t$.

Incumbent firms' production technology (3) is unchanged, while the R&D technology is slightly modified. Following Impullitti and Licandro (2018), we assume:

24 The same logic applies to a change in τ_D and τ_D^*.

25 Allen and Arkolakis (2014) and Tabuchi et al. (2018), also obtain that a lower inter-regional transport cost does not necessarily lead to agglomeration, while Martin and Ottaviano (1999) point out that transport costs are inconsequential for growth if spillovers are global rather than local.

$$\dot{A}_{q_j,t} = \delta L^A_{q_j,t} K_t,$$

where K_t is an external factor (knowledge spillover) defined as follow:

$$K_t = \overline{A}_{j,t} D_t,$$

and $\overline{A}_{j,t} = \sum_{q_j=1}^{Q_{j,t}} A_{q_j,t}/Q$ is the average productivity of those producing in the same sector j.[26] The term $D_t = \tilde{A}_t/\overline{A}_{j,t}$ denotes the difficulty of innovation, and is related to the distance between the average productivity of the overall economy and the average productivity of firms belonging to the same sector. Average productivity of the overall economy is given by: [27]

$$\tilde{A}_t = \frac{1}{N_t}\left[\sum_{j=1}^{\gamma N_t} (\overline{A}_{j,t}) + \mu \sum_{j=\gamma N_t+1}^{N_t} (\overline{A}^*_{j,t})\right].$$

This specification implies that innovation is harder for firms belonging to sectors which use a greater amount of knowledge in their production process. [28]

Two comments are in order. First, compared to technology (4), the production of new pieces of knowledge which spill over is an average stock across sectors, rather than an entire stock. As it will become clear later, this feature is instrumental to avoid explosive growth in steady state. Furthermore, income per capita growth is now positively related to the average firm size, thereby eliminating scale effects (see, Laincz and Peretto (2006) and Bond-Smith (2019)).

Second, firms are heterogeneous. In line with Impullitti and Licandro (2018), heterogeneity between firms stems from the amount of knowledge required to produce a given variety of good j. Specifically, firms belonging to a given sector are identical, while firm productivity differs across sectors. Notation-wise, we have: $A_{q_j,t} = A_{j,t}/Q = \overline{A}_{j,t}$ for any $q_j = 1, \ldots, Q$, and $A_{j,t} \neq A_{k,t}$ for any $j, k = 1, \ldots, N_t$.

With regard to potential entrants, building on Impullitti and Licandro (2018), we assume that, at any time, a new variety among the non-operative varieties can be introduced at zero cost by Q firms associated to it. At the time of entry, each firm producing a new variety draws a common stock of knowledge, $A_{q_j,t} = A_{j,t}/Q = \overline{A}_{j,t}$, from a time-invariant distribution. Following Klette and Kortum (2004), we assume a discrete uniform distribution with support $(A_t^{\min}, A_t^{\max})$ for the North, and support $(A_t^{*\min}, A_t^{*\max})$ for the South, respectively.[29] We posit that the northern region is more productive implying that, at any moment, the average stock of knowledge in the North is greater than in the South, i.e., $\overline{A}_t = \sum_{j=1}^{\gamma N_t} (\overline{A}_{j,t})/(\gamma N_t) > \overline{A}^*_t = \sum_{j=1}^{(1-\gamma)N_t} (\overline{A}^*_{j,t})/[(1-\gamma)N_t]$.

Finally, to ensure a steady state in which every firm (incumbents and entrants) grows at the same common growth rate, the distributions of knowledge must *de facto* depend on the endogenous growth rate, g. Formally, this means that, at any time t, in any sector j, northern (southern) firms draw a stock of knowledge given by: $A_{q_j,t} = \overline{A}_{j,0} e^{gt}$ ($A^*_{q_j,t} = \overline{A}^*_{j,0} e^{gt}$).

26 As firms are identical within a sector, we have $\overline{A}_{j,t} = A_{q_j,t}$ for any $q_j = 1, \ldots, Q$.
27 For the South, the amount of knowledge spillover is defined as: $\tilde{A}^*_t = \frac{1}{N_t}[\mu^* \sum_{j=1}^{\gamma N} (\overline{A}_{j,t}) + \sum_{j=\gamma N+1}^{N} (\overline{A}^*_{j,t})]$.
28 Impullitti and Licandro (2018) also assume decreasing returns to innovation (see p.195, and empirical evidence cited therein).
29 Akcigit and Kerr (2018) also extend the analysis by Klette and Kortum (2004), by developing a model in which heterogeneous firms perform both internal and external R&D. Their setup provides a characterisation of innovation behaviour across different-sized firms and its implications for growth. Here we stick to a simpler formulation to focus on the implications for the geography of knowledge of firm-specific R&D.

As noted by Impullitti and Licandro (2018), this assumption is akin to a spillover from incumbents to entrants. Furthermore, since distributions are time-invariant, it follows that $\overline{A}_t = \overline{A}_0 e^{gt}$ and $\overline{A}_t^* = \overline{A}_0^* e^{gt}$ at any time, where $\overline{A}_0 > \overline{A}_0^* > 0$ are the average productivity levels at date zero in the North and South, respectively. It, therefore, follows that firm heterogeneity translates into differences in average knowledge between regions. Repeating the same analysis of Section 3, we solve for the intra-regional equilibrium under heterogeneity and population growth. As shown in Appendix 9.9, we recover a set of equations reminiscent of the model with homogeneous firms, but for the terms ω and $\overline{A}_t/\overline{A}_t^* \equiv \Theta$. In the spirit of conciseness, in what follows we focus on the impact of heterogeneity on our main results; then, on the role of demographic shocks on the model outcomes.[30]

Firm heterogeneity strengthens the productivity advantage of the North and, consequently, it weakens the relocation of firms from North to South when the degree of spillovers in the South increase, i.e., $\frac{d^2\gamma}{d\mu^* d\Theta} < 0$. Similarly, we find that the the impact on inter-regional utility differential is weakened, i.e., $\frac{d^2\Delta U}{d\mu^* d\Theta} < 0$. This is because heterogeneity mitigates the change in the relative wage between North and South and, thereby, the utility gain associated with higher knowledge spillover in the South. In conclusion, firm heterogeneity translates into differences in average knowledge between regions, mitigating the scale of firm relocation and inter-regional income and utility changes associated with a reduction in the gap in knowledge spillovers between regions.[31]

Finally, let us analyse the role of the rate of population growth. By direct inspection of (A.11) it can be checked that changes in the population growth rate have no effect on γ, since ω is common to both regions. For the same reason, the equilibrium wage is independent of ω; therefore, changes in the rate of growth of population have no impact on the spatial equilibrium. On the other hand, higher population growth allows for a greater share of the workforce to be engaged in R&D. As a consequence, the number of firms in each sector decreases and the growth rate increases.[32]

8. Concluding Remarks

In this paper, we have constructed a two-region growth model in which firms operate under Cournot competition and innovate through in-house R&D. Drawing from the evidence that both intra and inter-regional spillovers matter, our main aim was to study how regional disparities in innovation enhancing activities, coupled with migration costs, shape the geography of economic activities and the market structure.

We have shown that disparities in knowledge spillovers between regions lead to spatial concentration of industries, and the latter is associated with fewer, but more innovative

30 See Appendix 9.9 for details on analytical computations.

31 Note that the effect of Θ on $dg/d\mu^*$ (and $dQ/d\mu^*$) is neutral. This is because the assumption of increasing innovation difficulty, which equalises growth rates, offsets the positive effect of greater productivity on innovation efforts.

32 Recall that population growth enters in the utility function, and the net rate of time preference is now given by $\rho - \omega < \rho$. Thus, as population growth increases, individuals put a higher weight on future consumption relative to present consumption. This means that a lower amount of labour is allocated to production of commodities and a greater amount is allocated to R&D, leading to higher growth. Note that, in contrast to our model, in Peretto (1998) firms operate under monopolistic competition (implying that there is one firm per sector as if $Q = 1$), and since the number of firms grows at the same rate as population, productivity growth is independent of the population growth rate. In our model, with Cournot competition within each sector, productivity growth is affected by population growth because firms' market share in a given sector $(1/Q)$ is itself affected by population growth.

firms. Frictions in the movement of workers, on the other hand, limit the geographic concentration of economic activities in the most productive region. In this context, a weakening of the spatial disparities in knowledge spillovers between the advanced and the lagging region reduces income inequality, while preserving the positive effect on growth. This occurs because reductions in productivity advantages make firms more sensitive to wage differentials, leading to a rise in the share of industries operating in the lagging region. Welfare increases because the growth gains associated with fewer and more innovative firms outweigh the static losses associated with higher mark ups, and because the gains associated with the change in the spatial concentration of industries are likely to outweigh the losses associated with lower relative wages in the advanced region.

We have also considered whether transport costs play any role in shaping the distribution of activities across regions and found that the latter do not influence the equilibrium location of industries. This is consistent with the growing body of literature emphasising frictions in knowledge flows as source of agglomeration economies. Finally, extending the model to consider heterogeneous firms and population growth shows the robustness of our results.

