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Strong coupling expansion of $\frac{1}{2}$ BPS Wilson loop in SYM theory and 2-loop Green-Schwarz string in $AdS_5 \times S^5$

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ABSTRACT

The exact localization result for the expectation value of the $\frac{1}{2}$ BPS circular Wilson loop in $\mathcal{N}=4$ SYM theory is given in the planar limit by the famous Bessel function expression: $\langle W\rangle = \frac{2N}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$. Expanded in large λ and expressed in terms of the $AdS_5\times S^5$ string tension $T=\frac{\sqrt{\lambda}}{2\pi}$ this gives $\langle W\rangle = \frac{\sqrt{T}}{2\pi g_s} e^{2\pi T}(1-\frac{3}{16\pi}T^{-1}+\dots)$.The exponential is matched by the value of the action of the string with the AdS_2 world volume while the prefactor comes from the 1-loop GS string correction. Here we address the question of how the subleading T^{-1} term could be reproduced by the 2-loop correction in the corresponding partition function of the $AdS_5\times S^5$ GS string expanded near the AdS_2 minimal surface. We find that the string correction contains a non-zero UV logarithmic divergence implying that comparison with the SYM result requires a particular subtraction prescription. We discuss implications of this conclusion for checking the AdS/CFT duality at strong coupling.

1. Introduction

Even in the most symmetric example of AdS/CFT duality – between $\mathcal{N}=4$ SYM and $AdS_5\times S^5$ superstring – the two sides are not on an equal footing. While the $\mathcal{N}=4$ SYM has a well-defined perturbation theory, the $AdS_5\times S^5$ superstring in the Green-Schwarz description is represented by a formally non-renormalizable 2d action and a priori is not unambiguously defined at the quantum level beyond the semiclassical 1-loop approximation.

Indeed, already in the case of t he flat target space the 2-loop world-sheet S-matrix of the GS string expanded near the long-string vacuum contains UV poles. One is thus required to add some specific (divergent and finite) counterterms order by order in loop expansion in order to maintain its consistency with quantum integrability [1].[1]

Postulating that quantum $AdS_5\times S^5$ GS theory should have a formulation consistent with integrability implies that its classical action should be expected to be supplemented with a particular set of 2-loop, etc., higher-derivative counterterms. While the GS action itself (or the string tension) should not be renormalized due to symmetries of the action [3], the logarithmic divergences do appear in the 2-loop diagrams [4].

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1 Some alternative approaches to definition of string theory in $AdS_5\times S^5$ are discussed in [2] and refs. there.

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One might expect that in some particular cases the GS string partition function may still be UV finite. Indeed, that was found to happen in the case of the 2-loop correction to the null cusp anomalous dimension [5–7]. This example, however, is special in having flat induced 2d metric (implying that possible covariant counterterms may vanish). As we shall find below, the 2-loop UV poles do not cancel in the case of the expansion near a minimal surface with curved AdS_2 metric.

This raises the general question of how strong-coupling predictions of SYM theory (coming from localization or assumption of quantum integrability) can be directly compared to perturbative results on the $AdS_5 \times S^5$ string side, i.e. how to provide independent checks of AdS/CFT at strong coupling beyond the semiclassical 1-loop approximation. There is no known direct way of implementing quantum integrability in the string perturbation theory that would fix in general the required 2-loop and higher counterterms allowing to get finite string predictions that can be compared to the SYM side. One may then need to compute several observables in order to "measure" the coefficients of the counterterms before being able to make unambiguous (scheme-independent) predictions that would check AdS/CFT at strong coupling.[2]

In this paper we will focus on the case of the $\frac{1}{2}$ BPS circular Wilson loop observable in the planar limit represented by the $AdS_5 \times S^5$ string partition function on the disk expanded near the AdS_2 minimal surface. We will find that the 2-loop UV divergences do not cancel automatically and thus matching the known strong-coupling result on the SYM side requires a particular prescription of how to subtract the UV poles and fix the finite part. It would be interesting to see if there are other observables (like particular derivatives of latitude WL, etc.) that may happen to be UV finite and thus can be directly compared to the SYM side.

1.1. The setup

One of the postulates of the AdS/CFT correspondence is the equivalence between the expectation value $\langle W\rangle$ of a Wilson loop in $\mathcal{N}=4$ SYM and the partition function of the quantum superstring in $AdS_5 \times S^5$ defined with appropriate boundary conditions [14–18]. The simplest example is provided by the $\frac{1}{2}$ BPS circular Wilson loop with known exact gauge theory expression [19–21] which in the planar limit is given by

$$\langle W\rangle = \frac{2N}{\sqrt{\lambda}} I_1(\sqrt{\lambda}). \tag{1.1}$$

Expanded in the limit of large 't Hooft coupling $\lambda = N g^2_{\rm YM}$ and expressed in terms of the $AdS_5 \times S^5$ string tension $T = \frac{L^2}{2\pi\alpha'} = \frac{\sqrt{\lambda}}{2\pi}$ and string coupling $g_s = \frac{g^2_{\rm YM}}{4\pi}$ (1.1) may be written as [22]

$$\langle W\rangle = \sqrt{\frac{2}{\pi}}\frac{N}{\lambda^{3/4}} e^{\sqrt{\lambda}}\Big[1 - \frac{3}{8\sqrt{\lambda}} + \dots\Big] = c_1 \frac{1}{g_s}\sqrt{\frac{T}{2\pi}} e^{2\pi T}\Big[1 - \frac{3}{16\pi}\frac{1}{T} + \dots\Big], \qquad c_1 = \frac{1}{\sqrt{2\pi}}. \tag{1.2}$$

This should represent the large string tension expansion of the disk partition function of the $AdS_5 \times S^5$ superstring near the AdS_2 minimal surface that ends on a circle at the boundary of AdS_5 [16–18].

Indeed, the term in the exponential in (1.2) is the same as the classical string action evaluated on the AdS_2 induced metric with the regularized volume $V = -2\pi$. The prefactor c_1 represents the 1-loop correction given by a product of quadratic fluctuation determinants [18] (see also [8,23,24]). The origin of the $\sqrt{T}$ factor was explained in [22]. The computation of the remaining $\frac{1}{\sqrt{2\pi}}$ factor in (1.2) remains an open problem as it depends on a precise normalization of the GS superstring measure (it was indirectly confirmed by matching the ratio of the $\frac{1}{2}$ and $\frac{1}{4}$ BPS WL expectation values [13]).

Our focus here will be on the subleading T^{-1} term in (1.2) that should correspond to the 2-loop contribution in the GS superstring path integral expanded near the AdS_2 minimal surface. Separating the prefactor $Z_1 = \frac{1}{g_s}\sqrt{\frac{T}{2\pi}}$ in (1.2) we may write it as

$$\langle W\rangle = Z_1\, e^{-F}, \qquad F = -f(T)V, \tag{1.3}$$

$$f(T) \equiv f_0 T + f_1 + f_2 T^{-1} + \dots = -T + \log c_1 + \frac{3}{32\pi^2} T^{-1} + \dots, \qquad f_2 = \frac{3}{32\pi^2}. \tag{1.4}$$

Viewing (1.3) as the string partition function we used that since AdS_2 is a homogeneous space the corresponding free energy should be proportional to its IR-renormalized volume $V = -2\pi$ in the case of the circular boundary ($V = 0$ in the case of the line). In general, $V_{\rm circle} = \frac{1}{z} - 2\pi$ and $V_{\rm line} = \frac{1}{z}$ where $z \to 0$ is an IR cutoff in AdS_2.

To avoid the step of renormalizing the IR divergence, i.e. to get a manifestly IR finite quantity one may consider the ratio $\langle W_{\rm circle}\rangle / \langle W_{\rm line}\rangle = Z_1\, e^{2\pi f(T)}$.[3] On the SYM side the expression for the circular WL in (1.1) corresponds to the normalization choice for which $\langle W_{\rm line}\rangle = 1$.

[2] Even ignoring the issue of UV divergences on the string side, to provide a scheme-independent comparison of the SYM and string predictions one should compare at least two observables, e.g. dimensions of two operators as functions of the 't Hooft coupling λ in the planar limit, and eliminate λ expressing one dimension as a function of the other. That function can then be matched to the one relating the AdS energies of one corresponding string states. Also, in some cases, to eliminate possible ambiguities in the definition of string path integral it may be useful to consider ratios of partition functions, see, e.g., [8–13].

[3] Let us note that like the $AdS_5 \times S^5$ radius, the shape and radius of the minimal surface AdS_2 metric can not be renormalized due to underlying symmetry of the theory. Thus AdS_2 should remain the solution of the quantum-corrected 2d effective action also the 2-loop and higher level.

The $AdS_5 \times S^5$ GS action [3] is non-linear and while its 1-loop semiclassical quantization is straightforward (see, e.g., [18,25–27]) there are issues related to potential UV divergences and scheme dependence at higher loop orders. The $AdS_5 \times S^5$ GS action itself should not be renormalized being protected by a high amount of symmetry and the presence of the fermionic WZ term [3], and the 1-loop finiteness of its on-shell effective action can be checked directly [18], there are potential higher loop divergences that may require introduction of higher-derivative counterterms with coefficients that need to be fixed in a specific way to be consistent with symmetries of the model.[4] Currently, there is no known way to directly fix such possible counterterms.[5]

It could still be that for some special observables the underlying symmetry of the theory may lead to a UV finite result thus allowing a direct comparison to the finite strong-coupling expansion on the dual gauge theory side. Indeed, examples of consistent 2-loop computations in $AdS_5 \times S^5$ GS theory matching the expected gauge theory results appeared in [5–7,28]. The challenge is to find a regularization scheme consistent with symmetries of the theory. Ideally, one would like to use a version of dimensional regularization to avoid dealing with power divergences and ambiguities in choice of local measure and local field redefinitions (and, in particular, gauge dependence). Even though the GS fermions are originally 2d scalars, a direct use of dimensional regularization is problematic as the GS action involves $\varepsilon^{\mu\nu}$ symbol in the fermionic WZ term.

Once the GS action is expanded near a particular bosonic background, the fermion kinetic term takes a 2d Dirac form so that the fermions may be interpreted as a collection of 2d spinors. Then the use of dimensional regularization would not be consistent with the effective 2d global AdS_2 supersymmetry [18] of the gauge-fixed world sheet theory. One may still try to use some version of dimensional reduction regularization [29] like it was done in [5–7].[6] In general, having curved AdS_2 background metric requires special care in choice of regularization prescription even in simpler 2d QFT models like the super-Liouville one [32,33] (see also [34–37]). The ambiguity in choice of quantization is to be fixed by extra constraints like preservation of hidden symmetries of the path integral or expected symmetries of boundary correlators.

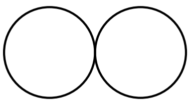
1.2. Results

Fixing the static gauge on bosons and the corresponding "adapted" κ-symmetry gauge on the fermions (generalizing the flat-space gauge used, e.g., in [1]) one finds that the Lagrangian for the physical 8+8 GS string fluctuations takes the following symbolic form [18,38][7]

$$L = (\partial x^i)^2 + m_b^2 x^2 + (\partial y^a)^2 + i\bar\theta(\nabla\!\!\!/ + m_f)\theta + T^{-1}\Big[(\partial x)^4 + x^2(\partial x)^2 + x^4 + (\partial x)^2(\partial y)^2 + (\partial y)^4 + y^2(\partial y)^2 + (\partial x\partial x + x^2)(\theta\nabla\theta + \theta^2) + \ldots + \theta\theta\theta\nabla\theta + \theta\nabla\theta\theta\nabla\theta + \ldots\Big] + \mathcal{O}(T^{-2}). \tag{1.5}$$

Here all indices are contracted with the induced AdS_2 metric and derivatives are 2d covariant so that there is a manifest AdS_2 symmetry with fermions θ treated as a set of 8 Majorana 2d fermions. x^i represent 3 transverse fluctuations in AdS_5 with mass $m_b^2 = 2$, y^a are massless fluctuations in S^5 and θ have mass $m_f^2 = 1$.

The important simplification (compared to the examples in [4–7]) is that in the static κ-symmetry gauge we will use below there are no cubic interaction terms in (1.5) and thus all 2-loop graphs contributing to the free energy will be just the "double-bubble" ones



with each of the two propagators being either bosonic or fermionic. As the AdS_2 propagators (and their derivatives) taken at the coinciding points are constant, the remaining integral over the bulk point is trivial, giving just a factor of the AdS_2 volume V as in (1.3).

We will use the dimensional reduction regularization, i.e. will keep the field indices and thus the tensor and spinor algebra contractions in their original 2d form while treating the arguments of the fields as points in $2-2\epsilon$ dimensional space, thus continuing the remaining scalar propagator factors to $AdS_{\rm d}$ with ${\rm d} = 1 + d = 2 - 2\epsilon$, $\epsilon \to 0$.[8]

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[4] This is similar to the case of $T\bar T$ deformation: to ensure that the classical integrability of a theory extends also to the quantum level one is to add specific counterterms at each order in the inverse tension expansion (see a discussion in [1]).

[5] If one imagines deriving GS action as a collective coordinate action for a fundamental string soliton in 10d supergravity from the supergravity action one may also get some specific higher derivative corrections suppressed by the inverse string tension. However, the supergravity action itself gets corrected by string α' corrections and which should be the resulting "bootstrapped" GS + corrections action is unclear.

[6] The examples considered in [4–7,30] were different from the present one in having flat induced metric and thus flat-space propagators for 2d fields (but lacking 2d Lorentz symmetry in the interaction terms). The special Catalan-constant contribution (matching gauge-theory integrability prediction [31]) was coming from non-trivial 2-loop integrals that could be isolated from UV divergent and scheme-dependent parts. In fact, the logarithmic UV divergences were found to cancel in the gauges used in [5–7]. This will no longer be so for the circular WL case discussed below where all terms will have rational coefficients and thus will be on an equal footing.

[7] We assume that the radius of AdS_2 is 1 and rescale the 2d fields by square root of the tension T.

[8] Note that there is only trivial Euler number 1-loop divergence (that should be cancelled by measure) at the 1-loop order [18,22]. Compared to the S-matrix case considered in [1] here the evanescent 1-loop $\frac{1}{\epsilon}\int R^{(\rm d)}$ counterterm does not contribute to the 2-loop free energy: expanded near AdS_2 it does not contain a non-trivial linear in $\partial x\partial x$ term.

All 2-loop contributions will be expressed in terms of the regularized values of the following constant factors representing the coincident values of the AdS_{1+d} propagators for a boson with mass $m_b^2 = 2$ and a fermion with mass $m_f = 1$ [9]

$$G = \frac{\Gamma(\frac{1-d}{2})\,\Gamma(\Delta)}{(4\pi)^{(d+1)/2}\Gamma(1-d+\Delta)} = \frac{1}{4\pi\varepsilon'} - \frac{a_b}{4\pi} + \mathcal{O}(\varepsilon)\,, \qquad \Delta(\Delta-d) = m_b^2 = 2\,, \qquad a_b = 2\,, \tag{1.6}$$

$$\frac{1}{\varepsilon'} \equiv \frac{1}{\varepsilon} + \ell, \qquad \ell \equiv \log(4\pi e^{\gamma_E})\,, \qquad \varepsilon = \tfrac{1}{2}(1-d)\,, \tag{1.7}$$

$$S = -\frac{\Gamma(\frac{1-d}{2})\,\Gamma(\frac{1}{2}+\Delta)}{(4\pi)^{(d+1)/2}\Gamma(\frac{1}{2}-d+\Delta)} = -\frac{1}{4\pi\varepsilon'} + \frac{a_f}{4\pi} + \mathcal{O}(\varepsilon)\,, \qquad \Delta = \frac{d}{2} + m_f, \quad m_f = 1\,, \quad a_f = 1. \tag{1.8}$$

The massless scalars y^a will not give a non-trivial 2-loop contribution in dimensional regularization.[10] The resulting 2-loop correction f_2 to the free energy in (1.3) will be found to have the form

$$f_2 = q_b\, G^2 + q_f\, G\, S + q_f\, S^2\,, \tag{1.9}$$

$$q_b = \frac{9(1-d)}{2(1+d)}\,, \qquad q_f = \frac{24(1-d)}{(1+d)^2}\,, \qquad q_f = \frac{2(1-d)(5-3d)}{(1+d)^2}. \tag{1.10}$$

Here the G^2 contribution comes from the quartic x-terms, the $G\,S$ one – from the mixed $x^2\theta^2$ terms and the S^2 one – from the quartic θ terms in (1.5).