9. Appendix

9.1 Proof of Proposition 1

Equation (19) draws from $g = g^*$ (Proposition 1 b) together with (4), (5) and (16), while equation (20) draws from $Q = Q^*$ (Proposition 1 b) together with (4), (12), (17) and (18) and the fact that, at the steady state, $r_t = \rho$. The proof is structured as follows. First we show that, in a *symmetric* equilibrium, the economy jumps immediately to a steady state where $g = g^*$. Second, that in the symmetric equilibrium, $Q = Q^*$. Finally we prove existence and uniqueness.

9.1.1 Step 1, showing that $g = g^*$

Using (3), (11) and (17), recall that the wage equation in the North is given by $\frac{\eta}{\gamma} w_t = \left(\frac{\eta E_t}{\tau_D} + \frac{(1-\eta)E_t^*}{\tau_I}\right)$ and an equivalent condition is satisfied for the South. Since $E_t^* = w_t^* = 1$ at each instant, and the consumer problem implies that $E_t^{\bullet}/E_t = E_t^{*\bullet}/E_t^* = r_t - \rho$ at each instant, it follows that $E_t = E$ and $r_t = \rho$. Thus, the wage equations in the North and in the South imply that the wage in the North, and the share of sectors locating in the North (South) jump immediately to their steady state values.

From the individual demand functions ($c_t = E_t/(N\tau_D p_t)$ and $\overline{c}_t = E_t/(N\tau_I p_t^*)$ for $0 < j \leq \gamma N$), it follows that $p_t c_t$ and $p_t^* \overline{c}_t$ must be constant. Therefore, at the aggregate level, $p_t c_t^d = p_t X_t$ is also constant. Since every firm in a given sector is identical, then $p_t X_t / Q_t$ ($p_t^* X_t^* / Q_t^*$) must be constant. Therefore, the number of firms Q (Q^*) in each sector $j = 1, \ldots, N$, jumps immediately to its steady-state value.

To complete the proof, we derive the dynamic equation. From equation (12), we have

$$w = \xi_t \delta \frac{A_{q,t}}{\gamma NQ} \left[\gamma NQ + \mu(1-\gamma) NQ^* \frac{A_{q,t}^*}{A_{q,t}} \right]. \tag{A.1}$$

Differentiating with respect to time yields

$$-\frac{\dot{\xi_t}}{\xi_t} = \frac{\delta\eta}{\gamma NQ^2}\left[\gamma NQ + \mu(1-\gamma)NQ^*\frac{1}{\hat{A}_t}\right] - \frac{\mu(1-\gamma)NQ^*}{\hat{A}_t[\gamma NQ + \mu(1-\gamma)NQ^*/\hat{A}_t]}\frac{\dot{\hat{A}_t}}{\hat{A}_t},$$

where we have denoted $\hat{A}_t = A_{q_j,t}/A^*_{q_j,t}$. Then, using (13), (12), (17) and (18),we obtain

$$-\frac{\dot{\xi_t}}{\xi_t} = -\rho + \delta\nu\eta\frac{Q-1}{\gamma NQ^2}\left[\gamma NQ + \mu(1-\gamma)NQ^*\frac{1}{\hat{A}_t}\right] + \frac{\delta\eta}{\gamma NQ^2}.$$

Combining the two previous equations yields

$$\frac{\dot{\hat{A}_t}}{\hat{A}_t} = \frac{\frac{\delta\eta}{\gamma NQ^2}\left[\gamma NQ + \mu(1-\gamma)NQ^*\frac{1}{\hat{A}_t}\right][1-\nu(Q-1)] + \rho - \frac{\delta\eta}{\gamma NQ^2}}{\frac{\mu(1-\gamma)NQ^*}{\hat{A}_t[\gamma NQ + \mu(1-\gamma)NQ^*/\hat{A}_t]}}.$$

Taking a first order Taylor approximation of the dynamic equation around the steady state, we obtain

$$\frac{\dot{\hat{A}_t}}{\hat{A}_t} = -\frac{\frac{[1-\nu(Q-1)]\delta\eta}{\gamma Q}\mu(1-\gamma)\frac{\hat{A}_t-\hat{A}}{\hat{A}^2}}{\frac{\mu(1-\gamma)NQ^*}{\hat{A}_t[\gamma NQ + \mu(1-\gamma)NQ^*/\hat{A}_t]}},$$

where we have dropped the subscript t to indicate the steady-state value of $\hat{A}$. Direct inspection of this dynamic equation shows that, if $1-\nu(Q-1) < 0$ (i.e., $Q > 1/\nu + 1$) the economy jumps immediately to its steady state where $g = g^*$.

9.1.2 Step 2, showing that $Q = Q^*$

Using the fact that the growth rates are the same in both regions, together with (4), (5) and (15), we obtain

$$0 = \frac{\eta}{\gamma}[\gamma NQ + \mu(1-\gamma)NQ^*] - \frac{(1-\eta)}{(1-\gamma)}[(1-\gamma)NQ^* + \mu^*\gamma NQ]. \tag{A.2}$$

Using (3), (4), (12), (13), (17) and (18), we obtain

$$\rho = \delta\nu[\gamma NQ + \mu(1-\gamma)NQ^*]\frac{\eta(Q-1)}{\gamma NQ^2} - \delta[\gamma NQ + \mu(1-\gamma)NQ^*]\frac{\eta}{\gamma NQ^2}. \tag{A.3}$$

Noting that we can derive an equivalent expression for the South, the two equations above imply that $Q = Q^*$.

9.1.3 Step 3, Existence and uniqueness

From equation (A.2), using $Q = Q^*$, we obtain

$$\eta\left[1 + \mu\frac{(1-\gamma)}{\gamma}\right] = (1-\eta)\left[1 + \mu^*\frac{\gamma}{(1-\gamma)}\right]. \tag{A.4}$$

It can be easily checked that the left hand side (hereafter LHS) is strictly decreasing, with $\lim_{\gamma\to 0} LHS = +\infty$ and $\lim_{\gamma\to 1} LHS = \eta$. Similarly, the right hand side (hereafter, RHS) is strictly increasing, with $\lim_{\gamma\to 0} RHS = (1-\eta)$ and $\lim_{\gamma\to 1} RHS = +\infty$. Since $0 \le \mu^* < \mu \le 1$, it follows that there exists a *unique* solution for γ verifying $0 \le \gamma \le 1$.

Rearranging (A.4) yields the following quadratic equation

$$[(1-\eta)\mu^* - (1-2\eta) - \mu\eta]\gamma^2 + [1-2\eta+2\mu\eta]\gamma - \mu\eta = 0\,.$$

Simple computations show that the solution is given by

$$\gamma = \frac{-(1-2\eta+2\mu\eta) + \sqrt{(1-2\eta)^2 + 4\mu\eta(1-\eta)\mu^*}}{2[(1-\eta)\mu^* - (1-2\eta) - \mu\eta]}\,, \tag{A.5}$$

where it can easily be verified that $0 < 1/2 \le \eta < \gamma \le 1$ for any population distribution verifying $\eta \in [1/2, 1]$.

To compute Q, using $Q = Q^*$ in (A.3) and re-arranging terms, we obtain (20).

Finally, plugging (20) in (24), we obtain:

$$g = \frac{\nu\delta\eta[\gamma + \mu(1-\gamma)] - \gamma\rho}{\gamma(1+\nu)}\,.$$

The above expression clearly shows that $\nu > \frac{\rho}{\delta\eta}$ is a necessary and sufficient condition for $g > 0$ (cf. Assumption 1), since the RHS is strictly increasing in γ.

9.2 Gross-profit and business-stealing effects

By use of equation (3) and the equilibrium value $L^X_{q_j,t} = \frac{L^X}{\gamma QN} = \frac{\eta}{\gamma QN}\left(1 - \frac{1}{Q}\right)$, the business stealing effect ($\nu(1 - X_{q_j,t}/X_{j,t})$) and the gross profit effect ($\frac{w_t L^A_{q_j,t}}{X_{q_j,t}/X_{j,t}}$) are given by, respectively: $\nu\left(\frac{Q-1}{Q}\right)$ and $w_t\frac{\eta}{\gamma QN}$. Recall that the gross profit effect is increasing in firms' market power (decreasing in the number of firms) while the reverse applies to the business stealing effect. In equilibrium the product of the two effects, hereafter denoted by Γ, is given by (for any $N > 2$)

$$\Gamma = \nu\left(\frac{Q-1}{Q}\right) w_t \frac{\eta}{\gamma QN}\,.$$

Normalising $\tau_D = \tau_D^* = \tau_I = 1$, the equilibrium wage is $w = \frac{\gamma}{1-\gamma}\frac{1-\eta}{\eta} > 1$, which is independent of Q. Differentiating the above with respect to Q, we obtain

$$\frac{d\Gamma}{dQ} = -w\frac{\nu\eta}{\gamma N}\left(\frac{Q-1}{Q^3}\right) < 0\,,$$

implying that the gross profit effect dominates the business stealing effect.

9.3 Comparative statics

9.3.1 Change in γ with respect to μ, μ^* and η

Applying the implicit function theorem to (A.4), we obtain

$$\frac{d\gamma}{d\mu} = \frac{\frac{(1-\gamma)}{\gamma}\frac{\eta}{(1-\eta)}}{\frac{\mu}{\gamma^2}\frac{\eta}{(1-\eta)} + \frac{\mu^*}{(1-\gamma)^2}} > 0,$$

$$\frac{d\gamma}{d\mu^*} = \frac{-\frac{\gamma}{(1-\gamma)}}{\frac{\mu}{\gamma^2}\frac{\eta}{(1-\eta)} + \frac{\mu^*}{(1-\gamma)^2}} < 0,$$

$$\frac{d\gamma}{d\eta} = \frac{\left[1 + \mu\frac{(1-\gamma)}{\gamma} + 1 + \mu^*\frac{\gamma}{(1-\gamma)}\right]\frac{1}{(1-\eta)}}{\frac{\mu}{\gamma^2}\frac{\eta}{(1-\eta)} + \frac{\mu^*}{(1-\gamma)^2}} > 0.$$

9.3.2 Change in Q with respect to μ, μ^* and η

Note that a change in Q with respect to any parameter x is given by $\frac{dQ}{dx} = \frac{\partial Q}{\partial x} + \frac{\partial Q}{\partial \gamma}\frac{d\gamma}{dx}$. From (20) it readily follows that $\frac{\partial Q}{\partial \gamma} = \frac{\eta\delta(\nu+1)\rho\mu}{[\nu\eta\delta[\gamma+\mu(1-\gamma)]-\gamma\rho]^2} > 0$ and $\frac{\partial Q}{\partial \mu} = \frac{-(1+\nu)\delta\eta\rho(1-\gamma)\gamma}{\{\delta\nu[\gamma+\mu(1-\gamma)]\eta-\rho\gamma\}^2} < 0$. Accordingly, after replacing $\frac{d\gamma}{d\mu}$ and rearranging terms, we obtain,

$$\frac{dQ}{d\mu} = \frac{\partial Q}{\partial \mu} + \frac{\partial Q}{\partial \gamma}\frac{d\gamma}{d\mu} = \frac{\delta\eta\rho(1-\gamma)\gamma(1+\nu)}{\{\delta\nu[\gamma+\mu(1-\gamma)]\eta-\rho\gamma\}^2}\left\{\frac{\mu\frac{\eta}{(1-\eta)}}{\mu\frac{\eta}{(1-\eta)} + \frac{\gamma^2\mu^*}{(1-\gamma)^2}} - 1\right\} < 0\,.$$

Similarly,

$$\frac{dQ}{d\mu^*} = \frac{\partial Q}{\partial \gamma}\frac{d\gamma}{d\mu^*} < 0.$$

To compute $dQ/d\eta$, let us recall that $L^A = \eta/Q$ (see (18)). Thus, we have

$$\frac{dQ}{d\eta} = \frac{1 - \frac{\eta}{L^A}\frac{dL^A}{d\eta}}{L^A}.$$

Therefore, the sign of $dQ/d\eta$ is the same as the sign of $1 - \frac{\eta}{L^A}\frac{dL^A}{d\eta}$. Using (20), we obtain

$$\frac{dL^A}{d\eta}\frac{\eta}{L^A} = \eta\frac{\left[\nu\delta - \left(\frac{\gamma\rho\mu(1-\gamma)^2}{\gamma^2(1-\eta)\mu^*+\eta\mu(1-\gamma)^2}\right)\frac{1}{[\gamma+\mu(1-\gamma)](1-\eta)}\right]}{\left[\nu\eta\delta - \frac{\gamma\rho}{[\gamma+\mu(1-\gamma)]}\right]}.$$

Simplifying the above expression, $1 - \frac{\eta}{L^A}\frac{dL^A}{d\eta} < 0$ is equivalent to the following condition

$$\frac{\mu(1-\gamma)}{\gamma}\frac{\eta}{(1-\eta)^2} < \frac{\gamma}{(1-\gamma)}\mu^* + \frac{\eta\mu(1-\gamma)}{(1-\eta)\gamma}\,.$$

Using the equilibrium condition (19), substitute $\mu^* \frac{\gamma}{1-\gamma} = \frac{\eta}{1-\eta}\left[1 + \mu\frac{(1-\gamma)}{\gamma}\right] - 1$ in the equation above to obtain, after simple manipulations

$$\frac{\mu(1-\gamma)}{\gamma}\frac{\eta}{(1-\eta)} < 1,$$

which is always satisfied since $0 < \mu < 1$ and $\gamma > \eta$. Therefore, $dQ/d\eta < 0$.

9.3.3 Change in L_A with respect to η

In a similar way as before, we can use $L^A = \eta/Q$ (see (18)) and (20) to obtain

$$\frac{dL^A}{d\eta} = \frac{1}{\delta(\nu+1)}\left[\nu\delta - \frac{\rho}{[\gamma+\mu(1-\gamma)]}\frac{\mu}{[\gamma+\mu(1-\gamma)]}\frac{d\gamma}{d\eta}\right].$$

After substituting for $d\gamma/d\eta > 0$, we obtain

$$\frac{dL^A}{d\eta} = \frac{1}{\delta(\nu+1)}\left[\nu\delta - \left(\frac{\gamma\rho\mu(1-\gamma)^2}{\gamma^2(1-\eta)\mu^* + \eta\mu(1-\gamma)^2}\right)\frac{1}{[\gamma+\mu(1-\gamma)](1-\eta)}\right].$$

From (20), we know that $\nu\delta > \frac{\gamma\rho}{\eta[\gamma+\mu(1-\gamma)]}$. Therefore, let us show

$$\left(\frac{\gamma\rho\mu(1-\gamma)^2}{\gamma^2(1-\eta)\mu^* + \eta\mu(1-\gamma)^2}\right)\frac{1}{[\gamma+\mu(1-\gamma)](1-\eta)} < \frac{\gamma\rho}{\eta[\gamma+\mu(1-\gamma)]},$$

implying that $dL^A/d\eta > 0$. Simplifying the previous expression, we obtain the following condition

$$\frac{\eta^2\mu(1-\gamma)}{(1-\eta)^2\gamma} < \frac{\gamma}{(1-\gamma)}\mu^*$$

Using the equilibrium condition (19), substitute $\mu^* \frac{\gamma}{1-\gamma} = \frac{\eta}{1-\eta}\left[1 + \mu\frac{(1-\gamma)}{\gamma}\right] - 1$ into the equation above to obtain the following inequality

$$\frac{\mu(1-\gamma)}{\gamma}\frac{\eta}{(1-\eta)} < 1,$$

which is always satisfied as $\gamma > \eta$ and $0 < \mu < 1$.Therefore, $dL^A/d\eta > 0$.

9.3.4 Change in g with respect to μ and μ^*

Recall that the equilibrium growth rate is given by

$$g = \frac{\delta\nu[\gamma+\mu(1-\gamma)]\frac{\eta}{\gamma} - \rho}{(1+\nu)}.$$

Differentiating with respect to μ and μ^* yields, respectively

$$\frac{dg}{d\mu}=\frac{\nu}{1+\nu}\left\{\delta\eta\frac{\mu^*}{\mu^*\frac{\gamma}{1-\gamma}+\mu\frac{\eta}{1-\eta}\frac{1-\gamma}{\gamma}}\right\}>0,\frac{dg}{d\mu^*}=\frac{\nu}{1+\nu}\left\{\delta\eta\frac{\mu}{\mu^*\frac{\gamma}{1-\gamma}+\mu\frac{\eta}{1-\eta}\frac{1-\gamma}{\gamma}}\right\}>0.$$

9.4 Wage

9.4.1 Proof that $w > w^* = 1$

Equation (21) readily shows that its LHS is strictly increasing with $\lim_{w\to 0} LHS = \eta\tau_I/\tau_D^* < \lim_{w\to 0} RHS = +\infty$ and $\lim_{w\to+\infty} LHS = +\infty > \lim_{w\to+\infty} RHS = \gamma\eta\tau_I/[(1-\gamma)(1-\eta)\tau_D]$. Therefore, the solution for the level of wage is unique. Moreover, under the assumption $\tau_I/\tau_D > \tau_I/\tau_D^*$ and the property $\gamma > \eta \geq 1/2$, we can check

$$\lim_{w\to 1} LHS = \frac{\eta}{(1-\eta)}\left[\frac{\eta}{(1-\eta)}+\frac{\tau_I}{\tau_D^*}\right] < \lim_{w\to 1} RHS = \frac{\gamma}{1-\gamma}\left[\frac{\eta}{(1-\eta)}\frac{\tau_I}{\tau_D}+1\right].$$

i.e., the intersection between LHS and RHS in (21) necessarily occurs at $w > 1$.