As each of the three terms in (1.9) is multiplied by $1-d=2\varepsilon$, the double-pole ($\log^2$ UV divergences) cancel independently in each of the three types of contributions.[11] However, the single pole $\frac{1}{\varepsilon}$ term does not cancel:

$$f_2 = -\frac{11}{32\pi^2\bar{\varepsilon}} + f_{2,\rm fin} + \mathcal{O}(\varepsilon)\,, \qquad f_{2,\rm fin} = \frac{6a_b + 16a_f - 19}{32\pi^2}\,, \tag{1.11}$$

where $\frac{1}{\bar\varepsilon} = \frac{1}{\varepsilon} + 2\ell = \frac{1}{\varepsilon'} + \ell$ (cf. (1.7)). The remaining log UV divergence means the finite part is a priori scheme dependent and cannot be directly compared to the gauge-theory result in (1.4).[12]

One may try several options of how to interpret the result (1.9),(1.10):

(i) One may assume a "hard" version of dimensional reduction regularization in which contractions of all of the world-volume indices (both ones of derivatives and the spinor ones) to be done first in 2 dimensions and after that the resulting scalar propagators are to be continued to $1+d=2-2\varepsilon$ dimensions. In this case one is to set $d=1$ in the coefficients in (1.9) first so that the total 2-loop result will be just zero. The issue will then be of how to reconcile this with a non-zero value of f_2 in the gauge theory expression in (1.4). In any case, the question of how to make this prescription consistent is a priori unclear.[13]

(ii) One may conjecture that the 2d UV pole in (1.11) should be subtracted out or cancelled by an appropriate counterterm. This interpretation was used in [1] in the case of the 2-loop correction to scattering of fluctuations on an infinite GS string in flat target space. In the "modified minimal subtraction" (where one drops the $\frac{1}{\bar\varepsilon}$ term in (1.11)) the finite part in (1.11) will be given by (see (1.6),(1.8))

$$f_{2,\rm fin}\Big|_{a_b=2,a_f=1} = \frac{9}{32\pi^2}. \tag{1.12}$$

This is 3 times larger than the gauge theory coefficient in (1.4), implying that exact matching would require some additional finite subtraction .

To try to modify the value of the finite part in (1.12) one may take into account that the finite constants in the coincident values of the bosonic (1.6) and fermionic (1.8) propagators in AdS_2 are, in general, scheme-dependent and thus special choices of them may be required for consistency with underlying 2d supersymmetry (cf. [33]). One may conjecture that the required modification of the

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[9] Here we are not indicating dependence on a normalization mass μ and radius r of AdS_2 (in general, $\frac{1}{\varepsilon} \to \frac{1}{\varepsilon} + 2\log(\mu r)$) assuming that $\mu r = 1$. In general, a redefinition of μ corresponds to a scheme change (for example, we may absorb ℓ into $\log\mu$).

[10] Note that we are assuming that masses of the fields in (1.5) are kept fixed while replacing AdS_2 by $AdS_{2-2\varepsilon}$. One could consider an alternative prescription of keeping the boundary dimensions of the fields fixed to $\Delta_x = 2$, $\Delta_y = 1$, $\Delta_\theta = \frac{3}{2}$ but that approach does not appear to lead to a consistent picture.

[11] The cancellation of the 2-loop double-pole divergences in the bosonic contribution is due to the constant curvature of the AdS_5 space. This is a general fact for a 2d sigma model on a constant curvature space (see a discussion in [4]). The same cancellation found also for the fermionic contribution may be viewed as a consequence of the residual global supersymmetry relating the bosonic contribution to the fermionic ones.

[12] Note that if we use dimensional regularization to regularize also the AdS_2 volume V in (1.3) as $\mathrm{vol}(AdS_{1+d}) = \mathrm{vol}(AdS_{2-2\varepsilon}) = \pi^{d/2}\Gamma(-d/2) = -2\pi[1+(2-\ell)\varepsilon+\ldots]$ then the presence of a pole in (1.11) will lead to an extra finite contribution to f_2. However, this will be due to mixing the UV and IR regularizations; this does not appear to be a consistent procedure in general.

[13] Note that compared to standard version of dimensional reduction regularization [29] were the main concern was to maintain balance of degrees of freedom to maintain supersymmetry here we have a separate complication of how to continue ϵ^{ab} terms.

dimensional reduction regularization that leads to (1.6),(1.8) is the one for which the coincident values of the bosonic and fermionic propagators differ only by the sign[14]

$$G = -S = \frac{1}{4\pi\epsilon'} + \frac{a_b}{4\pi} + \mathcal{O}(\epsilon) , \qquad \text{i.e.} \qquad a_b = a_f = 1. \tag{1.13}$$

Assuming this "supersymmetry relation" (1.13) we then conclude that the finite part of (1.11) is given by

$$f_{2,\text{fin}}\Big|_{a_f=a_b=1} = \frac{3}{32\pi^2}. \tag{1.14}$$

We thus observe that the formal subtraction of the $\frac{1}{\varepsilon}$ pole in (1.11) combined with a scheme choice implying (1.13) allows one to reproduce the gauge-theory value of f_2 in (1.4). The structure of the rest of the paper is as follows. In Section 2 we will discuss the expansion of the bosonic and fermionic parts of the $\text{AdS}_5 \times S^5$ GS action near AdS_2 minimal surface to quartic order in the fluctuation fields fixing the bosonic static gauge and its κ-symmetry counterpart. In Section 3 we will present the expressions for the bosonic and fermionic kinetic operators and their derivatives required in computing the 2-loop correction in dimensional regularization, i.e. in AdS_d with $d = 2 - 2\epsilon$. The resulting contributions to the 2-loop coefficient f_2 will be presented in Section 4. Some concluding remarks will be made in Section 5.

In Appendix A we will present some details about the computation of fermionic correlators. The expression for the 2-loop correction to free energy directly in 2 dimensions (and specifically in the ζ-function regularization) will be discussed in Appendix B. In Appendix C we will compute the 1-loop correction to the 2-point function of x^i fluctuations. A review of strong coupling expansion of $\frac{1}{2}$ BPS WL in ABJM theory and some comments on the corresponding string theory computation will be given in Appendix D.

2. Expansion of $\text{AdS}_5 \times S^5$ GS action near AdS_2 minimal surface

2.1. Bosonic part of the action in static gauge

Following [38] we shall parametrize the $\text{AdS}_5 \times S^5$ metric as

$$ds^2_{10} = \frac{(1+\frac{1}{4}\boldsymbol{x}^2)^2}{(1-\frac{1}{4}\boldsymbol{x}^2)^2} ds^2_2 + \frac{d\boldsymbol{x}\cdot d\boldsymbol{x}}{(1-\frac{1}{4}\boldsymbol{x}^2)^2} + \frac{d\boldsymbol{y}\cdot d\boldsymbol{y}}{(1+\frac{1}{4}\boldsymbol{y}^2)^2} , \tag{2.1}$$

where $\boldsymbol{x} = (x^i)$, $i = 1,2,3$; $\boldsymbol{y} = (y^a)$, $a = 1,\dots,5$ and ds^2_2 is the AdS_2 metric. The result of our computation will depend on explicit parametrization or topology of AdS_2 factor only through the value of the volume V factor so that we may choose it, e.g., to be the Poincare half-plane (with metric $\frac{1}{z^2}(-dx_0^2 + dz^2)$) having infinite line as its boundary. The minimal surface corresponding to the Wilson line at the boundary is represented by

$$x^0 = \xi^0, \qquad z = \xi^1 , \qquad x^i = 0, \qquad y^a = 0, \qquad \xi^\mu = (\tau,\sigma) , \tag{2.2}$$

$$ds^2_2 = g_{\mu\nu}(\xi)\, d\xi^\mu d\xi^\nu = \frac{1}{\sigma^2}(-d\tau^2 + d\sigma^2), \qquad g_{\mu\nu}(\xi) = \frac{1}{\sigma^2}\eta_{\mu\nu}. \tag{2.3}$$

The bosonic part of the $\text{AdS}_5 \times S^5$ action in the static gauge where the fluctuations of z and x^0 are set to zero has the form

$$S_b = -T\int d^2\xi\, \sqrt{-\text{h}} \equiv T\int d^2\xi\, \sqrt{-g}\,(-1 + \mathcal{L}_b) , \tag{2.4}$$

$$\text{h}_{\mu\nu} = \frac{(1+\frac{1}{4}\boldsymbol{x}^2)^2}{(1-\frac{1}{4}\boldsymbol{x}^2)^2} g_{\mu\nu}(\xi) + \frac{\partial_\mu \boldsymbol{x}\cdot\partial_\nu \boldsymbol{x}}{(1-\frac{1}{4}\boldsymbol{x}^2)^2} + \frac{\partial_\mu \boldsymbol{y}\cdot\partial_\nu \boldsymbol{y}}{(1+\frac{1}{4}\boldsymbol{y}^2)^2}. \tag{2.5}$$

The action has global $SO(2,1)\times[SO(3)\times SO(5)]$ symmetry. Expanding in powers of the fields we find the following interacting Lagrangian for the 3 massive and 5 massless fluctuation fields

$$\mathcal{L}_b = \mathcal{L}_2 + \mathcal{L}_{4b} + \cdots , \qquad \mathcal{L}_{4b} = \mathcal{L}_{4x} + \mathcal{L}_{2x,2y} + \mathcal{L}_{4y} , \tag{2.6}$$

$$\mathcal{L}_2 = -\tfrac{1}{2}(\partial\boldsymbol{x}\cdot\partial\boldsymbol{x}) - \boldsymbol{x}^2 - \tfrac{1}{2}(\partial\boldsymbol{y}\cdot\partial\boldsymbol{y}), \tag{2.7}$$

$$\mathcal{L}_{4x} = -\tfrac{1}{8}(\partial\boldsymbol{x}\cdot\partial\boldsymbol{x})^2 + \tfrac{1}{4}(\partial x^i\partial x^j)(\partial x^i\partial x^j) - \tfrac{1}{4}\boldsymbol{x}^2(\partial\boldsymbol{x}\cdot\partial\boldsymbol{x}) - \tfrac{1}{2}(\boldsymbol{x}^2)^2, \tag{2.8}$$

$$\mathcal{L}_{2x,2y} = -\tfrac{1}{4}(\partial\boldsymbol{x}\cdot\partial\boldsymbol{x})(\partial\boldsymbol{y}\cdot\partial\boldsymbol{y}) + \tfrac{1}{2}(\partial x^i\partial y^a)(\partial x^i\partial y^a), \tag{2.9}$$

$$\mathcal{L}_{4y} = \tfrac{1}{4}y^2(\partial\boldsymbol{y}\cdot\partial\boldsymbol{y}) - \tfrac{1}{8}(\partial\boldsymbol{y}\cdot\partial\boldsymbol{y})^2 + \tfrac{1}{4}(\partial y^a\partial y^b)(\partial y^a\partial y^b), \tag{2.10}$$

where all derivatives are contracted with the AdS_2 metric.

[14] As also emphasised in [32], the regularization required to remove the coincident-point singularities of the bosonic and fermionic propagators is not unique. One may select a particular scheme by demanding consistency with the underlying symmetries of the theory. In the present case, one may expect that a preferred regularisation is fixed by demanding compatibility with the (non-linear) OSp(4*|4) action on the worldsheet fluctuations. Similar relations between the bosonic and fermionic propagators in AdS_2 that should be a consequence of maintaining AdS_2 supersymmetry appeared in [39] (see also [22]) and in [33].

2.2. Fermionic part of the action in static κ-symmetry gauge

Here we shall follow the same notation as in [4] using Lorentzian signature and define the following combinations of 10d Dirac matrices (with indices $A=(a,a')$, $a=0,.,4;\ a'=5,.,9; i=1,2,3$)

$$\{\Gamma_A,\Gamma_B\}=2\eta_{AB}\ ,\qquad \eta_{AB}=(-1,+1,\dots,+1),\qquad \Gamma_{11}=-\Gamma_0\dots\Gamma_9\ ,\quad \Gamma_{11}^2=1\ , \tag{2.11}$$

$$\Gamma_*=i\Gamma_0\Gamma_1\Gamma_2\Gamma_3\Gamma_4\ ,\quad \Gamma_*^2=1\ ,\quad \Gamma_*\Gamma_a=\Gamma_a\Gamma_*\ ,\quad \Gamma_*\Gamma_{a'}=-\Gamma_{a'}\Gamma_*\ , \tag{2.12}$$

$$\Gamma'_*=i\Gamma_5\Gamma_6\Gamma_7\Gamma_8\Gamma_9\ ,\qquad \Gamma'^2_*=-1\ ,\qquad \Gamma_{11}=\Gamma_*\Gamma'_*=-\Gamma'_*\Gamma_*. \tag{2.13}$$

The two GS fermions θ^I $(I=1,2)$ satisfy the Majorana-Weyl conditions

$$\theta^I=\Gamma_{11}\theta^I\ ,\qquad \bar\theta^I=(\theta^I)^\dagger\Gamma^0=(\theta^I)^TC\ ,\qquad C^T=-C\ ,\qquad \Gamma_A=-C^{-1}\Gamma_A^TC. \tag{2.14}$$

One may choose a representation of Dirac matrices in which $C=\Gamma^0$ and θ^I are thus real. The 10d Majorana fermions θ,θ' satisfy

$$\bar\theta\Gamma_{A_1\dots A_n}\theta'=(-1)^{\frac{n(n+1)}{2}}\bar\theta'\Gamma_{A_1\dots A_n}\theta\ ,\qquad \bar\theta\Gamma_B\Gamma_*\Gamma_A\theta=-\bar\theta\Gamma_A\Gamma_*\Gamma_B\theta. \tag{2.15}$$

In the static gauge we will split the target space coordinates X^M in 2 + 3 + 5 way (cf. (2.2)) here labelling them as (interchanging 1 and 4 directions compared to (2.1))

$$X^M=(\xi^0,x^i,\xi^1;y^{a'})\ ,\qquad i=1,2,3\ ,\qquad a'=5,\dots,9. \tag{2.16}$$

We also split the corresponding tangent-space components of 10d Dirac matrices as $\Gamma_{\hat\alpha},\Gamma_i,\Gamma_{a'}$ defining "parallel" components $\Gamma_{\hat\alpha}$ as[15]

$$\Gamma_{\hat\alpha}=(\Gamma_0,\Gamma_4)\ ,\qquad \hat\Gamma=\Gamma_0\Gamma_4\ ,\qquad \hat\Gamma^2=1,\qquad \Gamma^{\hat\alpha}\Gamma^{\hat\beta}=\eta^{\hat\alpha\hat\beta}-\epsilon^{\hat\alpha\hat\beta}\hat\Gamma\ , \tag{2.17}$$

$$\hat\Gamma\Gamma_{\hat\alpha}=-\Gamma_{\hat\alpha}\hat\Gamma\ ,\qquad \hat\Gamma\Gamma_i=\Gamma_i\hat\Gamma\ ,\qquad \hat\Gamma\Gamma_{a'}=\Gamma_{a'}\hat\Gamma\ , \tag{2.18}$$

$$\Gamma_{123}=\Gamma_1\Gamma_2\Gamma_3\ ,\qquad \Gamma_\alpha\Gamma_{123}=-\Gamma_{123}\Gamma_\alpha\ ,\qquad \Gamma_i\Gamma_{123}=\Gamma_{123}\Gamma_i\ ,\qquad \Gamma_{a'}\Gamma_{123}=-\Gamma_{123}\Gamma_{a'}\ , \tag{2.19}$$

$$\Gamma_*=-i\hat\Gamma\Gamma_{123}=-i\Gamma_{123}\hat\Gamma\ ,\qquad \hat\Gamma\Gamma_*=\Gamma_*\hat\Gamma=-i\Gamma_{123}\ ,\qquad \hat\Gamma^TC=\Gamma_4^T\Gamma_0^TC=-C\hat\Gamma. \tag{2.20}$$