9.4.2 Proof that $dw/d\eta > 0$

Manipulating (21), we obtain:

$$\frac{\eta}{1-\eta}w^2+\left(\frac{\tau_I}{\tau_D^*}-\frac{\gamma}{1-\gamma}\frac{\tau_I}{\tau_D}\right)w-\left(\frac{1-\eta}{\eta}\right)\frac{\gamma}{1-\gamma}=0\,. \tag{A.6}$$

Thus, the solution for the wage is given by

$$w=\frac{\frac{\gamma}{1-\gamma}\frac{\tau_I}{\tau_D}-\frac{\tau_I}{\tau_D^*}+\left[\left(\frac{\gamma}{1-\gamma}\frac{\tau_I}{\tau_D}-\frac{\tau_I}{\tau_D^*}\right)^2+\frac{4\gamma}{1-\gamma}\right]^{1/2}}{2\frac{\eta}{1-\eta}}\,. \tag{A.7}$$

Applying the implicit function theorem to (A.6), and substituting w by its value given by (A.7) in the denominator, we obtain

$$\frac{dw}{d\eta}=-\frac{\frac{\partial F}{\partial\eta}}{\left[\left(\frac{\gamma}{1-\gamma}\frac{\tau_I}{\tau_D}-\frac{\tau_I}{\tau_D^*}\right)^2+\frac{4\gamma}{1-\gamma}\right]^{1/2}},$$

where

$$\frac{\partial F}{\partial\eta}=-\frac{w}{(1-\gamma)^2}\frac{\tau_I}{\tau_D}\left[\frac{d\gamma}{d\eta}-\frac{(1-\gamma)^2}{(1-\eta)^2}\frac{\tau_D}{\tau_I}w\right]-\frac{(1-\eta)}{\eta}\frac{1}{(1-\gamma)^2}\left[\frac{d\gamma}{d\eta}-\frac{1-\gamma}{\eta}\frac{\gamma}{1-\eta}\right].$$

Note that, if $\frac{d\gamma}{d\eta} > \frac{1-\gamma}{\eta}\frac{\gamma}{1-\eta}$ and $\frac{(1-\gamma)^2}{(1-\eta)^2}\frac{\tau_D}{\tau_I}w < \frac{1-\gamma}{\eta}\frac{\gamma}{1-\eta}$, then $\frac{\partial F}{\partial\eta}$ would be negative.

Starting from the latter, where we substitute w by its value given by (A.7), after some manipulations, we obtain the following inequality

$$-4\frac{\gamma}{1-\gamma}\frac{\tau_I}{\tau_D}\frac{\tau_I}{\tau_D^*}+\frac{4\gamma}{1-\gamma} < 0,$$

which is always satisfied as $\tau_I > \tau_D^* > \tau_D$.

Next, let us show that $\frac{d\gamma}{d\eta} > \frac{1-\gamma}{\eta}\frac{\gamma}{1-\eta}$. Noting that we can write

$$\frac{d\gamma}{d\eta} = \frac{\left[1+\mu\frac{(1-\gamma)}{\gamma}+1+\mu^*\frac{\gamma}{(1-\gamma)}\right]\frac{1}{(1-\eta)}}{\frac{\mu}{\gamma^2}\frac{\eta}{(1-\eta)}+\frac{\mu^*}{(1-\gamma)^2}},$$

after some computations, we obtain

$$\frac{d\gamma}{d\eta} > \frac{1-\gamma}{\eta}\frac{\gamma}{1-\eta} \iff \frac{\eta}{[(1-\eta)\mu^*+\eta]} > \gamma.$$

To show that the above condition is always verified, let us recall that (A.4) is given by

$$(1-\eta)\left[1+\mu^*\frac{\gamma}{(1-\gamma)}\right] = \eta\left[1+\mu\frac{(1-\gamma)}{\gamma}\right].$$

Now let us define

$$LHS(\gamma) = (1-\eta)\left[1+\mu^*\frac{\gamma}{(1-\gamma)}\right];$$
$$RHS(\gamma) = \eta\left[1+\mu\frac{(1-\gamma)}{\gamma}\right].$$

We know that $LHS(\gamma)$ is strictly increasing and $RHS(\gamma)$ is strictly decreasing. Moreover, we can easily check that

$$RHS\left(\frac{\eta}{\mu^*(1-\eta)+\eta}\right) < LHS\left(\frac{\eta}{\mu^*(1-\eta)+\eta}\right).$$

As in equilibrium we must have $RHS(\gamma) = LHS(\gamma)$, this implies that $\gamma < \eta/[(1-\eta)\mu^* + \eta]$. That is, the intersection between $RHS(\gamma)$ and $LHS(\gamma)$ occurs for a value of γ such that: $\gamma < \frac{\eta}{[(1-\eta)\mu^*+\eta]}$. Therefore, $dw/d\eta > 0$.

9.4.3 Evaluating $\lim_{\eta\to 0} w$ *and* $\lim_{\eta\to 1} w$

In this section, we compute the level of wage in the limit cases where η tends to 0 or 1. Using (A.5) we can obtain

$$\lim_{\eta\to 0}\frac{d\gamma}{d\eta} = \mu < 1,$$

and

$$\lim_{\eta \to 1} \frac{d\gamma}{d\eta} = \mu^* < 1 .$$

Then, using the above results, along with l'Hôpital's rule applied to (A.7), we obtain

$$\lim_{\eta \to 0} w = \lim_{\eta \to 0} \frac{d\gamma}{d\eta} \left(\frac{\tau_I}{\tau_D^*} \right)^{-1} = \mu \frac{\tau_D^*}{\tau_I} < 1 ,$$

and

$$\lim_{\eta \to 1} w = \left(\frac{d\gamma}{d\eta} \right)^{-1} \frac{\tau_I}{\tau_D} = \frac{1}{\mu^*} \frac{\tau_I}{\tau_D} > 1 .$$

9.4.4 Change in w with respect to μ, μ^*, τ_I, τ_D, τ_D^*

Recall that the wage is implicitly given by (see (21))

$$\frac{\eta}{(1-\eta)} \left[\frac{\eta}{(1-\eta)} w + \frac{\tau_I}{\tau_D^*} \right] = \frac{\gamma}{1-\gamma} \left[\frac{\eta}{(1-\eta)} \frac{\tau_I}{\tau_D} + \frac{1}{w} \right].$$

Applying the implicit function theorem to the above, we obtain

$$\frac{dw}{d\mu} = \frac{\frac{1}{(1-\gamma)^2} \left[\frac{\eta}{(1-\eta)} \frac{\tau_I}{\tau_D} + \frac{1}{w} \right] \frac{d\gamma}{d\mu}}{\frac{\eta^2}{(1-\eta)^2} + \frac{\gamma}{1-\gamma} \frac{1}{w^2}} > 0 ,$$

$$\frac{dw}{d\mu^*} = \frac{\frac{1}{(1-\gamma)^2} \left[\frac{\eta}{(1-\eta)} \frac{\tau_I}{\tau_D} + \frac{1}{w} \right] \frac{d\gamma}{d\mu^*}}{\frac{\eta^2}{(1-\eta)^2} + \frac{\gamma}{1-\gamma} \frac{1}{w^2}} < 0 ,$$

$$\frac{dw}{d\tau_I} = -\frac{\frac{\eta}{(1-\eta)} \left(\frac{1}{\tau_D^*} - \frac{\gamma}{1-\gamma} \frac{1}{\tau_D} \right)}{\frac{\eta^2}{(1-\eta)^2} + \frac{\gamma}{1-\gamma} \frac{1}{w^2}} > 0 ,$$

$$\frac{dw}{d\tau_D} = -\frac{\frac{\gamma}{1-\gamma} \frac{\eta}{(1-\eta)} \frac{\tau_I}{(\tau_D)^2}}{\frac{\eta^2}{(1-\eta)^2} + \frac{\gamma}{1-\gamma} \frac{1}{w^2}} < 0 ,$$

$$\frac{dw}{d\tau_D^*} = \frac{\frac{\eta}{(1-\eta)} \frac{\tau_I}{(\tau_D^*)^2}}{\frac{\eta^2}{(1-\eta)^2} + \frac{\gamma}{1-\gamma} \frac{1}{w^2}} > 0 .$$

9.5 Proof of Proposition 4

Using (26), define the following function,

$$F(\underline{\eta}) = \frac{1}{\rho}\left[\log w + \gamma \log\left(\frac{\tau_I}{\tau_D}\right) + (1-\gamma)\log\left(\frac{\tau_D^*}{\tau_I}\right)\right] - m = 0.$$

Since μ is set higher than μ^* by default (and it needs to be fulfilled at all times), and both parameters are bounded from above, to study the effect of changes in spillovers we simply look at a decrease in μ^* for a given μ. This is as if spillovers are (in relative terms) more potent in the North. Thus, applying the implicit function theorem to $F(\bullet)$ above, we obtain,

$$\frac{d\underline{\eta}}{d\mu^*} = -\frac{\frac{1}{w}\frac{\partial w}{\partial \mu^*} + \frac{\partial \gamma}{\partial \mu^*}\log\frac{\tau_I}{\tau_D}\frac{\tau_I}{\tau_D^*}}{\frac{1}{w}\frac{\partial w}{\partial \underline{\eta}} + \frac{\partial \gamma}{\partial \underline{\eta}}\log\left(\frac{\tau_I}{\tau_D}\frac{\tau_I}{\tau_D^*}\right)} > 0\,.$$

The denominator of $\frac{d\underline{\eta}}{d\mu^*}$ is positive while the numerator is negative and, overall, the effect is positive.