The Minkowski signature GS Lagrangian may be written as $(I,J=1,2)$

$$\mathcal{L}=-\sqrt{-h}-2i\epsilon^{\alpha\beta}\int_0^1 ds\, L^A_{\alpha s}\big(\mathrm{s}^{\mathrm{IJ}}\bar\theta^{\mathrm{I}}\Gamma_{\mathrm{A}}\mathrm{L}^{\mathrm{J}}_{\beta\mathrm{s}}\big)\ , \tag{2.21}$$

$$h_{\alpha\beta}=L^A_\alpha L_{A\beta}\ ,\qquad \mathrm{s}^{\mathrm{IJ}}=(1,-1),\qquad \mathrm{L}^{\mathrm{A}}_\alpha=(\mathrm{L}^{\mathrm{A}}_\alpha)_{\mathrm{s}=1}\ ,\qquad \mathrm{L}^{\mathrm{I}}_\alpha=(\mathrm{L}^{\mathrm{I}}_\alpha)_{\mathrm{s}=1}\ , \tag{2.22}$$

$$L^A_{s\alpha}=E^A_M\partial_\alpha X^M-4i\bar\theta^I\Gamma^A\Big[\frac{\sinh^2(\frac{s}{2}\mathcal{M})}{\mathcal{M}^2}\Big]^{IJ}D_\alpha\theta^J\ ,\qquad L^I_{s\alpha}=\Big[\frac{\sinh(s\mathcal{M})}{\mathcal{M}}D_\alpha\theta\Big]^I\ , \tag{2.23}$$

$$D_\alpha\theta^I=\nabla_\alpha\theta^I-\frac{i}{2}\epsilon^{IJ}E^A_M\partial_\alpha X^M\Gamma_*\Gamma_A\theta^J\ ,\qquad \nabla_\alpha\theta^I=\partial_\alpha\theta^I+\frac14\Omega^{AB}_M\partial_\alpha X^M\Gamma_{AB}\theta^I\ , \tag{2.24}$$

$$(\mathcal{M}^2)^{IJ}=-\epsilon^{IK}\Gamma_*\Gamma^A\theta^K\bar\theta^J\Gamma_A+\frac12\big(-\Gamma^{ab}\theta^I\bar\theta^K\Gamma_{ab}\Gamma_*+\Gamma^{a'b'}\theta^I\bar\theta^K\Gamma_{a'b'}\Gamma'_*\big)\epsilon^{KJ}. \tag{2.25}$$

We will fix the κ-symmetry in the following static-gauge adapted way ("κ-static" gauge)[16]

$$\hat\Gamma\theta^1=\theta^1\ ,\qquad \hat\Gamma\theta^2=-\theta^2\ ,\qquad \hat\Gamma=\Gamma_0\Gamma_4\ ,\quad \hat\Gamma^2=1. \tag{2.26}$$

Thus θ^1 and θ^2 will have opposite chirality with respect to $\hat\Gamma$, so that (index p stands for i or a')

$$\bar\theta^I\Gamma_{p_1}\dots\Gamma_{p_n}\theta^I=0\ ,\qquad \bar\theta^1\Gamma_{\hat\alpha_1}\dots\Gamma_{\hat\alpha_n}\theta^2=(-1)^n\bar\theta^1\Gamma_{\hat\alpha_1}\dots\Gamma_{\hat\alpha_n}\theta^2. \tag{2.27}$$

Let us define

$$\theta\equiv\theta^1+\theta^2\ ,\qquad \theta^1=\mathcal{P}\theta\ ,\qquad \theta^2=(1-\mathcal{P})\theta\ ,\qquad \mathcal{P}\equiv\tfrac12(1+\hat\Gamma). \tag{2.28}$$

Our aim is to expand the Lagrangian (2.21) to quartic order in the independent MW variable θ. Since we will be interested only up to quartic terms in bosons and fermions we will use that in the θ^4 terms we may replace the vielbein 1-form E^A by its AdS_2 value using that according to (2.1) ($e^{\hat\alpha}$ is the AdS_2 1-form, cf. (2.36))

$$E^{\hat\alpha}=\frac{(1+\frac14x^2)}{(1-\frac14x^2)}e^{\hat\alpha}\ ,\qquad E^i=\frac{dx^i}{(1-\frac14x^2)}\ ,\qquad E^{a'}=\frac{dy^{a'}}{(1+\frac14y^2)}\ , \tag{2.29}$$

$$\Omega^{\hat\alpha\hat\beta}=\omega^{\hat\alpha\hat\beta}\ ,\quad \Omega^{\hat\alpha i}=\frac{e^{\hat\alpha}x^i}{(1-\frac14x^2)}\ ,\quad \Omega^{ij}=-\frac12\,\frac{x^idx^j-x^jdx^i}{1-\frac14x^2}\ ,\quad \Omega^{a'b'}=\frac12\,\frac{y^{a'}dy^{b'}-y^{b'}dy^{a'}}{1+\frac14y^2}. \tag{2.30}$$

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[15] We will always assume that Γ_A matrices have tangent-space indices, i.e. are constant. Here $\Gamma^0=-\Gamma_0$ and $\epsilon^{01}=1$. We will use the Greek letters $\alpha,\beta,.$ for 2d world-sheet indices and $\hat\alpha,\hat\beta,\dots$ for the corresponding tangent-space indices.

[16] The same gauge was used in other contexts, e.g., in [1,40–42].

Then we get for the $\hat\alpha$ and $p=(i,a')$ components of the 1-forms $L^A \equiv L^A_M dX^M$

$$L^{\hat\alpha}_s = E^{\hat\alpha} - is^2(\bar\theta^I\Gamma^{\hat\alpha}D\theta^I) - \tfrac{s^4}{12}(\bar\theta\Gamma_{123}\theta)(\bar\theta\Gamma^{\hat\alpha}\mathfrak{D}\theta) - \tfrac{is^4}{24}(\bar\theta\Gamma^{\hat\alpha}\Gamma^{ij}\theta)(\bar\theta\Gamma\Gamma_{ij}\Gamma_*\mathfrak{D}\theta) + \tfrac{is^4}{24}(\bar\theta\Gamma^{\hat\alpha}\Gamma^{a'b'}\theta)(\bar\theta\Gamma\Gamma_{a'b'}\Gamma'_*\mathfrak{D}\theta) + \mathcal{O}(\theta^6)\,, \tag{2.31}$$

$$L^p_s = E^p - is^2(\bar\theta^I\Gamma^p D\theta^I) + \mathcal{O}(\theta^6)\,, \qquad E^A = E^A_M dX^M\,, \tag{2.32}$$

$$\mathfrak{D}\theta \equiv \nabla\theta - \tfrac{i}{2}e^{\hat\alpha}\hat\Gamma\Gamma_*\Gamma_{\hat\alpha}\theta = \nabla\theta + \tfrac12 e^{\hat\alpha}\Gamma_{\hat\alpha}\Gamma_{123}\theta\,, \tag{2.33}$$

where we used that $\bar\theta\Gamma_*\theta=0$, $\epsilon^{JI}\bar\theta^J\Gamma_{\hat\alpha}\Gamma_*\Gamma_{\hat\beta}\theta^I = i\eta_{\hat\alpha\hat\beta}\bar\theta\Gamma_{123}\theta$, $\bar\theta^I\Gamma_{\hat\alpha}\Gamma_{\hat\beta\hat\gamma}\theta^I=0$. Expanding in powers of the bosonic fields using (2.29),(2.30) we have

$$\begin{aligned}\bar\theta^I\Gamma^{\hat\alpha}D\theta^I &= \bar\theta^I\Gamma^{\hat\alpha}(\nabla\theta^I - \tfrac{i}{2}\epsilon^{IJ}E^{\hat\beta}\Gamma_*\Gamma_{\hat\beta}\theta^J + \tfrac14\Omega^{ij}\Gamma_{ij}\theta^I + \tfrac14\Omega^{a'b'}\Gamma_{a'b'}\theta^J)\\ &= \bar\theta\Gamma^{\hat\alpha}\mathfrak{D}\theta - \tfrac14 x^2 e^{\hat\beta}(\bar\theta\Gamma^{\hat\alpha}\Gamma_{123}\Gamma_{\hat\beta}\theta) - \tfrac14 x^i dx^j(\bar\theta\Gamma^{\hat\alpha}\Gamma_{ij}\theta) + \tfrac14 y^{a'}dy^{b'}(\bar\theta\Gamma^{\hat\alpha}\Gamma_{a'b'}\theta) + \dots\,,\end{aligned} \tag{2.34}$$

$$\begin{aligned}\bar\theta^I\Gamma^p D\theta^I &= \bar\theta^I\Gamma^p(\tfrac12\Omega^{\hat\alpha j}\Gamma_{\hat\alpha}\Gamma_j\theta^I - \tfrac{i}{2}\epsilon^{IJ}E^j\Gamma_*\Gamma_j\theta^J - \tfrac{i}{2}\epsilon^{IJ}E^{a'}\Gamma_*\Gamma_{a'}\theta^I)\\ &= \tfrac12 e^{\hat\alpha}x^j(\bar\theta\Gamma^p\Gamma_{\hat\alpha}\Gamma_j\theta) + \tfrac{i}{2}dx^j(\bar\theta\Gamma\Gamma^p\Gamma_*\Gamma_j\theta) + \tfrac{i}{2}dy^{a'}(\bar\theta\Gamma\Gamma^p\Gamma_*\Gamma_{a'}\theta) + \dots\,.\end{aligned} \tag{2.35}$$

Using the AdS_2 2-bein $e^{\hat\alpha}_\mu$ we may define the world-volume projectors of Γ_α as

$$\gamma_\mu \equiv e^{\hat\alpha}_\mu\Gamma_{\hat\alpha}\,, \qquad \{\gamma_{\hat\alpha},\gamma_{\hat\beta}\} = 2g_{\hat\alpha\hat\beta}\,, \qquad g_{\mu\nu} = e^{\hat\alpha}_\mu e^{\hat\beta}_\nu\eta_{\hat\alpha\hat\beta}. \tag{2.36}$$

Then we find that

$$\begin{aligned}h_{\alpha\beta} = L^A_\alpha L_{A\beta} &= E^A_\alpha E^A_\beta - 2i(1+\tfrac12x^2)(\bar\theta\gamma_{(\alpha}\mathfrak{D}_{\beta)}\theta) + \tfrac{i}{2}x^2\bar\theta\gamma_{(\alpha}\Gamma_{123}\gamma_{\beta)}\theta\\ &+ \tfrac{i}{2}x^i\partial_{(\alpha}x^j(\bar\theta\gamma_{\beta)}\Gamma_{ij}\theta) - \tfrac{i}{2}y^{a'}\partial_{(\alpha}y^{b'}(\bar\theta\gamma_{\beta)}\Gamma_{a'b'}\theta) - (\bar\theta\gamma^\mu\mathfrak{D}_\alpha\theta)(\bar\theta\gamma_\mu\mathfrak{D}_\beta\theta)\\ &- iE^p_{(\alpha}\Big[x^j(\bar\theta\Gamma^p\gamma_{\beta)}\Gamma_j\theta) + \partial_{\beta)}x^j(\bar\theta\Gamma^p\Gamma_{123}\Gamma_j\theta) + \partial_{\beta)}y^{a'}(\bar\theta\Gamma^p\Gamma_{123}\Gamma_{a'}\theta)\Big]\\ &- \tfrac16(\bar\theta\Gamma_{123}\theta)(\bar\theta\gamma_{(\alpha}\mathfrak{D}_{\beta)}\theta) - \tfrac{i}{12}(\bar\theta\gamma_{(\alpha}\Gamma^{ij}\theta)(\bar\theta\hat\Gamma\Gamma_{ij}\Gamma_*\mathfrak{D}_{\beta)}\theta) + \tfrac{i}{12}(\bar\theta\gamma_{(\alpha}\Gamma^{a'b'}\theta)(\bar\theta\hat\Gamma\Gamma_{a'b'}\Gamma'_*\mathfrak{D}_{\beta)}\theta) + \dots\,.\end{aligned} \tag{2.37}$$

Thus the first term in (2.21) is given by

$$\begin{aligned}&-\sqrt{-h} = -\sqrt{-\mathrm{h}} + i\sqrt{-\mathrm{h}}\,\mathrm{h}^{\alpha\beta}\bar\theta\gamma_\alpha\mathfrak{D}_\beta\theta + \sqrt{-g}\Big[\tfrac{i}{2}x^2\bar\theta\rlap{\,/}\mathfrak{D}\theta + \tfrac{i}{2}x^2\bar\theta\Gamma_{123}\theta + \tfrac{i}{4}(x^i\partial_\alpha x^j)(\bar\theta\gamma^\alpha\Gamma_{ij}\theta)\\ &+ \tfrac{i}{4}(y^{a'}\partial_\alpha y^{b'})(\bar\theta\gamma^\alpha\Gamma_{a'b'}\theta) + \tfrac12(\bar\theta\gamma^\rho\mathfrak{D}_\alpha\theta)(\bar\theta\gamma_\rho\mathfrak{D}^\alpha\theta) + \tfrac12(\bar\theta\rlap{\,/}\mathfrak{D}\theta)^2 - g^{\alpha\beta}g^{\gamma\delta}(\bar\theta\gamma_{(\alpha}\mathfrak{D}_{\gamma)}\theta)(\bar\theta\gamma_{(\beta}\mathfrak{D}_{\delta)}\theta)\\ &+ \tfrac{1}{12}(\bar\theta\Gamma_{123}\theta)(\bar\theta\rlap{\,/}\mathfrak{D}\theta) + \tfrac{i}{24}(\bar\theta\gamma^\alpha\Gamma^{ij}\theta)(\bar\theta\Gamma\Gamma_{ij}\Gamma_*\mathfrak{D}_\alpha\theta) - \tfrac{i}{24}(\bar\theta\gamma^\alpha\Gamma^{a'b'}\theta)(\bar\theta\Gamma\Gamma_{a'b'}\Gamma'_*\mathfrak{D}_\alpha\theta)\\ &+ \tfrac{i}{2}\partial^\alpha x^i\partial_\alpha x^j\,\bar\theta\Gamma_i\Gamma_{123}\Gamma_j\theta\Big] + \dots\,.\end{aligned} \tag{2.38}$$

Here $\mathrm{h}_{\alpha\beta}$ is the full bosonic part of the induced metric in (2.5) while $g_{\alpha\beta}$ is the AdS_2 metric, so that (cf. (2.6))

$$\sqrt{-\mathrm{h}}\,\mathrm{h}^{\alpha\beta} = \sqrt{-g}\Big[g^{\alpha\beta} - \partial^\alpha x^i\partial^\beta x^i - (\partial^\alpha y^{a'})(\partial^\beta y^{a'}) + \tfrac12 g^{\alpha\beta}\big((\partial x)^2 + (\partial y)^2\big) + \dots\Big]. \tag{2.39}$$

To find the contribution of the WZ term in (2.11) we note that (see (2.23))

$$\begin{aligned}s^{IJ}\bar\theta^I\Gamma^{\hat\alpha}L^J_s &= s\,s^{IJ}\bar\theta^I\Gamma^{\hat\alpha}D\theta^J - \tfrac{is^3}{6}\epsilon^{\hat\alpha\hat\beta}(\bar\theta\Gamma_{123}\theta)(\bar\theta\Gamma_{\hat\beta}\mathfrak{D}\theta)\\ &+ \tfrac{s^3}{12}(\bar\theta\Gamma^{\hat\alpha}\Gamma^{ij}\hat\Gamma\theta)(\bar\theta\hat\Gamma\Gamma_{ij}\Gamma_*\mathfrak{D}\theta) - \tfrac{s^3}{12}(\bar\theta\Gamma^{\hat\alpha}\Gamma^{a'b'}\hat\Gamma\theta)(\bar\theta\hat\Gamma\Gamma_{a'b'}\Gamma'_*\mathfrak{D}\theta) + \mathcal{O}(\theta^6)\,,\end{aligned} \tag{2.40}$$

$$s^{IJ}\bar\theta^I\Gamma^p L^J_s = s\,s^{IJ}\bar\theta^I\Gamma^p D\theta^J + \mathcal{O}(\theta^6). \tag{2.41}$$

Thus

$$\begin{aligned}&-2is^{IJ}\int_0^1 ds\,\epsilon^{\alpha\beta}L^A_{\alpha s}\bar\theta^I\Gamma^A L^J_{\beta s} = \sqrt{-g}\Big\{i(1+\tfrac12x^2+\dots)\bar\theta\gamma^\beta\Big[\mathfrak{D}_\beta\theta - \tfrac14x^4\Gamma_{123}\gamma_\beta\theta\\ &\quad - \tfrac14x^i\partial_\beta x^j\,\Gamma_{ij}\theta + \tfrac14y^{a'}\partial_\beta y^{b'}\,\Gamma_{a'b'}\theta + \dots\Big] + \tfrac{1}{12}(\bar\theta\Gamma_{123}\theta)(\bar\theta\rlap{\,/}\mathfrak{D}\theta)\\ &\quad + \tfrac{i}{24}(\bar\theta\gamma^\beta\Gamma^{ij}\theta)(\bar\theta\hat\Gamma\Gamma_{ij}\Gamma_*\mathfrak{D}_\beta\theta) - \tfrac{i}{24}(\bar\theta\gamma^\beta\Gamma^{a'b'}\theta)(\bar\theta\hat\Gamma\Gamma_{a'b'}\Gamma'_*\mathfrak{D}_\beta\theta)\\ &+ \tfrac12\Big[(\bar\theta\rlap{\,/}\mathfrak{D}\theta)^2 - (\bar\theta\gamma_\beta\mathfrak{D}_\alpha\theta)(\bar\theta\gamma^\alpha\mathfrak{D}^\beta\theta)\Big] + \tfrac{i}{2}x^j\partial_\alpha x^p(\bar\theta\gamma^\alpha\Gamma_p\Gamma_j\theta)\Big\} - \tfrac12\epsilon^{\alpha\beta}\partial_\alpha x^i\partial_\beta x^j(\bar\theta\Gamma_i\Gamma_*\Gamma_j\theta) + \dots\,,\end{aligned} \tag{2.42}$$

where we used that $\epsilon^{\alpha\beta}(\bar\theta\gamma_\delta\mathfrak{D}_\alpha\theta)(\bar\theta\gamma^\delta\hat\Gamma\mathfrak{D}_\beta\theta) = -\sqrt{-g}\big[(\bar\theta\rlap{\,/}\mathfrak{D}\theta)^2 - (\bar\theta\gamma_\beta\mathfrak{D}_\alpha\theta)(\bar\theta\gamma^\alpha\mathfrak{D}^\beta\theta)\big]$.