9.6 Proof of Proposition 6

Apply the implicit function theorem to (21) to obtain $\frac{dw}{d\gamma} = \frac{\frac{1}{(1-\gamma)^2}\left[\frac{\eta}{(1-\eta)}\frac{\tau_I}{\tau_D} + \frac{1}{w}\right]}{\left(\frac{\eta}{1-\eta}\right)^2 + \frac{\gamma}{1-\gamma}\frac{1}{w^2}} > 0$. Since $\frac{d\gamma}{d\mu} > 0$ and $\frac{d\gamma}{d\mu^*} < 0$, then w is positively correlated with μ and negatively correlated with μ^*. Furthermore, since $\frac{d\gamma}{d\eta} > 0$ it follows that w is also positively correlated with η.

9.7 Welfare

Using (6), (14), $E = w$ and $E^* = w^* = 1$, in steady state, the individual lifetime utility in the North reads

$$\rho U = \log\left[w^{1-\gamma}\frac{(A_{q_j,0})^\nu (Q-1)}{NQ}\left(\frac{1}{\tau_D}\right)^\gamma \left(\frac{1}{\tau_I}\right)^{1-\gamma}\right] + \nu\frac{g}{\rho}\,,$$

Similarly, for the South, we have

$$\rho U^* = \log\left[w^{-\gamma}\frac{(A_{q_j,0})^\nu (Q-1)}{NQ}\left(\frac{1}{\tau_I}\right)^\gamma \left(\frac{1}{\tau_D^*}\right)^{1-\gamma}\right] + \nu\frac{g}{\rho}\,.$$

Differentiating the above expressions with respect to μ^* yields (30) and (31) in the main text.

9.7.1 Proof that $\frac{dU^*}{d\mu^*} > 0$

$$\frac{dU^*}{d\mu^*} = \frac{1}{\rho}\left\{\frac{1/Q^2}{1-1/Q}\frac{dQ}{d\mu^*} + \frac{1-\gamma}{w}\frac{dw}{d\mu^*} - \frac{d\gamma}{d\mu^*}\log w + \frac{d\gamma}{d\mu^*}\left(\log\frac{1}{\tau_I} - \log\frac{1}{\tau_D^*}\right) + \frac{\nu}{\rho}\frac{dg}{d\mu^*}\right\},$$

where $1 < \tau_D^* < \tau_I$, $Q = \frac{(1+\nu)[\gamma+\mu(1-\gamma)]^{\frac{\delta\eta}{\gamma}}}{\nu[\gamma+\mu(1-\gamma)]^{\frac{\delta\eta}{\gamma}}-\rho} > 1$, $\frac{dQ}{d\mu^*} = \frac{\partial Q}{\partial\gamma}\frac{d\gamma}{d\mu^*} < 0$, $\frac{dw}{d\mu^*} = \frac{\frac{1}{(1-\gamma)^2}\left[\frac{\eta}{(1-\eta)}\frac{\tau_I}{\tau_D}+\frac{1}{w}\right]\frac{d\gamma}{d\mu^*}}{\frac{\eta^2}{(1-\eta)^2}+\frac{\gamma}{1-\gamma}\frac{1}{w^2}} < 0$, $\frac{d\gamma}{d\mu^*} < 0$ and $\frac{dg}{d\mu^*} = -\frac{g}{Q}\frac{dQ}{d\mu^*} - \delta\eta\frac{1}{Q}\frac{\mu}{\gamma^2}\frac{d\gamma}{d\mu^*} > 0$. Therefore, for $\frac{dU^*}{d\mu^*} > 0$ to hold it suffices to show that $\frac{1/Q^2}{1-1/Q}\frac{dQ}{d\mu^*} - \frac{g}{Q}\frac{dQ}{d\mu^*} > 0$, that is the dynamic competition effect outweighs the static competition effect. Since $\frac{dQ}{d\mu^*} < 0$, we need to show that

$$\frac{1}{Q-1} < \frac{\nu}{\rho}g.$$

Substituting for $\frac{1}{Q-1} = \frac{\nu[\gamma+\mu(1-\gamma)]\delta\eta-\gamma\rho}{[\gamma+\mu(1-\gamma)]\delta\eta+\gamma\rho}$ and $g = \frac{\nu[\gamma+\mu(1-\gamma)]^{\frac{\delta\eta}{\gamma}}-\rho}{(1+\nu)}$, we obtain

$$\nu > \left(\frac{\rho(1+\nu)\gamma}{\nu[\gamma+\mu(1-\gamma)]\delta\eta-\rho\gamma}\right)\left(\frac{\nu[\gamma+\mu(1-\gamma)]\delta\eta-\gamma\rho}{[\gamma+\mu(1-\gamma)]\delta\eta+\gamma\rho}\right).$$

Simplifying yields,

$$\nu > \frac{\gamma\rho}{[\gamma+\mu(1-\gamma)]\delta\eta}.$$

This is always satisfied under assumption 1.

9.7.2 Proof that $\frac{dU}{d\mu^*} > 0$

Assuming negligible transport costs ($\tau_D = \tau_D^* = \tau_I \rightarrow 1$) we have

$$\frac{dU}{d\mu^*} = \frac{1}{\rho}\left[\frac{1/Q^2}{1-1/Q}\frac{\partial Q}{\partial\gamma}\frac{d\gamma}{d\mu^*} + \frac{1-\gamma}{w}\frac{dw}{d\mu^*} - \frac{d\gamma}{d\mu^*}\log w + \frac{\nu}{\rho}\frac{dg}{d\mu^*}\right],$$

where $\frac{dg}{d\mu^*} = -\frac{g}{Q}\frac{dQ}{d\mu^*} - \delta\eta\frac{1}{Q}\frac{\mu}{\gamma^2}\frac{d\gamma}{d\mu^*}$. Under assumption 1 (cf. proof above) we know that $\frac{1/Q^2}{1-1/Q}\frac{dQ}{d\mu^*} - \frac{g}{Q}\frac{dQ}{d\mu^*} > 0$. Hence for $\frac{dU}{d\mu^*} > 0$ to hold it suffices to show that $\frac{1-\gamma}{w}\frac{dw}{d\mu^*} \leq \log w\frac{d\gamma}{d\mu^*} + \frac{\nu}{\rho}\delta\eta\frac{1}{Q}\frac{\mu}{\gamma^2}\frac{d\gamma}{d\mu^*}$, that is the gains associated with the re-location of sectors outweighs the losses associated with the decrease in the relative wage.

Under negligible transport costs the wage equation (21) implies $w = \frac{\left(\frac{2\gamma-1}{1-\gamma}\right)+\left[\frac{4\gamma^2+1-4\gamma}{(1-\gamma)^2}+\frac{4\gamma}{1-\gamma}\right]^{1/2}}{2\frac{\eta}{1-\eta}} = \frac{\gamma}{1-\gamma}\frac{1-\eta}{\eta} > 1$ and $\frac{dw}{d\mu^*} = \frac{1}{(1-\gamma)^2}\frac{1-\eta}{\eta}\frac{d\gamma}{d\mu^*}$, hence we need to show that

$$\frac{1}{\gamma} - 1 + \frac{1-\gamma}{\gamma}\frac{\eta}{1-\eta} - \frac{\nu}{\rho}\delta\eta\frac{1}{Q}\frac{\mu}{\gamma^2} \leq 0,$$

which simplifies to

$$\frac{1-\gamma}{1-\eta} \leq \frac{\nu}{\rho}\delta\eta\frac{1}{Q}\frac{\mu}{\gamma}.$$

Replacing Q and simplifying

$$\frac{c}{d}\frac{1-\gamma}{1-\eta}\frac{(1+\nu)[\gamma+\mu(1-\gamma)]}{\nu[\gamma+\mu(1-\gamma)]\frac{\delta\eta}{\gamma}-\rho}\mu \Longleftrightarrow \frac{\gamma\rho(1-\gamma)(1+\nu)[\gamma+\mu(1-\gamma)]}{\nu(1-\eta)\mu\{\nu[\gamma+\mu(1-\gamma)]\delta\eta-\gamma\rho\}} \leq 1.$$

Denote by $\hat{\nu}$ the solution of the implicit function $\frac{\gamma\rho(1-\gamma)(1+\hat{\nu})[\gamma+\mu(1-\gamma)]}{\hat{\nu}(1-\eta)\mu\{\hat{\nu}[\gamma+\mu(1-\gamma)]\delta\eta-\gamma\rho\}} = 1$; then $\frac{dU}{d\mu^*} > 0$for any $\nu \leq \hat{\nu}$.

9.8 Asymmetries in productivity

Here we postulate $\delta > \delta^*$ (i.e., R&D productivity in the North is greater than in the South) and $\mu = \mu^* < 1$. In this case, the condition equalising growth rates ($g = g^*$) reads as

$$\delta\eta\left[1+\frac{\mu(1-\gamma)}{\gamma}\right] = \delta^*(1-\eta)\left[1+\frac{\gamma\mu}{(1-\gamma)}\right].$$

Accordingly, the location decision of sectors depends on R&D productivity.