As a result, the fermionic part of the GS Lagrangian expanded to quartic order in the fluctuation fields is found (after rescaling $\theta\to\frac12\theta$ and factoring out $\sqrt{-g}$) to be:

$$\mathcal{L}_{\rm f} = \tfrac12 i\bar\theta\rlap{\,/}\mathfrak{D}\theta + \mathcal{L}_{4\rm f} + \dots\,, \tag{2.43}$$

$$\mathcal{L}_{4\rm f} = \tfrac14 i\Big[x^2g^{\alpha\beta} - \partial^\alpha x^i\partial^\beta x^i - \partial^\alpha y^{a'}\partial^\beta y^{a'} + \tfrac12g^{\alpha\beta}\big((\partial x)^2 + (\partial y)^2\big)\Big]\bar\theta\gamma_\alpha\mathfrak{D}_\beta\theta$$

$$+ \tfrac{i}{8}\big(\partial^\alpha x^i \partial_\alpha x^i + 2x^2 - \partial^\alpha y^{a'} \partial_\alpha y^{a'}\big)\bar\theta\Gamma_{123}\theta$$
$$+ \tfrac{1}{96}(\bar\theta\Gamma_{123}\theta)(\bar\theta \not{\mathfrak{D}}\theta) + \tfrac{i}{192}(\bar\theta\gamma^\alpha\Gamma^{ij}\theta)(\bar\theta\hat\Gamma\Gamma_{ij}\Gamma_*\mathfrak{D}_\alpha\theta) - \tfrac{i}{192}(\bar\theta\gamma^\alpha\Gamma^{a'b'}\theta)(\bar\theta\hat\Gamma\Gamma_{a'b'}\Gamma'_*\mathfrak{D}_\alpha\theta)$$
$$+ \tfrac{1}{16}(\bar\theta \not{\mathfrak{D}}\theta)^2 - \tfrac{1}{16}(\bar\theta\gamma^\alpha\mathfrak{D}^\beta\theta)(\bar\theta\gamma_\beta\mathfrak{D}_\alpha\theta) + \dots \ , \tag{2.44}$$

$$\mathfrak{D}_\alpha\theta \equiv \nabla_\alpha\theta + \tfrac{1}{2}\gamma_\alpha\Gamma_{123}\theta \ , \qquad \not{\mathfrak{D}} = \not\nabla + \Gamma_{123}. \tag{2.45}$$

Note that we have dropped the terms in (2.38),(2.42)

$$\Delta\mathcal{L} = \tfrac{i}{2}(x^i\partial_\alpha x^j)(\bar\theta\gamma^\alpha\Gamma_{ij}\theta) + \tfrac{i}{2}(y^{a'}\partial_\alpha y^{b'})(\bar\theta\gamma^\alpha\Gamma_{a'b'}\theta) - \frac{1}{2\sqrt{-g}}\epsilon^{\alpha\beta}\partial_\alpha x^i\partial_\beta x^j(\bar\theta\Gamma_i\Gamma_*\Gamma_j\theta) \ , \tag{2.46}$$

which will not contribute to the 2-loop free energy.

3. AdS propagators

Below we will use the dimensional reduction regularization by replacing AdS_2 with $AdS_{\rm d}$ with ${\rm d} = 1 + d$, $d = 1 - 2\varepsilon$ but not changing the number of fermionic components and keeping ${\rm d} = 2$ in gamma matrix contractions. Let us first review the standard expressions for the scalar and fermion propagators in AdS_{1+d}.

3.1. Scalar propagator in AdS_{1+d}

In Euclidean AdS_{1+d} with the Poincare coordinates ${\rm w}^\mu = (z, w^r)$ we have for a massive scalar field with the standard Dirichlet boundary conditions (see, e.g., [43])[17]

$$\hat S = \frac12\int d^{d+1}{\rm w}\,\sqrt{g}(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2), \qquad ds^2 = g_{\mu\nu}({\rm w})d{\rm w}^\mu d{\rm w}^\nu = \frac{1}{z^2}(dz^2 + dw_r dw^r) \ , \tag{3.1}$$

$$\langle\phi({\rm w})\phi({\rm w}')\rangle = G_\Delta({\rm w},{\rm w}'), \qquad \Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + m^2}, \qquad \Delta(\Delta - d) = m^2 \ , \tag{3.2}$$

$$(-\nabla^2 + m^2)G_\Delta({\rm w},{\rm w}') = \frac{1}{\sqrt{g}}\delta^{(d+1)}({\rm w} - {\rm w}') \ , \tag{3.3}$$

$$G_\Delta({\rm w},{\rm w}') = \frac{C_\Delta}{2\Delta - d}\left[\frac{v({\rm w},{\rm w}')}{2}\right]^\Delta {}_2F_1\Big(\frac{\Delta}{2}, \frac{\Delta}{2} + \frac12; \Delta - \frac{d}{2} + 1; v^2({\rm w},{\rm w}')\Big), \tag{3.4}$$

$$C_\Delta = \frac{\Gamma(\Delta)}{\pi^{d/2}\Gamma(\Delta - \frac{d}{2})} \ , \qquad v({\rm w},{\rm w}') \equiv \frac{2zz'}{z^2 + z'^2 + (w^r - w'^r)^2}. \tag{3.5}$$

In Minkowski AdS_{1+d} we get the same propagator with the replacement of the δ_{rs} metric in v in (3.5) by the Minkowski one. Using that

$${}_2F_1(a,b,c;1) = \frac{\Gamma(c)\Gamma(c-b-a)}{\Gamma(c-b)\Gamma(c-a)}, \qquad \Re(c-a-b) > 0, \quad c \neq 0,-1,-2,\dots, \tag{3.6}$$

and doing the analytic continuation in d to extend beyond the above conditions on the parameters a, b, c we get for the scalar propagator at the coinciding points

$$G_\Delta({\rm w},{\rm w}) = \frac{1}{(4\pi)^{(d+1)/2}}\frac{\Gamma(\frac12 - \frac{d}{2})\Gamma(\Delta)}{\Gamma(1 - d + \Delta)}. \tag{3.7}$$

The first derivatives of the propagator taken at the coinciding points vanish, $\partial_\mu G_\Delta({\rm w},{\rm w}')\big|_{{\rm w}={\rm w}'} = 0$ (note that for v in (3.5) one has $\partial_\mu v({\rm w},{\rm w}')\big|_{{\rm w}={\rm w}'} = 0$). We need also the value of the second derivative[18] in the limit ${\rm w}' \to {\rm w}$. We get using again ((3.6))

$$\nabla_\mu\nabla'^\mu G_\Delta({\rm w},{\rm w}')\big|_{{\rm w}={\rm w}'} = \frac{d+1}{2}\frac{1}{(4\pi)^{(d+1)/2}}\frac{\Gamma(-\frac12 - \frac{d}{2})\Gamma(\Delta + 1)}{\Gamma(-d + \Delta)} = -m^2 G_\Delta({\rm w},{\rm w}) \ , \tag{3.8}$$

where we used (3.7) and that $\Delta(\Delta - d) = m^2$. This relation is in agreement with (3.3) and $\delta^{(1+d)}({\rm w},{\rm w}) = 0$ in dimensional regularization. Similarly, one finds that

$$\nabla_\mu\nabla'_\nu G_\Delta({\rm w},{\rm w}')\big|_{{\rm w}={\rm w}'} = \frac12 g_{\mu\nu}\frac{1}{(4\pi)^{(d+1)/2}}\frac{\Gamma(-\frac12 - \frac{d}{2})\Gamma(\Delta + 1)}{\Gamma(-d + \Delta)} = -\frac{m^2}{d+1}g_{\mu\nu}G_\Delta({\rm w},{\rm w}). \tag{3.9}$$

Setting $d = 1 - 2\varepsilon$ and expanding in small ε with Δ defined as in (3.2) we find

$$G_\Delta({\rm w},{\rm w}) = \frac{1}{4\pi\varepsilon} + \frac{1}{4\pi}\big[\log(4\pi) - \gamma_{\rm E} - 2\psi(\Delta_0)\big] + \cdots, \qquad \Delta_0 \equiv \Delta\big|_{\varepsilon=0} = \frac12 + \sqrt{\frac14 + m^2}. \tag{3.10}$$

[17] The Dirichlet conditions correspond to a supersymmetric (Maldacena-Wilson loop) case. If one uses the Neumann boundary conditions for the S^5 massless modes that would correspond to the case of the standard (non-BPS) WL [44].

[18] Here and below ∂'_μ stands for the derivative over ${\rm w}'$ with index μ.

For the special values of the masses we are interested in we get

$$m^2 = 0: \quad \Delta_0 = 1, \quad \psi(\Delta_0) = -\gamma_E \; ; \qquad m^2 = 2: \quad \Delta_0 = 2, \quad \psi(\Delta_0) = 1 - \gamma_E. \tag{3.11}$$

Note that for the massless field the expressions in (3.8),(3.9) vanish; as a result, the correlators of the y^a field in (2.6) will not contribute to the 2-loop free energy.

For the propagator of the $m^2 = 2$ scalar we will introduce the special notation

$$\mathrm{G}(\mathrm{w}, \mathrm{w}') \equiv G_\Delta(\mathrm{w}, \mathrm{w}')\Big|_{m^2=2}, \qquad \Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + 2}. \tag{3.12}$$

Expanding in $\varepsilon = \frac{1}{2}(1-d) \to 0$ we get[19]

$$\mathrm{G} \equiv \mathrm{G}(\mathrm{w}, \mathrm{w}) = \frac{1}{4\pi\varepsilon'} - \frac{1}{2\pi} + \mathcal{O}(\varepsilon), \qquad \frac{1}{\varepsilon'} \equiv \frac{1}{\varepsilon} + \ell, \qquad \ell = \log(4\pi e^{\gamma_E}), \tag{3.13}$$

$$\nabla_\mu \nabla'^\mu \mathrm{G}(\mathrm{w}, \mathrm{w}')\Big|_{\mathrm{w}=\mathrm{w}'} = -2\mathrm{G} = -\frac{1}{2\pi\varepsilon'} + \frac{1}{\pi} + \mathcal{O}(\varepsilon), \tag{3.14}$$

$$\nabla_\mu \nabla'_\nu \mathrm{G}(\mathrm{w}, \mathrm{w}')\Big|_{\mathrm{w}=\mathrm{w}'} = -\frac{2}{d+1} g_{\mu\nu} \mathrm{G} = g_{\mu\nu}\Big[-\frac{1}{4\pi\varepsilon'} + \frac{1}{4\pi} + \mathcal{O}(\varepsilon)\Big]. \tag{3.15}$$

3.2. Spinor propagator in AdS_{1+d}

Let us consider the spinor field in Euclidean AdS_{1+d} in Poincare coordinates with the action

$$S_F = \int d^{d+1}\mathrm{w}\, \sqrt{g}\, \bar{\psi}(\nabla\!\!\!/ - M)\psi, \tag{3.16}$$

where $\nabla\!\!\!/ = e^\alpha_{\hat\alpha}\gamma^{\hat\alpha}(\partial_\alpha + \frac{1}{2}\omega_\alpha^{\hat\alpha\hat\beta}\Omega_{\hat\alpha\hat\beta})$, $\gamma^{(\hat\alpha}\gamma^{\hat\beta)} = \delta^{\hat\alpha\hat\beta}$, $\Omega^{\hat\alpha\hat\beta} = \frac{1}{4}[\gamma^{\hat\alpha}, \gamma^{\hat\beta}]$, $e^{\hat\alpha}_\alpha = \frac{1}{z}\delta^{\hat\alpha}_\alpha$, $\omega^{\hat\alpha\hat\beta}_{\hat\gamma} = \frac{1}{z}(\delta^{\hat\alpha}_0\delta^{\hat\beta}_{\hat\gamma} - \delta^{\hat\beta}_0\delta^{\hat\alpha}_{\hat\gamma})$. Explicitly (cf., e.g., [45])

$$\nabla\!\!\!/ = z\gamma^{\hat\alpha}\partial_{\hat\alpha} - \tfrac{d}{2}\gamma_{\hat 0}, \qquad \partial_{\hat\alpha} = \delta^\alpha_{\hat\alpha}\partial_\alpha, \tag{3.17}$$

where $\hat 0$ denotes the tangent index in the z-direction. Let us consider the corresponding propagator defined by

$$(\nabla\!\!\!/ - M)S(\mathrm{w}, \mathrm{w}') = \frac{1}{\sqrt{g}}\delta^{(d+1)}(\mathrm{w}, \mathrm{w}'). \tag{3.18}$$

In the standard case when M commutes with $\gamma_{\hat\alpha}$, e.g. if $M = mI$, the propagator is given by (see, e.g., [46])

$$S(\mathrm{w}, \mathrm{w}') = -\frac{1}{2^{m+(d+3)/2}\pi^{d/2}} \frac{\Gamma(m + \frac{d+1}{2})}{\Gamma(m + \frac{1}{2})} \frac{1}{\sqrt{zz'}} \frac{1}{(u+2)^{m+(d+1)/2}}$$
$$\times \Big[(\mathrm{w}\!\!\!/\gamma_{\hat 0} + \gamma_{\hat 0}\mathrm{w}\!\!\!/')\,\mathrm{F}_1(u) - (\mathrm{w}\!\!\!/ - \mathrm{w}\!\!\!/')\,\mathrm{F}_2(u)\Big], \tag{3.19}$$

$$u(\mathrm{w}, \mathrm{w}') \equiv \frac{(z-z')^2 + (w^r - w'^r)^2}{2zz'}, \qquad \mathrm{w}\!\!\!/ = \gamma_\alpha \mathrm{w}^\alpha, \tag{3.20}$$

$$\mathrm{F}_1(u) = {}_2F_1(m + \tfrac{d+1}{2}, m, 2m+1, \tfrac{2}{u+2}), \qquad \mathrm{F}_2(u) = {}_2F_1(m + \tfrac{d+1}{2}, m+1, 2m+1, \tfrac{2}{u+2}). \tag{3.21}$$

One can check that if the mass term M anticommutes with γ_α, i.e.

$$M = -m\bar\gamma, \qquad \{\gamma^{\hat\alpha}, \bar\gamma\} = 0, \qquad \bar\gamma^2 = -1, \qquad m > 0, \tag{3.22}$$

eq. (3.18) is solved by a similar matrix function

$$S(\mathrm{w}, \mathrm{w}') = -\frac{1}{2^{m+(d+3)/2}\pi^{d/2}} \frac{\Gamma(m + \frac{d+1}{2})}{\Gamma(m + \frac{1}{2})} \frac{1}{\sqrt{zz'}} \frac{1}{(u+2)^{m+(d+1)/2}}$$
$$\times \Big[\bar\gamma\,(\mathrm{w}\!\!\!/\gamma_{\hat 0} + \gamma_{\hat 0}\mathrm{w}\!\!\!/')\,\mathrm{F}_1(u) - (\mathrm{w}\!\!\!/ - \mathrm{w}\!\!\!/')\,\mathrm{F}_2(u)\Big]. \tag{3.23}$$

The same expression is found in the case of Minkowski signature AdS_{1+d} space provided one uses the Minkowski metric in contracting indices of w^r in $u(\mathrm{w}, \mathrm{w}')$ in (3.20).