It can be easily checked that the left hand side (hereafter LHS) is strictly decreasing, with $\lim_{\gamma\to 0} LHS = +\infty$ and $\lim_{\gamma\to 1} LHS = \delta\eta$. Similarly, the right hand side (hereafter, RHS) is strictly increasing, with $\lim_{\gamma\to 0} RHS = \delta^*(1-\eta)$ and $\lim_{\gamma\to 1} RHS = +\infty$. Under the assumption $0 \leq \delta\eta \leq 1$, it follows that there exists a unique solution for γ verifying $0 \leq \gamma \leq 1$. Notice, also, that $\delta\eta < 1$ is not stringent as $\eta \leq 1$ and we can expect δ to be much lower than 1.

Note that when $\eta = 1/2$, the equation above can be written as

$$\delta^*\mu\left(\frac{\gamma}{1-\gamma}\right)^2 - (\delta-\delta^*)\left(\frac{\gamma}{1-\gamma}\right) - \delta\mu = 0.$$

Solving for $\frac{\gamma}{1-\gamma}$ we obtain

$$\frac{\gamma}{1-\gamma} = \frac{(\delta-\delta^*)+\sqrt{(\delta-\delta^*)^2+4\delta^*\mu(\delta-\delta^*)}}{2\delta^*\mu}.$$

Therefore when $\eta = 1/2, \gamma = X$ where

$$X = \frac{(\delta-\delta^*)+\sqrt{(\delta-\delta^*)^2+4\delta^*\mu(\delta-\delta^*)}}{2\delta^*\mu+(\delta-\delta^*)+\sqrt{(\delta-\delta^*)^2+4\delta^*\mu(\delta-\delta^*)}}.$$

Furthermore, $X > 1/2$ if, after some simplifications,$(\delta-\delta^*)+\sqrt{(\delta-\delta^*)^2+4\delta^*\mu(\delta-\delta^*)} > 2\delta^*\mu$.

Since $\delta > \delta^*$, we can as well show that $(\delta-\delta^*)+\sqrt{(\delta-\delta^*)^2+4\delta^*\mu(\delta-\delta^*)} > \delta^*\mu+\delta\mu$. Simplifying, the latter condition becomes $\delta(1-\mu)-\delta^*(1-\mu)+\sqrt{(\delta-\delta^*)^2+4\delta^*\mu(\delta-\delta^*)} > 0$, which is always satisfied. This proves that $\gamma > 1/2$ when $\eta = 1/2$ as in the case analysed in the main section of the paper.

Finally, by applying the implicit function theorem, it can be easily checked that $\frac{d\gamma}{d\delta^*} < 0$. Therefore, the properties of the model described for μ^* apply to δ^*.

9.9 Intra-regional equilibrium under heterogeneity and population growth

The problem of individuals remains the same as in the main text. For firms, the current value Hamiltonian becomes

$$CVH_{q_j,t} = \left\{ \begin{array}{c} X_{q_j,t}[\Omega_t(\sum_{q_j=1}^{Q_{j,t}} X_{q_j,t})^{-1} - w_t(A_{q_j,t})^{-\nu}] - w_t L^A_{q_j,t} \\ +\xi_t \delta L^A_{q_j,t} K_t \end{array} \right\}.$$

The first order conditions are given by: $\partial CVH_{q_j,t}/\partial X_{q_j,t} = 0$, $\partial CVH_{q_j,t}/\partial L^A_{q_j,t} = 0$ and $\partial CVH_{q_j,t}/\partial A_{q_j,t} = -\xi_t^{\bullet} + r_t \xi_t$. The transversality condition is $lim_{t\to\infty}\, \xi_t A_{q_j,t} e^{-\int_0^t r_u du} = 0$. After simple manipulations, the price of variety j is given by

$$p_{j,t} = \frac{w_t(A_{q_j,t})^{-\nu}}{(1 - X_{q_j,t}/X_{j,t})}. \tag{A.8}$$

The zero profit condition reads as

$$\frac{w_t L^X_{q_j,t}}{(1 - X_{q_j,t}/X_{j,t})} - w_t L^X_{q_j,t} - w_t L^A_{q_j,t} \leq 0. \tag{A.9}$$

The rate of return to R&D is now given by

$$r_t = \delta \nu L^X_{q_j,t} \frac{\tilde{A}_t}{(A_{q_j,t})} + \frac{\xi_t^{\bullet}}{\xi_t}. \tag{A.10}$$

Equations giving the relation between the amounts of labour allocated to the production of varieties, L^X_t, to R&D, L^A_t, and the number of firms in each sector, Q, are respectively

$$\frac{QL^X_t}{(Q-1)} - L^X_t - L^A_t = 0,$$
$$L^X_t = \eta L^w_t \left(1 - \frac{1}{Q}\right),$$

and

$$L^A_t = \eta L^w_t \frac{1}{Q}.$$

Then, we can compute the number of sectors located in the North, γ, the number of firms in each sector, Q, and the steady-state growth rate, g, respectively

$$\eta \left[1 + \mu \frac{(1-\gamma)}{\gamma} \frac{1}{\overline{A}_t/\overline{A}^*_t}\right] = (1-\eta)\left[1 + \mu^* \frac{\gamma}{(1-\gamma)} \overline{A}_t/\overline{A}^*_t\right], \tag{A.11}$$

$$Q = \frac{(1+\nu)\delta \frac{\eta L^w_t}{\gamma N_t}\left[\gamma + \mu(1-\gamma)\frac{1}{\overline{A}_t/\overline{A}^*_t}\right]}{\delta\nu \frac{\eta L^w_t}{\gamma N_t}\left[\gamma + \mu(1-\gamma)\frac{1}{\overline{A}_t/\overline{A}^*_t}\right] - (\rho - \omega)}, \tag{A.12}$$

$$g = \frac{1}{1+\nu}\left\{\nu\frac{\eta\delta L_t^w}{\gamma N_t}\left[\gamma + \mu(1-\gamma)\frac{1}{\overline{A}_t/\overline{A}_t^*}\right] - (\rho - \omega)\right\}. \tag{A.13}$$

Finally, the wage in the North and South verify

$$w_t = \frac{\gamma}{L_t^w}\left[\frac{E_t}{\tau_D} + \frac{(1-\eta)}{\eta}\frac{E^*}{\tau_I}\right],$$

and

$$w_t^* = \frac{(1-\gamma)}{L_t^w}\left[\frac{\eta}{(1-\eta)}\frac{E}{\tau_I} + \frac{E^*}{\tau_D^*}\right],$$

implying that w is solution of the following equation

$$\frac{\eta}{(1-\eta)}\left[\frac{\eta}{(1-\eta)}w + \frac{\tau_I}{\tau_D^*}\right] = \frac{\gamma}{1-\gamma}\left[\frac{\eta}{(1-\eta)}\frac{\tau_I}{\tau_D} + \frac{1}{w}\right]. \tag{A.14}$$

It is worth emphasising here that the equation is exactly the same as in the main text, although we should keep in mind that γ now depends on the relative average amount of knowledge between regions, $\overline{A}_t/\overline{A}_t^* \equiv \Theta$. Equally, the comparative statics identified in section 9.3 retain the same signs, as shown below.

9.9.1 Comparative statics

Applying the implicit function theorem to (A.11), we obtain

$$\frac{d\gamma}{d\mu} = \frac{\frac{(1-\gamma)}{\gamma}\frac{\eta}{(1-\eta)}\frac{1}{\Theta}}{\frac{\mu}{\gamma^2}\frac{\eta}{(1-\eta)}\frac{1}{\Theta} + \frac{\mu^*}{(1-\gamma)^2}\Theta} > 0,$$

$$\frac{d\gamma}{d\mu^*} = -\frac{\frac{\gamma}{1-\gamma}\Theta}{\frac{\mu}{\gamma^2}\frac{\eta}{(1-\eta)}\frac{1}{\Theta} + \frac{\mu^*}{(1-\gamma)^2}\Theta} < 0,$$

$$\frac{d\gamma}{d\Theta} = -\frac{\mu\frac{(1-\gamma)}{\gamma}\frac{\eta}{(1-\eta)}\frac{1}{\Theta^2} + \mu^*\frac{\gamma}{1-\gamma}}{\frac{\mu}{\gamma^2}\frac{\eta}{(1-\eta)}\frac{1}{\Theta} + \frac{\mu^*}{(1-\gamma)^2}\Theta} = -\frac{\gamma(1-\gamma)}{\Theta} < 0.$$

Note that $\frac{d\gamma}{d\Theta} < 0$ is consistent with the assumption of decreasing returns to innovation, implying that an increase in average knowledge in the North relative to the South makes the advanced region relative less productive at producing new knowledge and, thereby, a relatively less attractive location for firms. This also implies that, $\gamma > \eta$ might not be satisfied if the difference in average knowledge between regions is large enough. To rule out such a case, we impose the following restriction: $\frac{-\left[1-2\eta+2\eta\mu\frac{1}{\Theta}\right]+\sqrt{\left[1-2\eta+2\mu\eta\frac{1}{\Theta}\right]^2+4\mu\eta\frac{1}{\Theta}\left[(1-\eta)\mu^*\Theta-(1-2\eta)-\eta\mu\frac{1}{\Theta}\right]}}{2\left[(1-\eta)\mu^*\Theta-(1-2\eta)-\eta\mu\frac{1}{\Theta}\right]} > \eta$, which is fulfilled for Θ not too large.