In the GS string case we are interested in (see (2.45)) we have

$$\bar\gamma = \Gamma_{123}, \qquad \gamma_{\hat 0} = \Gamma_4. \tag{3.24}$$

Note that in the special case of $d = 1$ and $m = 1$ one finds explicitly

$$S(\mathrm{w}, \mathrm{w}') = -\frac{1}{4\pi}\frac{1}{\sqrt{zz'}}\frac{1}{(u+2)^2}\Big[\Gamma_{123}(\mathrm{w}\!\!\!/\Gamma_4 + \Gamma_4\mathrm{w}\!\!\!/')\,\mathrm{F}_1(u) - (\mathrm{w}\!\!\!/ - \mathrm{w}\!\!\!/')\,\mathrm{F}_2(u)\Big], \tag{3.25}$$

[19] For the massless field with $\Delta = d$ we have from (3.10),(3.11) $G_\Delta(\mathrm{w}, \mathrm{w})\Big|_{m^2=0} = \frac{1}{4\pi\varepsilon'} + \mathcal{O}(\varepsilon)$.

$$\mathrm{F}_1(u) = -\tfrac{1}{2}(u+2)(2+(u+2)\log\tfrac{u}{u+2})\,, \qquad \mathrm{F}_2(u) = \tfrac{(u+2)^2}{2u}(2+u\log\tfrac{u}{u+2}). \tag{3.26}$$

In the limit of coinciding points, i.e. $\mathrm{w}=\mathrm{w}'$ and $u=0$, using ((3.6)) we get from (3.21)

$$\mathrm{F}_1(0) = {}_2F_1(m+\tfrac{d+1}{2}, m, 2m+1, 1) = \frac{2^{2m}\Gamma(m+\frac{1}{2})}{\sqrt{\pi}}\frac{\Gamma(\frac{1-d}{2})}{\Gamma(\frac{1-d}{2}+m)}\,, \tag{3.27}$$

$$\mathrm{F}_2(0) = {}_2F_1(m+\tfrac{d+1}{2}, m+1, 2m+1, 1) = \frac{\Gamma(1+2m)}{\Gamma(m)}\frac{\Gamma(-\frac{d+1}{2})}{\Gamma(\frac{1-d}{2}+m)}. \tag{3.28}$$

The factor in the second line in ((3.23)) reduces to $\Gamma_{123}\{\not{w},\Gamma_4\}\ \mathrm{F}_1(0) \to 2\sigma\Gamma_{123}\ \mathrm{F}_1(0)$ (where $\sigma = w^4$) so that we get (cf. (1.8))

$$\mathcal{S}(\mathrm{w},\mathrm{w}) = \mathrm{S}\,\Gamma_{123}, \qquad \mathrm{S} = -\frac{1}{(4\pi)^{(d+1)/2}}\,\frac{\Gamma(\frac{1-d}{2})\Gamma(\frac{1+d}{2}+m)}{\Gamma(\frac{1-d}{2}+m)}. \tag{3.29}$$

In the case of $m=1$ and $d=1-2\varepsilon$ taking $\varepsilon\to 0$ (and using the same notation as in (3.13)) we find

$$\mathrm{S} = -\frac{1}{4\pi\varepsilon'} + \frac{1}{4\pi} + \mathcal{O}(\varepsilon). \tag{3.30}$$

We will also need the expressions for the derivative of the spinor propagator

$$\nabla_\alpha \mathcal{S}(\mathrm{w},\mathrm{w}')\Big|_{\mathrm{w}=\mathrm{w}'} = \frac{1}{2(4\pi)^{(d+1)/2}}\frac{\Gamma(-\frac{d+1}{2})\Gamma(\frac{d+1}{2}+m)}{\Gamma(\frac{1-d}{2}+m)}\,\frac{m}{z}\Gamma_{\hat\alpha} = \frac{1}{1+d}\,\frac{m}{z}\Gamma_{\hat\alpha}\,\mathrm{S}. \tag{3.31}$$

As a check, we may compute $\not{\nabla}\mathcal{S}(\mathrm{w},\mathrm{w}')\Big|_{\mathrm{w}=\mathrm{w}'}$ and find that the resulting expression is consistent with the definition of the Dirac propagator ($\Gamma_{123}^2=-1$)

$$\not{\nabla}\mathcal{S}(\mathrm{w},\mathrm{w}')\Big|_{\mathrm{w}=\mathrm{w}'} = -m\Gamma_{123}\mathcal{S}(\mathrm{w},\mathrm{w}) = m\,\mathrm{S}. \tag{3.32}$$

Note that S in (3.29),(3.30) representing the $m^2=1$ fermion propagator at the coinciding points is related to the $m^2=2$ and $m^2=0$ values of the bosonic propagators given in (3.13) and the footnote (19) as

$$\mathrm{S} = -\tfrac{1}{2}\Big[G_\Delta(\mathrm{w},\mathrm{w})\Big|_{m^2=2} + G_\Delta(\mathrm{w},\mathrm{w})\Big|_{m^2=0}\Big]. \tag{3.33}$$

As was already mentioned above, for the correlators of the GS fermions we will use the dimensional reduction regularization prescription: we will not continue the world-volume components of the gamma-matrices to $d=1-2\varepsilon$ doing all spinor index contractions and traces in $1+d=2$. In particular, we will use that $\gamma_\alpha\gamma^\alpha=2$ rather than $1+d$ so that (3.32) will be modified to

$$\not{\nabla}\mathcal{S}(\mathrm{w},\mathrm{w}')\Big|_{\mathrm{w}=\mathrm{w}'} = \frac{2}{1+d}m\,\mathrm{S}. \tag{3.34}$$

4. 2-loop contribution to string free energy

Our aim is to compute the 2-loop coefficient f_2 in the string free energy in (1.3),(1.4) starting with the quartic terms (2.6),(2.44) in the GS string Lagrangian. In the dimensional regularization there will be no contributions from possible local measure and ghost determinants for the (algebraic) static bosonic and fermionic κ-symmetry gauges.

4.1. Bosonic contribution

In the Euclidean notation where instead of (2.4) we have the path integral with $e^{-\hat S}$ where $\hat S = \hat S_\mathrm{b} + \hat S_\mathrm{f}$ and $\hat S_\mathrm{b} = T\int d^2\xi\,\sqrt{g}\,(1-\mathcal{L}_\mathrm{b}) = TV - T\int d^2\xi\,\sqrt{g}\,\mathcal{L}_\mathrm{b}$. Here $\mathcal{L}_\mathrm{b} = -\frac{1}{2}(\partial x_i)^2 + \ldots + \mathcal{L}_{4x} + \ldots$ with the same signs as in (2.6) but now with the Euclidean $g_{\mu\nu}$ comparing to (1.4). Then we have for the 2-loop coefficient in (1.3),(1.4)

$$f_2 = \langle \mathcal{L}_{4x} + \ldots\rangle. \tag{4.1}$$

Here dots stand for other quartic vertices. We used that the expectation value of the Lagrangian (computed with the Euclidean AdS_2 propagator $\langle xx\rangle$ after rescaling by T) will be constant so the AdS_2 volume V can be factored out.

Since the y^a dependent terms in the Lagrangian in (2.6)–(2.10) always contain factors with derivatives and since in dimensional regularization the derivatives of the massless field propagator taken at coinciding points vanish (cf. (3.9)) only the $m^2=2$ field x^i correlators will contribute to f_2 in (4.1).

Let us add for generality some coefficients b_k in front of each quartic term in ((2.8)), i.e. consider

$$\mathcal{L}_{4x} = L_{4x}\Big|_{b_{k=1}}\,, \qquad L_{4x} = -\tfrac{1}{8}b_1(\partial\boldsymbol{x}\cdot\partial\boldsymbol{x})^2 + \tfrac{1}{4}b_2(\partial x^i\partial x^j)(\partial x^i\partial x^j) - \tfrac{1}{4}b_3\boldsymbol{x}^2(\partial\boldsymbol{x}\cdot\partial\boldsymbol{x}) - \tfrac{1}{2}b_4(\boldsymbol{x}^2)^2. \tag{4.2}$$

Then using the dimensional regularization relations (3.12)-(3.15) we find

$$\langle L_{4x}\rangle = q_\mathrm{b}\,\mathrm{G}^2\,, \quad q_\mathrm{b} = -b_1\tfrac{N_x(2+(d+1)N_x)}{2(d+1)} + b_2\tfrac{N_x(2+d+N_x)}{d+1} + \tfrac{1}{2}b_3N_x^2 - \tfrac{1}{2}b_4N_x(N_x+2)\,, \quad N_x=3\,, \tag{4.3}$$

where N_x stands for the number of x^i fields.[20]

Expanding the coefficient $q_{\rm b}$ for $d = 1 - 2\varepsilon \to 1$ we get

$$q_{\rm b}\Big|_{d\to 1} = \tfrac{1}{2}N_x\big[-b_1(1+N_x)+b_2(3+N_x)+b_3N_x-b_4(2+N_x)\big]+\tfrac{1}{2}N_x\big[-b_1+b_2(N_x+1)\big]\varepsilon+\mathcal{O}(\varepsilon^2). \tag{4.4}$$

Since according to (3.13) $\mathrm{G}^2 = (\frac{1}{4\pi\varepsilon}+\ldots)^2$ we conclude that the double pole term in (4.3) cancels out for the relevant values of $b_k = 1$ for *any* value of N_x. The coefficient of the remaining single pole is then proportional to $\frac{1}{2}N_x\big[-b_1+b_2(N_x+1)\big] = \frac{1}{2}N_x^2$.

We conclude that in the case of $b_k = 1$ the bosonic contribution to f_2 in (4.1) is given by

$$f_{2\,\rm b} = \langle \mathcal{L}_{4x}\rangle = q_{\rm b}\,\mathrm{G}^2\ , \qquad\qquad q_{\rm b}\Big|_{b_k=1} = \frac{N_x^2(1-d)}{2(1+d)} = \frac{9(1-d)}{2(1+d)}. \tag{4.5}$$

4.2. Fermionic contribution

In computing the expectation values of the fermionic terms we will assume the formal continuation to the 2d Euclidean signature in the GS action and thus in the fermionic propagator $\langle\theta(\mathrm{w})\bar\theta(\mathrm{w}')\rangle = S(\mathrm{w},\mathrm{w}')$ (cf. (3.23)). The fermionic contributions to f_2 are given by

$$f_{2\,\rm bf} = i\langle\mathcal{L}_{x^2\theta^2}\rangle\ , \qquad\qquad f_{2\,\rm f} = \langle\mathcal{L}_{\theta^4}\rangle\ , \tag{4.6}$$

where the relevant terms in the fermionic part of the GS Lagrangian in (2.44) may be written as[21]

$$\mathcal{L}_{\rm f} = \mathcal{L}_{\theta^2}+\mathcal{L}_{x^2\theta^2}+\mathcal{L}_{\theta^4}\ ,\qquad \mathcal{L}_{x^2\theta^2} = L_{x^2\theta^2}\Big|_{c_1=1}\ ,\qquad \mathcal{L}_{\theta^4} = L_{\theta^4}\Big|_{c_k=1} \tag{4.7}$$

$$\mathcal{L}_{\theta^2} = \tfrac{i}{2}\bar\theta\,\mathfrak{D}\!\!\!/\,\theta\ ,\qquad \mathfrak{D}_\alpha = \nabla_\alpha+\tfrac{1}{2}\gamma_\alpha\Gamma_{123},\qquad \mathfrak{D}\!\!\!/ = \nabla\!\!\!/+\Gamma_{123}\ , \tag{4.8}$$

$$L_{x^2\theta^2} = \tfrac{i}{4}(\boldsymbol{x}^2+\tfrac{1}{2}\partial_\alpha\boldsymbol{x}\cdot\partial^\alpha\boldsymbol{x})\,\bar\theta\mathfrak{D}\!\!\!/\theta+\tfrac{i}{4}(\boldsymbol{x}^2+\tfrac{1}{2}\partial_\alpha\boldsymbol{x}\cdot\partial^\alpha\boldsymbol{x})\,\bar\theta\Gamma_{123}\theta-\tfrac{i}{4}c_1\partial^\alpha\boldsymbol{x}\cdot\partial^\beta\boldsymbol{x}\,\bar\theta\gamma_\alpha\mathfrak{D}_\beta\theta\ , \tag{4.9}$$

$$\begin{aligned} L_{\theta^4} =\ & \tfrac{1}{96}c_2\bar\theta\Gamma_{123}\theta\,\bar\theta\mathfrak{D}\!\!\!/\theta+\tfrac{i}{192}c_3\bar\theta\gamma^\alpha\Gamma^{ij}\theta\,\bar\theta\hat\Gamma\Gamma_{ij}\Gamma_*\mathfrak{D}_\alpha\theta-\tfrac{i}{192}c_4\bar\theta\gamma^\alpha\Gamma^{a'b'}\theta\,\bar\theta\hat\Gamma\Gamma_{a'b'}\Gamma'_*\mathfrak{D}_\alpha\theta \\ & +\tfrac{1}{16}c_5\,(\bar\theta\mathfrak{D}\!\!\!/\theta)^2-\tfrac{1}{16}c_6\,\bar\theta\gamma_\alpha\mathfrak{D}_\beta\theta\,\bar\theta\gamma^\beta\mathfrak{D}^\alpha\theta. \end{aligned} \tag{4.10}$$

We have omitted mixed boson-fermion terms with the massless field $\partial y\partial y$ factors as they will not contribute to the 2-loop free energy (cf. (3.9)). As in (4.2) we introduced some auxiliary coefficients $c_1,\ldots,c_6$ (equal to 1 in the GS Lagrangian in (2.44)) as this will allow us to trace the contributions of the individual quartic vertices.

Let us first observe that the first two terms in (4.9) will not contribute to (4.6) since due to (3.9) and (3.14) (or, equivalently, the $m^2 = 2$ scalar equation of motion) we have

$$\langle\partial_\alpha\boldsymbol{x}\cdot\partial_\beta\boldsymbol{x}\rangle = -\tfrac{2}{d+1}g_{\alpha\beta}\langle\boldsymbol{x}^2\rangle\ ,\qquad\qquad \langle\partial_\alpha\boldsymbol{x}\cdot\partial^\alpha\boldsymbol{x}\rangle = -2\langle\boldsymbol{x}^2\rangle. \tag{4.11}$$

As already mentioned, we will use the dimensional reduction prescription, i.e. will keep $\Gamma_{\hat\alpha} = \gamma_{\hat\alpha}$ in 2 dimensions, so that

$$\{\gamma_\alpha,\gamma_\beta\} = 2\bar g_{\alpha\beta},\qquad \bar g^{\alpha\beta}\bar g_{\alpha\beta} = 2\ ,\qquad g^{\alpha\beta}\bar g_{\alpha\beta} = 2\ , \tag{4.12}$$

where $\bar g_{\alpha\beta}$ is the metric of AdS_2 . Also, contracting γ_α with derivatives will select only the 2d components of the latter.

Note that under $\langle\ldots\rangle$ one may use (4.11) to rewrite the contribution of the remaining c_1 term in (4.9) using (4.8) as

$$\langle L_{x^2\theta^2}\rangle = -\tfrac{i}{4}c_1\langle\partial^\alpha\boldsymbol{x}\cdot\partial^\beta\boldsymbol{x}\,\bar\theta\gamma_\alpha\mathfrak{D}_\beta\theta\rangle = \tfrac{i}{2(d+1)}c_1\langle\boldsymbol{x}^2\bar\theta D\!\!\!/\theta\rangle. \tag{4.13}$$

This term is thus proportional to the expectation value of the fermion kinetic term suggesting that its contribution should be scheme-dependent (formally, it can be redefined away but it will still contribute under the dimensional reduction regularization, cf. (3.34)).

Using the definition of D_α and the Majorana spinor relation $\bar\theta\gamma_\alpha\gamma_\beta\Gamma_{123}\theta = \bar g_{\alpha\beta}\bar\theta\Gamma_{123}\theta$ we may rewrite (4.10) as

$$\begin{aligned} L_{\theta^4} =\ & \tfrac{1}{96}(c_2+12c_5-6c_6)\bar\theta\Gamma_{123}\theta\,\bar\theta\nabla\!\!\!/\theta+\tfrac{1}{96}(c_2+6c_5-3c_6)(\bar\theta\Gamma_{123}\theta)^2 \\ & +\tfrac{i}{192}c_3\bar\theta\gamma^\alpha\Gamma^{ij}\theta\,\bar\theta\hat\Gamma\Gamma_{ij}\Gamma_*\nabla_\alpha\theta-\tfrac{i}{192}c_4\bar\theta\gamma^\alpha\Gamma^{a'b'}\theta\,\bar\theta\hat\Gamma\Gamma_{a'b'}\Gamma'_*\nabla_\alpha\theta \\ & +\tfrac{i}{384}c_3\bar\theta\gamma^\alpha\Gamma^{ij}\theta\,\bar\theta\hat\Gamma\Gamma_{ij}\Gamma_*\gamma_\alpha\Gamma_{123}\theta-\tfrac{i}{384}c_4\bar\theta\gamma^\alpha\Gamma^{a'b'}\theta\,\bar\theta\hat\Gamma\Gamma_{a'b'}\Gamma'_*\gamma_\alpha\Gamma_{123}\theta \\ & +\tfrac{1}{16}c_5(\bar\theta\nabla\!\!\!/\theta)^2-\tfrac{1}{16}c_6\bar\theta\gamma_\alpha\nabla_\beta\theta\,\bar\theta\gamma^\beta\nabla^\alpha\theta. \end{aligned} \tag{4.14}$$

Using also that $\Gamma_{11}\theta = \theta$ and $(i,j = 1,2,3;\ a' = 5,\ldots,9)$

$$\hat\Gamma\Gamma_{ij}\Gamma_* = -i\Gamma_{ij}\Gamma_{123},\qquad \hat\Gamma\Gamma_{a'b'}\Gamma'_* = \hat\Gamma\Gamma_{a'b'}\Gamma_*\Gamma_{11} = -i\Gamma_{a'b'}\Gamma_{123}\Gamma_{11}\ , \tag{4.15}$$

we may also rewrite (4.14) as

$$L_{\theta^4} = \tfrac{1}{96}(c_2+12c_5-6c_6)\bar\theta\Gamma_{123}\theta\,\bar\theta\nabla\!\!\!/\theta+\tfrac{1}{96}(c_2+6c_5-3c_6)(\bar\theta\Gamma_{123}\theta)^2$$

[20] Note that in the dimensional reduction regularization the derivatives of scalar bosons are treated as in the standard dimensional regularization, i.e. they are assumed to be $d = 2-2\varepsilon$ dimensional ones. If their indices were directly contracted in 2 instead of $2-2\varepsilon$ dimensions that would be equivalent to setting $d = 1$ in the coefficient in (4.3) and then the final result in (4.5) would be zero.

[21] We follow the notation in section (2.2), i.e. $\hat\Gamma = \Gamma_{04}$, $\Gamma_* = i\Gamma_{01234}$, $\Gamma'_* = i\Gamma_{56789}$, $\Gamma_{11} = \Gamma_*\Gamma'_*$, γ_α is the AdS_2 2-bein projection of $\Gamma_{\hat\alpha} = (\Gamma_0,\Gamma_4)$, etc.

$$
\begin{aligned}
&+\tfrac{i}{192}c_3\bar\theta\gamma^\alpha\Gamma^{ij}\theta\,\bar\theta\hat\Gamma\Gamma_{ij}\Gamma_*\nabla_\alpha\theta-\tfrac{i}{192}c_4\bar\theta\gamma^\alpha\Gamma^{a'b'}\theta\,\bar\theta\hat\Gamma\Gamma_{a'b'}\Gamma'_*\nabla_\alpha\theta\\
&+\tfrac{i}{384}c_3\bar\theta\gamma^\alpha\Gamma^{ij}\theta\,\bar\theta\hat\Gamma\Gamma_{ij}\Gamma_*\gamma_\alpha\Gamma_{123}\theta-\tfrac{i}{384}c_4\bar\theta\gamma^\alpha\Gamma^{a'b'}\theta\,\bar\theta\hat\Gamma\Gamma_{a'b'}\Gamma'_*\gamma_\alpha\Gamma_{123}\theta\\
&+\tfrac{1}{16}c_5(\bar\theta\nabla\!\!\!\!/\,\theta)^2-\tfrac{1}{16}c_6\bar\theta\gamma_\alpha\nabla_\beta\theta\,\bar\theta\gamma^\beta\nabla^\alpha\theta. \qquad (4.16)
\end{aligned}
$$

To compute the expectation values of (4.13) and (4.14) we need to take into account that θ are Majorana. [22] As a result, we get switching to the Euclidean propagators in $\langle\ldots\rangle$ (using (3.12),(3.29),(3.34),(A.2) and that θ are subject to (2.28))[23]

$$
f_{2\,\rm bf}=i\langle L_{x^2\theta^2}\rangle=-\tfrac{c_1}{2(d+1)}\langle \boldsymbol{x}^2\rangle\,J=-\tfrac{c_1}{2(d+1)}N_x\mathrm{G}\,J\,,\qquad J=\langle\bar\theta(\nabla\!\!\!\!/+\Gamma_{123})\theta\rangle=J_1+J_2\,, \qquad (4.17)
$$

$$
J_1=\langle\bar\theta\nabla\!\!\!\!/\,\theta\rangle=-\tfrac{2}{d+1}N_\theta\,\mathrm{S}\,,\qquad J_2=\langle\bar\theta\Gamma_{123}\theta\rangle=N_\theta\,\mathrm{S}\,,\qquad J=\tfrac{d-1}{d+1}N_\theta\,\mathrm{S}\,, \qquad (4.18)
$$

$$
N_x=3\,,\qquad N_\theta\equiv \mathrm{tr}\,P_+=16. \qquad (4.19)
$$

Similarly, the contribution of the quartic fermionic terms in (4.14) is given by (see Appendix A)

$$
f_{2\,\rm f}=\langle L_{\theta^4}\rangle=\sum_{k=1}^{8}I_k\,, \qquad (4.20)
$$

$$
I_1=\tfrac{1}{96}(c_2+12c_5-6c_6)\bar I_1\,,\qquad \bar I_1=\langle\bar\theta\nabla\!\!\!\!/\,\theta\,\bar\theta\Gamma_{123}\theta\rangle=-\tfrac{2}{d+1}N_\theta(N_\theta-2)\,\mathrm{S}^2\,, \qquad (4.21)
$$

$$
I_2=\tfrac{1}{96}(c_2+6c_5-3c_6)\bar I_2\,,\qquad \bar I_2=\langle\bar\theta\Gamma_{123}\theta\,\bar\theta\Gamma_{123}\theta\rangle=N_\theta(N_\theta-2)\mathrm{S}^2\,, \qquad (4.22)
$$

$$
I_3=\tfrac{1}{192}c_3\bar I_3\,,\qquad \bar I_3=\langle\bar\theta\gamma^\alpha\Gamma^{ij}\theta\,\bar\theta\Gamma_{ij}\Gamma_{123}\nabla_\alpha\theta\rangle=-\tfrac{24}{d+1}N_\theta\mathrm{S}^2\,, \qquad (4.23)
$$

$$
I_4=-\tfrac{1}{192}c_4\bar I_4\,,\qquad \bar I_4=\langle\bar\theta\gamma^\alpha\Gamma^{a'b'}\theta\,\bar\theta\Gamma_{a'b'}\Gamma_{123}\nabla_\alpha\theta\rangle=-\tfrac{80}{d+1}N_\theta\mathrm{S}^2\,, \qquad (4.24)
$$

$$
I_5=\tfrac{1}{384}c_3\,\bar I_5\,,\qquad \bar I_5=\langle\bar\theta\gamma^\alpha\Gamma^{ij}\theta\,\bar\theta\Gamma_{ij}\gamma_\alpha\theta\rangle=24N_\theta\mathrm{S}^2\,, \qquad (4.25)
$$

$$
I_6=-\tfrac{1}{384}c_4\,\bar I_6\,,\qquad \bar I_6=\langle\bar\theta\gamma^\alpha\Gamma^{a'b'}\theta\,\bar\theta\Gamma_{a'b'}\gamma_\alpha\theta\rangle=80N_\theta\mathrm{S}^2\,, \qquad (4.26)
$$

$$
I_7=\tfrac{1}{16}c_5\,\bar I_7,\qquad \bar I_7=\langle\bar\theta\nabla\!\!\!\!/\,\theta\,\bar\theta\nabla\!\!\!\!/\,\theta\rangle=\Big[\tfrac{4}{(d+1)^2}(N_\theta-1)-\tfrac12+\tfrac{d(d+1)-4}{2(d+1)}\Big]N_\theta\,\mathrm{S}^2\,, \qquad (4.27)
$$

$$
I_8=-\tfrac{1}{16}c_6\,\bar I_8,\qquad \bar I_8=\langle\bar\theta\Gamma_\alpha\nabla_\beta\theta\,\bar\theta\Gamma^\beta\nabla^\alpha\theta\rangle=\Big[\tfrac{2}{(d+1)^2}(N_\theta-2)+\tfrac12+\tfrac{d(d+1)-4}{2(d+1)}\Big]N_\theta\,\mathrm{S}^2. \qquad (4.28)
$$

As a result, we get for (4.17),(4.20)

$$
f_{2\,\rm bf}=q_{\rm f}\,\mathrm{G}\,\mathrm{S}\,,\qquad q_{\rm f}=c_1\,\frac{24(1-d)}{(d+1)^2}\,, \qquad (4.29)
$$

$$
f_{2\,\rm f}=q_{\rm f}\,\mathrm{S}^2\,,\qquad q_{\rm f}=-\mathrm{c}\,\frac{1-d}{6\,(d+1)^2}\,,\qquad \mathrm{c}\Big|_{c_k=1}=-12(5-3d) \qquad (4.30)
$$

$$
\mathrm{c}=4(7c_2+3c_3-10c_4-6c_6)+2(7c_2+3c_3-10c_4+48c_5-30c_6)(d-1)+3(c_5-c_6)(d-1)^2. \qquad (4.31)
$$

Setting all $c_k=1$ we thus find the values of $q_{\rm f}$ and $q_{\rm f}$ given in (1.10).

Note that the contributions of the c_2, c_3 and c_4 terms in (4.10) effectively cancel each other in c, i.e. $f_{2\,\rm f}$ receives non-zero contributions only from the c_5 and c_6 terms. Also, the coefficient of the simple pole in $f_{2\,\rm f}$ (coming only from the first $4(7c_2+3c_3-10c_4-6c_6)$ term in c in (4.31)) does not depend on the contribution of the $c_5(\bar\theta D\!\!\!\!/\,\theta)^2$ term in (4.10).[24]

To conclude, the total value of the 2-loop coefficient is found to be

$$
f_2=q_{\rm b}\mathrm{G}^2+q\mathrm{f}\mathrm{G}\mathrm{S}+q_{\rm f}\mathrm{S}^2=\frac{9(1-d)}{2(1+d)}\mathrm{G}^2+\frac{24(1-d)}{(1+d)^2}\mathrm{G}\,\mathrm{S}+\frac{2(1-d)(5-3d)}{(1+d)^2}\mathrm{S}^2\,, \qquad (4.32)
$$

where G and S are given by (3.13) and (3.29) (or (1.6) and (1.8)).

5. Concluding remarks

As was already discussed in the Introduction (see (1.11)), expanding (4.32) in $\varepsilon=\frac12(1-d)\to 0$ we find that the pole term does not cancel while the finite remainder depends on the constant parts in the bosonic and fermionic propagators in (1.6),(1.8), i.e.

$$
f_2=-\frac{11}{32\pi^2\,\bar\varepsilon}+f_{2,\rm fin}\,,\qquad f_{2,\rm fin}=\frac{6a_{\rm b}+16a_{\rm f}-19}{32\pi^2}\,,\qquad f_{2,\rm fin}\Big|_{a_{\rm b}=2,\,a_{\rm f}=1}=\frac{9}{32\pi^2}. \qquad (5.1)
$$

[22] The propagator corresponds to the quadratic action factor in the path integral with $\exp(iS_M)\to\exp[-\frac12\int\bar\theta(\nabla\!\!\!\!/+\Gamma_{123})\theta]$ so that $\langle\theta\bar\theta\rangle=(\nabla\!\!\!\!/+\Gamma_{123})^{-1}$ (see also Appendix A). The Majorana action is $\frac12\int\bar\theta K\theta=\frac12\theta^T CK\theta$ (with CK operator being antisymmetric) so that the propagator should be same as that of the Dirac action $\int\bar\theta K\theta$.

[23] A check of normalization: we have from (4.8) and (4.13) $iL=\frac{i}{2}[1+\frac{c_1}{d+1}\boldsymbol{x}^2]\bar\theta D\!\!\!\!/\,\theta+\ldots$. Then integrating over θ the free energy contribution of this term will be given by $F=-\frac12\mathrm{tr}\log([1+\frac{c_1}{d+1}<\boldsymbol{x}^2>]D\!\!\!\!/)=-\frac12\frac{c_1}{d+1}<\boldsymbol{x}^2>N_\theta\delta(0)$ where $N_\theta=16$ and the fermionic $\delta(0)$ is given by $\delta(0)=\frac{d-1}{d+1}\mathrm{S}$ in dimensional reduction regularization.

[24] This is related to the fact that in the dimensional reduction regularization $\langle\bar\theta D\!\!\!\!/\,\theta\rangle=\mathcal{O}(1)$, cf. (3.34),(4.18).

As in the S-matrix computation in flat target space [1] it is natural to suggest that the pole term should be subtracted or cancelled by adding a particular 2-loop counterterm that should be required by underlying symmetries of the theory (supersymmetry and integrability). The regularization or scheme choice consistent with symmetries should then lead to the finite part of f_2 that should match the dual gauge theory prediction in (1.4).

Clearly, more is to be done to substantiate this suggestion. In particular, it would be important to clarify if the dimensional reduction regularization that we used actually preserves the residual AdS_2 supersymmetry that remains in the κ-symmetry gauge (2.26).

In general, one may use also other regularizations like ζ-function or heat kernel one applied directly in 2 dimensions (see Appendix B). In this case the $\delta^{(2)}(w,w)$ or "$\delta(0)$" terms may have a non-zero finite part. As a result, one may get additional 2-loop contributions from local path integral measure (cf. [47]) and determinant of local κ-symmetry ghost operator corresponding to the gauge (2.26). In general, when such "$\delta(0)$" terms are non-zero, the result may depend on field redefinitions and gauge choices unless one carefully takes into account all measure and algebraic ghost factors.[25]

To try to fix the regularization/subtraction scheme ambiguities it would be important to compute string world-sheet loop corrections to other related observables (like latitude WL or correlators of operators on an infinite WL) and compare them to data on the dual gauge theory side. A simple example of 1-loop correction to 2-point function of x^i fluctuations is discussed in Appendix C .

It would be interesting also to repeat a similar 2-loop computation in the case of $\frac{1}{2}$ BPS WL in ABJM theory which in the planar limit is dual to IIA string in $AdS_4 \times CP^3$ (the corresponding setup is reviewed in Appendix D).

Data availability

No data was used for the research described in the article.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Correlators of Majorana fermions

Our conventions for the 10d MW spinors were discussed in (2.11)–(2.15). Useful identities are, e.g.,

$$\bar\theta\Gamma_A\theta = 0, \qquad \bar\theta\Gamma_{ABC}\partial\theta = \partial\bar\theta\Gamma_{ABC}\theta \to \bar\theta\Gamma_{ABC}\partial\theta = \tfrac{1}{2}\partial(\bar\theta\Gamma_{ABC}\theta). \tag{A.1}$$

We need to compute the expectation values of the fermion operators of the form $\langle\bar\theta X\theta\rangle$ and $\langle\bar\theta X\nabla_\alpha\theta\rangle$ where X is a product of Γ_A matrices (with $(CX)^T = -CX$) and the fermion propagator is defined as

$$\langle\theta_u(\mathrm{w})\bar\theta_v(\mathrm{w}')\rangle = S_{uv}(\mathrm{w},\mathrm{w}')\ , \qquad \langle\theta_u\theta_v\rangle = (S\,C^{-1})_{uv}\ , \qquad (SC^{-1})^T = -SC^{-1}. \tag{A.2}$$

Here u, v are 10d spinor indices and the chirality projector $P_+ = \frac{1}{2}(1+\Gamma_{11})$ is implicit. Then

$$\langle\bar\theta X\theta\rangle = \mathrm{tr}[SC^{-1}(CX)^T] = -\mathrm{tr}[SX], \qquad \langle\bar\theta X\theta'\rangle = -\mathrm{tr}[S'X]\ , \tag{A.3}$$

where prime stands for a derivative over the second argument.