Differentiating $\frac{d\gamma}{d\mu^*}$ above with respect to Θ, after some manipulations, gives

$$\frac{d^2\gamma}{d\mu^* d\Theta} = \frac{\frac{\gamma}{1-\gamma}\Theta}{\frac{\mu}{\gamma^2}\frac{\eta}{(1-\eta)}\frac{1}{\Theta} + \frac{\mu^*}{(1-\gamma)^2}\Theta}\frac{2\gamma-1}{\gamma(1-\gamma)}\frac{d\gamma}{d\Theta} = \frac{d\gamma}{d\mu^*}\frac{(2\gamma-1)}{\Theta} < 0.$$

Moving on to the effect on the equilibrium number of firms, recall that a change in Q with respect to any parameter x is given by $\frac{dQ}{dx} = \frac{\partial Q}{\partial x} + \frac{\partial Q}{\partial \gamma}\frac{d\gamma}{dx}$. From (A.12) it readily follows that $\frac{\partial Q}{\partial \gamma} = \frac{(1+\nu)\delta\frac{\eta L_t^w}{N_t}\mu\frac{1}{\gamma^2}\frac{1}{\Theta}(\rho-\omega)}{\left[\delta\nu\frac{\eta L_t^w}{N_t}\left[1+\mu\frac{(1-\gamma)}{\gamma}\frac{1}{\Theta}\right]-(\rho-\omega)\right]^2} > 0$ and $\frac{\partial Q}{\partial \mu} = -\frac{(1+\nu)\delta\frac{\eta L_t^w}{N_t}\left[\frac{(1-\gamma)}{\gamma}\frac{1}{\Theta}\right](\rho-\omega)}{\left[\delta\nu\frac{\eta L_t^w}{N_t}\left[1+\mu\frac{(1-\gamma)}{\gamma}\frac{1}{\Theta}\right]-(\rho-\omega)\right]^2} < 0.$ Accordingly, after replacing $\frac{d\gamma}{d\mu}$ and rearranging terms, we obtain

$$\frac{dQ}{d\mu} = \frac{\partial Q}{\partial \mu} + \frac{\partial Q}{\partial \gamma}\frac{d\gamma}{d\mu} = -\frac{(1+\nu)\delta\frac{\eta L_t^w}{N_t}\frac{(1-\gamma)}{\gamma}(\rho-\omega)\frac{1}{\Theta}}{\left[\delta\nu\frac{\eta L_t^{\tilde{w}}}{N_t}\left[1+\mu\frac{(1-\gamma)}{\gamma}\frac{1}{\Theta}\right]-(\rho-\omega)\right]^2}\left[\frac{(1-\eta)\mu^*\frac{\gamma^2}{(1-\gamma)^2}\Theta}{\eta\mu\frac{1}{\Theta}+(1-\eta)\mu^*\frac{\gamma^2}{(1-\gamma)^2}\Theta}\right] < 0,$$

$$\frac{dQ}{d\mu^*} = \frac{\partial Q}{\partial \gamma}\frac{d\gamma}{d\mu^*} = \frac{(1+\nu)\delta\frac{\eta L_t^w}{N_t}\mu\frac{1}{\gamma^2}\frac{1}{\Theta}(\rho-\omega)}{\left[\delta\nu\frac{\eta L_t^w}{N_t}\left[1+\mu\frac{(1-\gamma)}{\gamma}\frac{1}{\Theta}\right]-(\rho-\omega)\right]^2}\frac{d\gamma}{d\mu^*} < 0,$$

and

$$\frac{dQ}{d\Theta} = \frac{(1+\nu)\delta\frac{\eta L_t^w}{\gamma N_t}\left(\frac{\mu}{\gamma}\right)\frac{1}{\Theta}(\rho-\omega)}{\left[\delta\nu\frac{\eta L_t^w}{\gamma N_t}\left[\gamma+\mu(1-\gamma)\frac{1}{\Theta}\right]-(\rho-\omega)\right]^2}\left[\frac{\gamma(1-\gamma)}{\Theta}+\frac{d\gamma}{d\Theta}\right] = 0,$$

since $\frac{d\gamma}{d\Theta} = -\frac{\gamma(1-\gamma)}{\Theta}$. Note that, firms' heterogeneity has no impact on the number of firms within each sector. Two effects cancel each other out: The positive direct effect from the change in relative average knowledge between regions (captured by the term in Θ) is offset by the negative indirect effect coming from the relocation of sectors (captured by $\frac{d\gamma}{d\Theta}$). Differentiating $\frac{dQ}{d\mu^*}$ above with respect to Θ, after some manipulations, gives

$$\frac{d^2Q}{d\mu^* d\Theta} = \frac{(1+\nu)\delta\frac{\eta L_t^w}{\gamma N_t}\left(\frac{\mu}{\gamma}\right)\frac{1}{\Theta}(\rho-\omega)}{\left[\delta\nu\frac{\eta L_t^w}{\gamma N_t}\left[\gamma+\mu(1-\gamma)\frac{1}{\Theta}\right]-(\rho-\omega)\right]^2}\left[-\frac{1}{\Theta}\frac{d\gamma}{d\mu^*}-\frac{2}{\gamma}\frac{d\gamma}{d\mu^*}\frac{d\gamma}{d\Theta}+\frac{d^2\gamma}{d\mu^* d\Theta}\right] = 0,$$

where we have used $\frac{d\gamma}{d\Theta} = -\frac{\gamma(1-\gamma)}{\Theta}$ and $\frac{d^2\gamma}{d\mu^* d\Theta} = \frac{d\gamma}{d\mu^*}\frac{(2\gamma-1)}{\Theta}$.$$

Similarly, differentiating the growth equation (A.13), we obtain

$$\frac{dg}{d\mu} = \frac{\nu}{\nu+1}\left[\frac{\delta\eta L_t^w}{N_t}\frac{\mu^*}{\mu^*\frac{\gamma}{1-\gamma}\Theta+\mu\frac{\eta}{1-\eta}\frac{1-\gamma}{\gamma}\frac{1}{\Theta}}\right] > 0,$$

$$\frac{dg}{d\mu^*} = -\frac{\nu}{\nu+1}\left[\delta\frac{\eta L_t^w}{N_t}\frac{1}{\Theta}\frac{\mu}{\gamma^2}\frac{d\gamma}{d\mu^*}\right] > 0,$$

$$\frac{dg}{d\Theta} = -\frac{\nu}{\nu+1}\delta\frac{\eta L_t^w}{N_t}\frac{1}{\Theta}\frac{\mu}{\gamma^2}\left[\frac{\gamma(1-\gamma)}{\Theta}+\frac{d\gamma}{d\Theta}\right] = 0.$$

And, differentiating $\frac{dg}{d\mu^*}$ above with respect to Θ, we obtain:

$$\frac{d^2g}{d\mu^* d\Theta} = -\frac{\nu}{\nu+1}\delta\frac{\eta L_t^w}{\gamma N_t}\left(\frac{\mu}{\gamma}\right)\frac{1}{\Theta}\left[-\frac{1}{\Theta}\frac{d\gamma}{d\mu^*} - \frac{2}{\gamma}\frac{d\gamma}{d\Theta}\frac{d\gamma}{d\mu^*} + \frac{d^2\gamma}{d\mu^* d\Theta}\right] = 0\,.$$

9.9.2 Effects on ΔU

To assess whether a change in Θ mitigates or reinforces the effect of μ^*on the utility differential, ΔU, we need first to look at the effect on the relative wage between regions. Applying the implicit function theorem to (A.14), we obtain

$$\frac{dw}{d\mu} = \frac{\frac{1}{(1-\gamma)^2}\left[\frac{\eta}{(1-\eta)}\frac{\tau_I}{\tau_D} + \frac{1}{w}\right]\frac{d\gamma}{d\mu}}{\frac{\eta^2}{(1-\eta)^2} + \frac{\gamma}{1-\gamma}\frac{1}{w^2}} > 0,$$

$$\frac{dw}{d\mu^*} = \frac{\frac{1}{(1-\gamma)^2}\left[\frac{\eta}{(1-\eta)}\frac{\tau_I}{\tau_D} + \frac{1}{w}\right]\frac{d\gamma}{d\mu^*}}{\frac{\eta^2}{(1-\eta)^2} + \frac{\gamma}{1-\gamma}\frac{1}{w^2}} < 0,$$

$$\frac{dw}{d\Theta} = \frac{\left(\frac{\eta}{1-\eta}\frac{\tau_I}{\tau_D} + \frac{1}{w}\right)\frac{1}{(1-\gamma)^2}\frac{d\gamma}{d\Theta}}{\frac{\eta^2}{(1-\eta)^2} + \frac{\gamma}{1-\gamma}\frac{1}{w^2}} = -\frac{w}{\Theta}\frac{\frac{\gamma}{w(1-\gamma)}\left(\frac{\eta}{1-\eta}\frac{\tau_I}{\tau_D} + \frac{1}{w}\right)}{\frac{\eta^2}{(1-\eta)^2} + \frac{\gamma}{1-\gamma}\frac{1}{w^2}} < 0.$$

Differentiating $\frac{dw}{d\mu^*}$ above with respect to Θ, yields:

$$\frac{d^2w}{d\mu^* d\Theta} = \frac{d\gamma}{d\mu^*}\left\{\begin{array}{l} \frac{\left(\frac{\eta}{1-\eta}\frac{\tau_I}{\tau_D} + \frac{1}{w}\right)\left[\frac{2}{(1-\gamma)^3}\left(\frac{\eta}{1-\eta}\right)^2 + \frac{2\gamma}{(1-\gamma)^4 w^2} - \frac{1}{(1-\gamma)^4 w^2}\right]}{\left[\left(\frac{\eta}{1-\eta}\right)^2 + \frac{\gamma}{1-\gamma}\frac{1}{w^2}\right]^2}\frac{d\gamma}{d\Theta} \\ + \frac{\frac{2\gamma}{(1-\gamma)^3 w^3}\left(\frac{\eta}{1-\eta}\frac{\tau_I}{\tau_D} + \frac{1}{w}\right) - \frac{1}{(1-\gamma)^2 w^2}\left[\left(\frac{\eta}{1-\eta}\right)^2 + \frac{\gamma}{1-\gamma}\frac{1}{w^2}\right]}{\left[\left(\frac{\eta}{1-\eta}\right)^2 + \frac{\gamma}{1-\gamma}\frac{1}{w^2}\right]^2}\frac{dw}{d\Theta} \end{array}\right\}$$

$$+ \frac{\frac{1}{(1-\gamma)^2}\left(\frac{\eta}{1-\eta}\frac{\tau_I}{\tau_D} + \frac{1}{w}\right)}{\left(\frac{\eta}{1-\eta}\right)^2 + \frac{\gamma}{1-\gamma}\frac{1}{w^2}}\frac{d^2\gamma}{d\mu^* d\Theta}.$$

Substituting $\frac{d^2\gamma}{d\mu^* d\Theta}$, $\frac{dw}{d\Theta}$, $\frac{d\gamma}{d\Theta}$ and $\frac{d\gamma}{d\mu^*}$ derived earlier, and simplifying, we obtain

$$\frac{d^2 w}{d\mu^* d\Theta} = -\frac{1}{\Theta}\frac{\left(\frac{\eta}{1-\eta}\right)^4 + \frac{2\gamma^2}{(1-\gamma)^2 w^3}\frac{\eta}{1-\eta}\frac{\tau_I}{\tau_D} + \frac{\gamma^2}{(1-\gamma)^2 w^4}}{\left[\left(\frac{\eta}{1-\eta}\right)^2 + \frac{\gamma}{1-\gamma}\frac{1}{w^2}\right]^2}\frac{dw}{d\mu^*} > 0.$$

Differentiating (28) with respect to Θ yields

$$\frac{d^2\Delta U}{d\mu^* d\Theta} = \frac{1}{\rho}\left[-\frac{1}{w^2}\frac{dw}{d\Theta}\frac{dw}{d\mu^*} + \frac{1}{w}\frac{d^2 w}{d\mu^* d\Theta} + \frac{d^2\gamma}{d\mu^* d\Theta}\log\left(\frac{\tau_I}{\tau_D}\frac{\tau_I}{\tau_D^*}\right)\right].$$

Substituting $\frac{d^2 w}{d\mu^* d\Theta}$, $\frac{dw}{d\Theta}$ and $\frac{dw}{d\mu^*}$ derived earlier, and simplifying, we obtain

$$\frac{d^2\Delta U}{d\mu^* d\Theta} = \frac{1}{\rho}\left\{-\frac{\frac{1}{\Theta}\frac{1}{w}\left(\frac{\eta}{1-\eta}\right)\frac{dw}{d\mu^*}\left[\frac{\gamma}{(1-\gamma)w^2} - \left(\frac{\eta}{1-\eta}\right)^2\right]\left[\frac{\gamma}{w(1-\gamma)}\frac{\tau_I}{\tau_D} - \frac{\eta}{1-\eta}\right]}{\left[\left(\frac{\eta}{1-\eta}\right)^2 + \frac{\gamma}{1-\gamma}\frac{1}{w^2}\right]^2} + \frac{d^2\gamma}{d\mu^* d\Theta}\log\left(\frac{\tau_I}{\tau_D}\frac{\tau_I}{\tau_D^*}\right)\right\}.$$

We know that $\frac{d^2\gamma}{d\mu^* d\Theta} < 0$ and $\frac{dw}{d\mu^*} < 0$. Moreover, we can easily show that $\frac{\gamma}{(1-\gamma)w^2} - \left(\frac{\eta}{1-\eta}\right)^2 < 0$ and $\frac{\gamma}{w(1-\gamma)}\frac{\tau_I}{\tau_D} - \left(\frac{\eta}{1-\eta}\right) > 0$. First, recall that $\frac{\gamma}{(1-\gamma)} > \frac{\eta}{1-\eta}$ because $\gamma > \eta > 1/2$. Moreover, we have: $\lim_{\tau_D = \tau_{D^*} = \tau_I} w = \frac{\gamma}{1-\gamma}\frac{1-\eta}{\eta} > 1$ and $\frac{dw}{d\tau_{D^*}} > 0$ and $\frac{dw}{d\tau_I} > 0$ implying that $w \geq \frac{\gamma}{1-\gamma}\frac{1-\eta}{\eta}$. Keeping these information in mind, note that $\frac{\gamma}{(1-\gamma)w^2} - \left(\frac{\eta}{1-\eta}\right)^2 < 0$ is equivalent to $\frac{1}{w^2} - \frac{1-\gamma}{\gamma}\left(\frac{\eta}{1-\eta}\right)^2 < 0$ which is always verified. Next, let us show that $\frac{\gamma}{w(1-\gamma)}\frac{\tau_I}{\tau_D} - \frac{\eta}{1-\eta} > 0$. Recall that the wage is implicitly given by (see A.14): $\frac{\eta}{(1-\eta)} + \frac{1}{w}\frac{\tau_I}{\tau_D^*} - \frac{1}{w}\frac{\gamma}{1-\gamma}\frac{\tau_I}{\tau_D} - \frac{\gamma}{1-\gamma}\frac{1-\eta}{\eta}\frac{1}{w^2} = 0 \Rightarrow \frac{1}{w}\frac{\gamma}{1-\gamma}\frac{\tau_I}{\tau_D} - \frac{\eta}{(1-\eta)} = \frac{1}{w}\frac{\tau_I}{\tau_D^*} - \frac{\gamma}{1-\gamma}\frac{1-\eta}{\eta}\frac{1}{w^2}$. Plugging the latter into the inequality $\left(\frac{\gamma}{w(1-\gamma)}\frac{\tau_I}{\tau_D} - \frac{\eta}{1-\eta} > 0\right)$ we obtain $\frac{1}{w}\frac{\tau_I}{\tau_D^*} - \frac{\gamma}{1-\gamma}\frac{1-\eta}{\eta}\frac{1}{w^2} > 0 \iff w\frac{\tau_I}{\tau_D^*} > \frac{\gamma}{1-\gamma}\frac{1-\eta}{\eta}$. Given that the latter is always verified, we can conclude that

$$\frac{d^2\Delta U}{d\mu^* d\Theta} < 0.$$

9.9.3 Demographic shocks

As Peretto (2003), we study the effects of demographic shocks on growth and market structure. I.e. we analyse how variables vary when the population growth rate, ω, varies.

Impact on γ

Inspection of (A.11) implies that any demographic shock (i.e. a change in population growth ω) has no impact on γ. Given that γ depends on η, this result is of course due to the fact that population growth is common to both regions.

Impact on Q

From equation (A.12), we have

$$\frac{dQ}{d\omega} = d\frac{\left\{\dfrac{(\nu+1)\delta\eta\frac{L_t^w}{N_t}\left[1+\mu\frac{(1-\gamma)}{\gamma}\frac{(\overline{A}_t^*)}{\overline{A}_t}\right]}{\delta\nu\eta\frac{L_t^w}{N_t}\left[1+\mu\frac{(1-\gamma)}{\gamma}\frac{(\overline{A}_t^*)}{\overline{A}_t}\right]-(\rho-\omega)}\right\}}{d\omega} < 0.$$

A greater population growth leads to a reduction of firms in each sector, i.e. less competition.

Impact on L_t^A/L_t^w
As $L_t^A = \eta L_t^w \frac{1}{Q}$, we obviously have $\frac{dL_t^A/L_t^w}{d\omega} > 0$.
As population growth increases, firms allocate a greater share of their workforce to R&D.

Impact on g
From equation (A.13), we obtain $\frac{dg}{d\omega} > 0$. A greater population growth leads to a higher level of growth.

Spatial equilibrium
Note that wage does not depend on population growth: $dw/d\omega = 0$. Therefore, from (A.14), we directly conclude that there is no impact of a demographic shock on the spatial equilibrium.

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