If X and $\widetilde X$ are products of Γ_A matrices then one finds that

$$\langle\bar\theta X\theta\ \bar\theta\widetilde X\theta\rangle = \mathrm{tr}[XS]\mathrm{tr}[\widetilde XS] - 2\mathrm{tr}[XS\widetilde XS]. \tag{A.4}$$

In the case when θ' and θ'' stand for derivatives of θ we get

$$\langle\bar\theta X\theta'\ \bar\theta\widetilde X\theta''\rangle = \mathrm{tr}[XS']\mathrm{tr}[\widetilde XS''] - \mathrm{tr}[XS'\widetilde XS''] - \mathrm{tr}[X\hat SC^{-1}\widetilde X^TCS], \tag{A.5}$$

$$\langle\theta'\theta\rangle = S'C^{-1}, \qquad \langle\theta''\theta\rangle = S''C^{-1}, \qquad \langle\theta'\theta''\rangle = \hat SC^{-1}. \tag{A.6}$$

In particular,

$$\langle(\nabla_\alpha\theta(\mathrm{w}))_u\theta_v(\mathrm{w}')\rangle = (\nabla_\alpha SC^{-1})_{uv}\ , \quad \langle(\nabla_\alpha\theta)_u\ (\nabla_\beta\theta)_v\rangle = (\hat S_{\alpha\beta}C^{-1})_{uv}\ , \quad \nabla_\alpha = \partial_\alpha + \frac{1}{2\sigma}\gamma_{\hat0\alpha}. \tag{A.7}$$

[25] In fact, these reservations apply also to our treatment of fermions in dimensional reduction regularization where the fermionic $\delta^{(2)}(w,w)$ is no longer zero: the equation of motion for the propagator is satisfied modulo $\mathcal{O}(\epsilon)$ term, see (3.34).

Using (3.23)–(3.29) we find that

$$\hat{S}_{\alpha\beta}(\mathrm{w},\mathrm{w}) = -\frac{1}{4z^2}\Big[\Gamma_{\hat\alpha\hat\beta} + \frac{4m^2-d(d+1)}{d+1}\eta_{\hat\alpha\hat\beta}\Big]\Gamma_{123}\mathrm{S}. \tag{A.8}$$

To compute the traces over the spinor indices we note that after the κ-symmetry gauge fixing (2.26) the remaining $D=10$ Majorana-Weyl fermion θ in (2.28) has $N_\theta = 16$ independent real components ($\mathrm{tr}\, I = 32,\ \mathrm{tr}\Gamma_{11}=0$) so that

$$\mathrm{tr}\, P_+ = N_\theta = 16, \qquad P_+ = \tfrac{1}{2}(1+\Gamma_{11}), \tag{A.9}$$
$$\mathrm{tr}(P_+\Gamma_A\Gamma_B) = \eta_{AB}\,\mathrm{tr}\, P_+, \qquad \mathrm{tr}(P_+\Gamma_A\Gamma_B\Gamma_C\Gamma_D) = (\eta_{AB}\eta_{CD} - \eta_{AC}\eta_{BD} + \eta_{AD}\eta_{BC})\,\mathrm{tr}\, P_+.$$

Then we find for the correlators in (4.18)

$$J_1 = \langle\bar\theta\nabla\!\!\!\!/\,\theta\rangle = -\mathrm{tr}[\gamma^\alpha P_+ \nabla_\alpha S] = -\frac{m}{d+1}\mathrm{S}\,\mathrm{tr}[P_+\gamma^\alpha\gamma_\alpha] = -\frac{2\,m}{d+1}\mathrm{S}\, N_\theta\ , \tag{A.10}$$

$$J_2 = \langle\bar\theta\Gamma_{123}\theta\rangle = -\mathrm{tr}[\Gamma_{123}P_+S] = \mathrm{S}\,N_\theta. \tag{A.11}$$

Similarly, we reproduce the expressions for the 4-fermion correlators in (4.21)–(4.28)[26]

$$\begin{aligned}
\bar I_1 &= \langle\bar\theta\nabla\!\!\!\!/\,\theta\theta\;\bar\theta\Gamma_{123}\theta\rangle = \mathrm{tr}[P_+\gamma^\alpha\nabla_\alpha S]\,\mathrm{tr}[P_+\Gamma_{123}S] - 2\mathrm{tr}[P_+\gamma^\alpha\nabla_\alpha S\Gamma_{123}S] = -\frac{2\,m}{d+1}\mathrm{S}^2 N_\theta(N_\theta-2),\\
\bar I_2 &= \langle\bar\theta\Gamma_{123}\theta\theta\;\bar\theta\Gamma_{123}\theta\rangle = \mathrm{tr}[P_+\Gamma_{123}S]\,\mathrm{tr}[P_+\Gamma_{123}S] - 2\mathrm{tr}[P_+\Gamma_{123}S\Gamma_{123}S] = \mathrm{S}^2 N_\theta(N_\theta-2),\\
\bar I_3 &= \langle\bar\theta\Gamma_{ij}\Gamma_{123}\nabla_\alpha\theta\;\bar\theta\gamma^\alpha\Gamma^{ij}\theta\rangle = \mathrm{tr}[P_+\Gamma_{ij}\Gamma_{123}\nabla_\alpha S]\,\mathrm{tr}[P_+\gamma^\alpha\Gamma^{ij}S] - 2\mathrm{tr}[P_+\Gamma_{ij}\Gamma_{123}\nabla_\alpha S\gamma^\alpha\Gamma^{ij}S]\\
&= \frac{m\,\mathrm{S}^2}{d+1}\Big(\mathrm{tr}[P_+\Gamma_{ij}\gamma_\alpha\Gamma_{123}]\,\mathrm{tr}[P_+\gamma^\alpha\Gamma^{ij}\Gamma_{123}] - 2\mathrm{tr}[P_+\Gamma_{ij}\Gamma_{123}\gamma_\alpha\gamma^\alpha\Gamma^{ij}\Gamma_{123}]\Big) = -24\frac{m}{d+1}\mathrm{S}^2 N_\theta,\\
\bar I_4 &= \langle\bar\theta\Gamma_{a'b'}\Gamma_{123}\nabla_\alpha\theta\;\bar\theta\gamma^\alpha\Gamma^{a'b'}\theta\rangle = 4\,\frac{m}{d+1}\mathrm{S}^2\mathrm{tr}[P_+\Gamma_{a'b'}\Gamma^{a'b'}] = -80\,\frac{m}{d+1}\mathrm{S}^2 N_\theta\ ,\\
\bar I_5 &= \langle\bar\theta\gamma_\alpha\Gamma_{ij}\theta\;\bar\theta\gamma^\alpha\Gamma^{ij}\theta\rangle = \mathrm{S}^2\Big(\mathrm{tr}[P_+\gamma_\alpha\Gamma_{ij}\Gamma_{123}]\mathrm{tr}[P_+\gamma^\alpha\Gamma^{ij}\Gamma_{123}] - 2\mathrm{tr}[P_+\gamma_\alpha\Gamma_{ij}\Gamma_{123}\gamma^\alpha\Gamma^{ij}\Gamma_{123}\Big)\\
&= 24\,\mathrm{S}^2\,N_\theta,\\
\bar I_6 &= \langle\bar\theta\gamma^\alpha\Gamma^{a'b'}\theta\;\bar\theta\Gamma_{a'b'}\gamma_\alpha\theta\rangle = \mathrm{S}^2\,\Big(\mathrm{tr}[P_+\gamma^\alpha\Gamma^{a'b'}\Gamma_{123}]\mathrm{tr}[P_+\gamma_\alpha\Gamma_{a'b'}\Gamma_{123}] - 4\mathrm{tr}[P_+\Gamma^{a'b'}\;\Gamma_{a'b'}]\Big)\\
&= 80\;\mathrm{S}^2\;N_\theta\ ,
\end{aligned} \tag{A.12}$$

where $\mathrm{tr}[P_+\gamma_\alpha\Gamma_{a'b'}\Gamma_{123}]=0,\ \mathrm{tr}[P_+\Gamma^{a'b'}\Gamma_{a'b'}] = -20\,N_\theta$. Also,

$$\begin{aligned}
\bar I_7 &= \langle\bar\theta\nabla\!\!\!\!/\,\theta\;\bar\theta\nabla\!\!\!\!/\,\theta\rangle\\
&= \mathrm{tr}[\gamma^\alpha P_+\nabla_\alpha S]\mathrm{tr}[\gamma^\beta P_+\nabla_\beta S] - \mathrm{tr}[\gamma^\alpha P_+\nabla_\alpha S\;\gamma^\beta P_+\nabla_\beta S] - \mathrm{tr}[\gamma^\alpha P_+\hat S_{\alpha\beta}C^{-1}(\gamma^\beta)^T C\,P_+S]\\
&= 4\Big(\frac{m}{d+1}\Big)^2\mathrm{S}^2 N_\theta(N_\theta-1) + \mathrm{S}^2\Big[-\tfrac{1}{2} + \frac{d(d+1)-4m^2}{2(1+d)}\Big]N_\theta\ ,
\end{aligned} \tag{A.13}$$

$$\begin{aligned}
\bar I_8 &= \langle\bar\theta\gamma_\alpha\nabla_\beta\theta\;\bar\theta\gamma^\beta\nabla^\alpha\theta\rangle\\
&= \mathrm{tr}[\gamma_\alpha P_+\nabla_\beta S]\mathrm{tr}[\gamma^\beta P_+\nabla^\alpha S] - \mathrm{tr}[\gamma_\alpha P_+\nabla_\beta S\;\gamma^\beta P_+\nabla^\alpha S] - \mathrm{tr}[\gamma^\alpha P_+\hat S_{\beta\alpha}C^{-1}(\gamma^\beta)^T C\,P_+S]\\
&= 2\Big(\frac{m}{d+1}\Big)^2\mathrm{S}^2 N_\theta(N_\theta-2) + \mathrm{S}^2\,\Big[\tfrac{1}{2} + \frac{d(d+1)-4m^2}{2(d+1)}\Big]N_\theta,
\end{aligned} \tag{A.14}$$

where we used that $\gamma^\alpha\gamma_{\alpha\beta}\gamma^\beta = 2$ and (A.8).

Appendix B. General form of 2-loop correction in 2d and ζ-function regularization

Here we shall not use dimensional regularization, i.e. assume that we regularize the theory directly in 2 dimensions. In particular, we may adopt the ζ-function regularization.

We shall use the following notation (ξ^α are coordinates in AdS_2 space)

$$\langle x^i(\xi)x^j(\xi')\rangle = \delta^{ij}G_x(\xi,\xi')\ ,\quad \langle y^{a'}(\xi)y^{b'}(\xi')\rangle = \delta^{a'b'}G_y(\xi,\xi')\ ,\quad \langle\theta(\xi)\bar\theta(\xi')\rangle = G_\theta(\xi,\xi'). \tag{B.1}$$

In the coincident-point limit we define[27]

$$\lim_{\xi'\to\xi} G_{x,y}(\xi,\xi') = \mathrm{G}_{x,y}\ ,\qquad \lim_{\xi'\to\xi}\partial_\alpha G_{x,y}(\xi,\xi') = \lim_{\xi'\to\xi}\partial'_\alpha G_{x,y}(\xi,\xi') = 0\ , \tag{B.2}$$

$$\lim_{\xi'\to\xi}\partial_\alpha\partial'_\beta G_{x,y}(\xi,\xi') = \hat G_{x,y}\,g_{\alpha\beta}\ , \tag{B.3}$$

$$\lim_{\xi'\to\xi} G_\theta(\xi,\xi') = \mathrm{G}_\theta M\ ,\qquad \lim_{\xi'\to\xi} i\mathfrak{D}_\alpha G_\theta(\xi,\xi') = \hat G_\theta\gamma_\alpha\ ,\qquad M = -i\Gamma_{123}\ , \tag{B.4}$$

[26] To recall, $\gamma_{\hat\alpha} = \Gamma_{\hat\alpha}$ where $\hat\alpha$ is the tangent-space 2d index with (0,4) values.

[27] Note that G_θ as defined in (B.4) corresponds to $-\mathrm{S}$ in (3.25),(3.29).

$$\lim_{\xi'\to\xi} \mathfrak{D}_\alpha \mathfrak{D}'_\beta G_\theta(\xi,\xi') = (H_\theta g_{\alpha\beta} + \tilde{G}_\theta \gamma_{\alpha\beta})M. \tag{B.5}$$

Then from (2.6),(2.44) we get for the bosonic and fermionic contributions to the 2-loop coefficient f_2

$$\langle \mathcal{L}_{4b}\rangle = N_x \hat{G}_x^2 - \tfrac{1}{2}N_x^2 \mathrm{G}_x \hat{G}_x - \tfrac{1}{2}N_x(N_x+2)\mathrm{G}_x^2 + N_y \hat{G}_y^2 + \tfrac{1}{2}N_y^2 \mathrm{G}_y \hat{G}_y\,, \tag{B.6}$$

$$\begin{aligned}\langle \mathcal{L}_{4f}\rangle =& -\tfrac{1}{2}\mathrm{G}_x \hat{G}_\theta\, N_x N_\theta + \tfrac{1}{4}\big[N_x(\hat{G}_x + \mathrm{G}_x) - N_y \hat{G}_y\big]\mathrm{G}_\theta N_\theta \\ &+ \tfrac{1}{48}\big[N_x(N_x-1) - N_y(N_y-1) + (N_\theta - 2)\big] N_\theta\, \hat{G}_\theta \mathrm{G}_\theta \\ &- \tfrac{1}{8}\hat{G}_\theta^2 N_\theta{}^2 - \tfrac{1}{4}\mathrm{G}_\theta \tilde{G}_\theta N_\theta.\end{aligned} \tag{B.7}$$

We used that

$$\begin{aligned}&\mathrm{tr}(\gamma_\alpha \Gamma_{ij}\Gamma_{123}P_+) = 0\,, \quad \mathrm{tr}(\gamma_\alpha \Gamma_{a'b'}\Gamma_{123}P_+) = 0\,, \quad \Gamma_{ij}\Gamma^{ij} = -N_x(N_x-1)\,, \quad \Gamma_{a'b'}\Gamma^{a'b'} = -N_y(N_y-1), \\ &\langle (\bar\theta\gamma_\alpha i\mathfrak{D}_\beta\theta)(\bar\theta\gamma_\rho i\mathfrak{D}_\lambda\theta)\rangle = \mathrm{tr}(\gamma_\beta\gamma_\alpha P_+)\mathrm{tr}(\gamma_\lambda\gamma_\rho P_+)\hat{G}_\theta^2 - \mathrm{tr}(\gamma_\lambda\gamma_\alpha\gamma_\beta\gamma_\rho P_+)\hat{G}_\theta^2 - \mathrm{tr}\big[\gamma_\alpha P_+ M\gamma_\rho(H_\theta g_{\lambda\beta} + \tilde{G}_\theta\gamma_{\lambda\beta})M\big]\mathrm{G}_\theta.\end{aligned} \tag{B.8}$$

Note that the second line in (B.7) vanishes identically for $N_x = 3,\ N_y = 5,\ N_\theta = 16$ while the third line is a covariant version of the same expression in flat space.

Let us introduce the following "equations of motion" combinations which are essentially the corresponding regularized "$\delta(0)$" or $\zeta(0)$ values[28]

$$E_x = 2(\hat{G}_x + \mathrm{G}_x)\,, \qquad E_y = 2\hat{G}_y\,, \qquad E_\theta = 2\hat{G}_\theta\,, \tag{B.9}$$

and assume that regularization is such that in (B.5) one has $\tilde{G}_\theta = 0$.[29] Also, let us set

$$N_x(N_x-1) - N_y(N_y-1) + (N_\theta - 2) = 0\,, \tag{B.10}$$

which is satisfied for the relevant values $N_x = 3,\ N_y = 5,\ N_\theta = 16$. Then from (B.6) and (B.7) we get

$$\begin{aligned}\langle \mathcal{L}_{4b} + \mathcal{L}_{4f}\rangle =& \tfrac{1}{4}N_x E_x\big[E_x - (N_x+4)\mathrm{G}_x\big] + \tfrac{1}{4}N_y E_y\big[E_y + N_y \mathrm{G}_y\big] \\ &- \tfrac{1}{4}N_\theta E_\theta\big[N_x \mathrm{G}_x + \tfrac{1}{8}N_\theta E_\theta\big] + \tfrac{1}{8}N_\theta(N_x E_x - N_y E_y)\mathrm{G}_\theta.\end{aligned} \tag{B.11}$$

Thus the total 2-loop contribution $\langle \mathcal{L}_{4b} + \mathcal{L}_{4f}\rangle$ is proportional to the $E_{x,y,\theta}$ or "$\delta(0)$" terms.

More explicitly, for $N_x = 3,\ N_y = 5,\ N_\theta = 16$ we get

$$\begin{aligned}\langle \mathcal{L}_{4b} + \mathcal{L}_{4f}\rangle =& (-\tfrac{21}{4}E_x - 12E_\theta)\mathrm{G}_x + \tfrac{25}{4}E_y \mathrm{G}_y + (6E_x - 10E_y)\mathrm{G}_\theta \\ &+ \tfrac{1}{4}(3E_x^2 + 5E_y^2 - 32E_\theta^2).\end{aligned} \tag{B.12}$$

This, however, may not represent the full 2-loop contribution to the free energy: there may be a non-trivial contribution of a local measure which is also proportional to $\delta(0)$ terms.[30] Here we will not attempt to fix the measure contribution (which was not relevant in the dimensional regularization used in the main part of the paper).

The E^2 terms in the second line may cancel in a supersymmetric regularization like the one discussed in [22] where

$$E_\theta = \tfrac{1}{2}E_x = \tfrac{1}{2}E_y \quad \to \quad 3E_x^2 + 5E_y^2 - 32E_\theta^2 = 0. \tag{B.13}$$

The G-terms in the first line of (B.12) will contain log UV divergences; both the divergent and finite part of the 2-loop result in (B.12) may still be changed by the contribution of the path integral measure.

Let us now consider the particular case of the spectral ζ-function regularization in AdS_2 (see, e.g., [18,22,24,49,50]). Here $\zeta(0;\xi,\xi')$ plays the role of the regularized δ-function (the AdS_2 radius is set to 1)

$$(-\nabla_\alpha\nabla^\alpha + 2)G_x = \delta_x^{(2)}(\xi,\xi') = \zeta_{m^2=2}(0\,;\xi,\xi'), \qquad -\nabla_\alpha\nabla^\alpha G_y = \delta_y^{(2)}(\xi,\xi') = \zeta_{m^2=0}(0;\xi,\xi')\,, \tag{B.14}$$

$$i\not{\mathfrak{D}} G_\theta = \mathrm{I}\delta_\theta^{(2)}(\xi,\xi') = \mathrm{I}\zeta_\theta(0;\xi,\xi')\,, \tag{B.15}$$

$$\hat{G}_x = -\mathrm{G}_x + \tfrac{1}{2}\zeta_{m^2=2}(0;\xi,\xi)\,, \qquad \hat{G}_y = \tfrac{1}{2}\zeta_{m^2=0}(0;\xi,\xi)\,, \qquad \hat{G}_\theta = \tfrac{1}{2}\zeta_\theta(0;\xi,\xi), \tag{B.16}$$

$$\zeta_{m^2}(0;\xi,\xi) = \tfrac{1}{4\pi}(\tfrac{1}{6}R^{(2)} - m^2) = -\tfrac{1}{12\pi} - \tfrac{1}{4\pi}m^2\,, \tag{B.17}$$

$$\zeta_\theta(0;\xi,\xi) = \tfrac{1}{4\pi}(\tfrac{1}{6}R^{(2)} - \tfrac{1}{4}R^{(2)} - m^2) = \tfrac{1}{24\pi} - \tfrac{1}{4\pi}m^2. \tag{B.18}$$

Then we get the following explicit expressions for the coefficients in (B.2)–(B.5)

$$\mathrm{G}_x = \tfrac{1}{2\pi}\log\bar\Lambda - \tfrac{1}{2\pi}\,, \qquad \mathrm{G}_y = \tfrac{1}{2\pi}\log\bar\Lambda\,, \qquad \log\bar\Lambda \equiv \log\Lambda + \gamma_{\mathrm{E}}\,, \tag{B.19}$$

[28] Note that in curved 2d space like AdS_2 the $\delta(0)$ terms contain, in addition to the quadratically divergent part, also a finite part so that their 2d regularized values are non-zero (see e.g. [48]).

[29] This should follow from integrability of Killing spinor derivative.

[30] For example, the standard bosonic measure $\prod_\xi \sqrt{G(x(\xi))}$ would lead to terms like $\int \delta(0)[x^2(\xi) + \ldots]$ contributing $\int \delta(0)\langle x^2\rangle \sim \delta(0)\mathrm{G}_x$ at the 2-loop order (see, e.g., [47]). The supersymmetric measure for the GS string may also contain terms like $\int \delta(0)[\bar\theta M\theta + \ldots]$, etc. In general, the contribution of the measure may depend on the induced metric and thus will not cancel in a ratio of two Wilson loop expectation values (like for circle and for latitude which have different induced metrics).

$$\hat{G}_x = -\mathrm{G}_x - \tfrac{7}{24\pi}\,, \qquad \hat{G}_y = -\tfrac{1}{24\pi}\,, \tag{B.20}$$

$$\mathrm{G}_\theta = -\tfrac{1}{2\pi}\log\bar{\Lambda} + \tfrac{1}{4\pi}\,, \qquad \hat{G}_\theta = -\tfrac{5}{48\pi}\,, \qquad \tilde{G}_\theta = 0\,, \qquad H_\theta = -\tfrac{5}{48\pi}\,, \tag{B.21}$$

where Λ is 2d UV cutoff. Comparing this to the dimensional regularization expressions in (1.6),(1.8) we conclude that G_x and G_θ are related by $\frac{1}{\varepsilon'} \leftrightarrow 2\log\bar{\Lambda}$ with the finite parts being the same.

Then for (B.6) and (B.7) we find

$$\langle \mathcal{L}_{4\mathrm{b}}\rangle = \tfrac{1}{96\pi^2} d_1 \log\bar{\Lambda} + \tfrac{1}{576\pi^2} d_2\,, \tag{B.22}$$

$$d_1 = (7N_x^2 + 28N_x - N_y^2)\Big|_{N_x=3,\,N_y=5} = 132\,, \quad d_2 = -(42N_x^2 + 119N_x - N_y)\Big|_{N_x=3,\,N_y=5} = -730\,,$$

$$\langle \mathcal{L}_{4\mathrm{f}}\rangle = \tfrac{1}{4608\pi^2} e_1 \log\bar{\Lambda} + \tfrac{1}{18432\pi^2} e_2\,, \tag{B.23}$$

$$e_1 = N_\theta\Big(5N_x^2 + 283N_x - 5N_y^2 - 19N_y + 5N_\theta - 10\Big)\Big|_{N_x=3,\,N_y=5,\,N_\theta=16} = 11904\,,$$

$$e_2 = N_\theta\Big(-10N_x^2 - 806N_x + 10N_y^2 + 38N_y - 35N_\theta + 20\Big)\Big|_{N_x=3,\,N_y=5,\,N_\theta=16} = -41728\,,$$

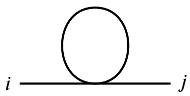
$$\langle \mathcal{L}_{4\mathrm{b}} + \mathcal{L}_{4\mathrm{f}}\ \rangle = \frac{95}{24\pi^2}\log\bar{\Lambda} - \frac{113}{32\pi^2}. \tag{B.24}$$

The coefficients of the divergent term and the finite term are not the same as in the dimensional regularization in (1.11),(1.12) which may be attributed to the still missing contribution of the measure.[31]

Appendix C. 1-loop correction to 2-point function

One could wonder if one is to take into account possible 1-loop renormalization of the 2-point functions of the fluctuation fields. They are, in fact, UV finite in agreement with the absence of non-trivial 1-loop divergences in the GS string partition function.

Let us illustrate this on the example of the 1-loop correction to the 2-point function $\langle x^i(\mathrm{w}_1)x^j(\mathrm{w}_2)\rangle$ of the massive scalar using dimensional regularization (with $\mathrm{d} = 1 + d = 2 - 2\varepsilon$).



We find (see (2.8),(3.12)–(3.15))

$$\langle x^i(\mathrm{w}_1)x^j(\mathrm{w}_2)\rangle_1 = \langle x^i(\mathrm{w}_1)x^j(\mathrm{w}_2)\int d^{\mathrm{d}}\mathrm{w}\,\sqrt{-g}\,\mathcal{L}_{4x}\rangle \tag{C.1}$$

$$= -\,\delta^{ij}\int d^{\mathrm{d}}\mathrm{w}\,\sqrt{-g}\,\Big[(N_x+4)\mathrm{G}(\mathrm{w}_1,\mathrm{w})\mathrm{G}(\mathrm{w},\mathrm{w}_2) + \tfrac{4+d(4-N_x)+3N_x}{2(d+1)}\,\partial_{\mathrm{w}\alpha}\mathrm{G}(\mathrm{w}_1,\mathrm{w})\partial^\alpha_{\mathrm{w}}\mathrm{G}(\mathrm{w},\mathrm{w}_2)\Big]\,\mathrm{G}.$$

Since $\mathrm{G} = \mathrm{G}(\mathrm{w},\mathrm{w})$ in (3.7) is constant we may integrate by parts and use (3.8) to obtain

$$\langle x^i(\mathrm{w}_1)x^j(\mathrm{w}_2)\rangle_1 = \delta^{ij} P\int d^{\mathrm{d}}\mathrm{w}\,\sqrt{-g}\,\mathrm{G}(\mathrm{w}_1,\mathrm{w})\mathrm{G}(\mathrm{w},\mathrm{w}_2)\,, \qquad P = P_x + \ldots\,, \qquad P_x = 2\,N_x\frac{1-d}{1+d}\,\mathrm{G}. \tag{C.2}$$

The fermionic loop contribution to this 2-point function contains a non-zero contribution coming from the c_1 term in (4.9).[32] As a result, the factor P in (C.2) is changed to[33]

$$P = \Big(2N_x\,\mathrm{G} + \frac{1}{1+d}N_\theta\,\mathrm{S}\Big)\frac{1-d}{1+d}. \tag{C.3}$$

This expression is finite in the limit $d \to 1$. Taking $d \to 1$ and setting $N_x = 3,\ N_\theta = 16$ (and reversing the overall sign) we get for the corresponding finite 1-loop mass counterterm

$$\delta\mathcal{L}_{2\,x} = \tfrac{1}{2}C_1\boldsymbol{x}^2, \qquad C_1 = -\lim_{d\to 1} P = \frac{1}{2\pi}. \tag{C.4}$$

Note that C_1 does not depend on the values of finite parts a_{b} and a_{f} in G (1.6) and S (1.8).[34]

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[31] In addition, there may be a non-trivial finite contribution of determinants of the local ghost operators that are proportional to "$\delta(0)$" terms.

[32] Here we ignore the terms in (2.46) that give trivial contributions after taking spinor traces.

[33] Note that $\langle x^i(\mathrm{w}_1)x^j(\mathrm{w}_2)(\boldsymbol{x}^2 + \frac{1}{2}\partial\text{-}\boldsymbol{x}\cdot\bar{\partial}\,\boldsymbol{x})\rangle = 0,\ \langle x^i(\mathrm{w}_1)x^j(\mathrm{w}_2)\partial_\alpha\boldsymbol{x}\cdot\partial_\beta\boldsymbol{x}\rangle = -\frac{4}{d+1}\eta_{\alpha\beta}\delta^{ij}$.

[34] Surprisingly, this value of C_1 does not match the coefficient in the subleading term in the Bremsstrahlung function $B(\lambda) = B_0 + B_1 + \ldots = \frac{\sqrt{\lambda}}{4\pi^2} - \frac{3}{8\pi^2} + \ldots$ (cf. [34,38,51]). Since x^i corresponds to the displacement operator (with the 2-point function proportional to $12B$) one would expect C_1 to be directly related (up to $\frac{1}{2\pi}$ factor) to $12B_1 = -\frac{9}{2\pi^2}$. This may be related to normalization of the boundary limit of the 2-point function. We leave the resolution of this puzzle to the future.

Appendix D. Strong coupling expansion of $\frac{1}{2}$ BPS Wilson loop in ABJM theory

The ABJM theory [52] is dual to M-theory in $AdS_4 \times S^7/\mathbb{Z}_k$ or, in the 't Hooft limit (large N with fixed $\lambda = N/k$), to IIA string on $AdS_4 \times CP^3$. One can try to compare the localization prediction for the $\frac{1}{2}$ BPS circular WL with its string dual given by the disk partition function of type IIA superstring in $AdS_4 \times CP^3$ expanded near AdS_2 minimal surface.

At 1-loop order that was discussed in [22] (see also [53,54]). A 2-loop computation should be possible using the same methods as in the $AdS_5 \times S^5$ case in this paper. In particular, since the S^5 modes do not appear to play much role at 2 loops, it is possible that the extension to the case of $AdS_4 \times CP^3$ may be straightforward.

Here we will only review the known expressions for the strong coupling expansion of the WL expectation value found from localization. From the general expression in [55,56], taking the large N, fixed $\lambda = N/k$ limit one finds

$$\langle W\rangle = \frac{N}{4\pi\lambda}e^{\pi\sqrt{2(\lambda-\frac{1}{24})}} + \mathcal{O}(N^0) = \frac{N}{4\pi\lambda}e^{\pi\sqrt{2\lambda}}\Big[1 - \frac{\pi}{24\sqrt{2}}\frac{1}{\sqrt{\lambda}} + \cdots\Big] + \mathcal{O}(N^0). \tag{D.1}$$

Here the natural relation for the dual string tension is[35]

$$T = \frac{1}{\sqrt{2}}\sqrt{\lambda - \tfrac{1}{24}}\,, \qquad T_0 \equiv \frac{\sqrt{\lambda}}{\sqrt{2}} = T\sqrt{1+\tfrac{1}{48}T^{-2}}. \tag{D.2}$$

Written in terms of T and $g_s = \sqrt{\pi}(2\lambda)^{5/4}N^{-1}$ as in [22] (D.1) becomes

$$\langle W\rangle = \frac{\sqrt{T_0}}{\sqrt{2\pi}\,g_s}\,e^{2\pi T} + \mathcal{O}\big((g_s)^0\big) = Z_1\sqrt{\frac{T_0}{T}}\,e^{2\pi T} + \mathcal{O}\big((g_s)^0\big)\,, \qquad Z_1 = \frac{\sqrt{T}}{\sqrt{2\pi}\,g_s}. \tag{D.3}$$

The corresponding free energy of the string theory on the disk should then have the form (cf. (1.3))

$$F = -\log\big(Z_1^{-1}\langle W\rangle\big) = -2\pi T + \tfrac{1}{2}\log\frac{T}{T_0} = -2\pi T - \tfrac{1}{192}T^{-2} + \mathcal{O}(T^{-3}). \tag{D.4}$$

This implies that the 2-loop string contribution here should be zero – the first non-trivial correction to the 1-loop result should come at 3 loops.

At fixed k the localization result expanded in large N or in the large M2 brane tension T_2 may be written as [59]

$$\langle W\rangle = \frac{1}{2\sin\frac{2\pi}{k}}\,e^{\frac{\pi^2}{k}T_2}\Big[1 - \frac{k^2+32}{24k}\frac{1}{T_2} + \mathcal{O}(T_2^{-2})\Big]\,, \qquad T_2 = \frac{1}{\pi}\sqrt{2kN}. \tag{D.5}$$

Here the prefactor is the 1-loop correction to partition function of M2 brane on $AdS_2 \times S^1$ inside of $AdS_4 \times S^7/\mathbb{Z}_k$ [59]. The subleading $1/T_2$ term should be the 2-loop M2 brane correction.

To compute the string or M2 brane partition function expanded near AdS_2 or $AdS_2 \times S^1$ minimal surface we will need to start with the corresponding GS or BST action. The type IIA GS string action in $AdS_4 \times CP^3$ expanded in the static gauge will have a similar form to the $AdS_5 \times S^5$ GS action.[36]

Let us note that there is a recent proposal in [68] suggesting that the comparison between the gauge theory localization result and the M2 brane partition function should be done differently and as a result

$\langle W\rangle$ should be 1-loop exact in the M2-brane sense, i.e. there should no M2 brane 2-loop correction (for fixed k). It would be interesting to check this by direct 2-loop $AdS_2 \times S^1$ M2 brane computation as in the recent example of M2 brane in $AdS_7 \times S^4$ expanded near AdS_3 minimal surface [69].

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[35] An indication that this is the correct relation for the string tension comes from the expressions for the magnon dispersion relation and the cusp anomaly at 2-loop order [30]: the cusp anomaly takes a natural form in terms of the effective tension given by $T = \frac{1}{\sqrt{2}}\sqrt{\lambda - \frac{1}{24} - \frac{\log 2}{2\pi}}$ (the $\log 2$ term is not relevant in the WL case as it leads just to an extra overall rescaling). This is also suggested by the structure of localization matrix model that implies that the combination $\lambda - \frac{1}{24} + \mathcal{O}(N^{-1})$ plays a special role. This follows from the Fermi gas representation $\exp(-\mu N + J)$, $J = \frac{1}{3}C\mu^3 + B\mu + A$, $B = \frac{k}{24} + \frac{1}{3k}$ (for a review see, e.g., [57]). This suggests an effective shift $N \to N - B = N - \frac{k}{24} + \frac{1}{3k}$. Also, the argument in [58] suggests that the AdS radius in IIA limit should be given by $L^4 = 2\pi^2\alpha'^2[\lambda - \frac{1}{24}(1-\frac{1}{k^2})]$ which at large N for fixed λ translates into the expression for the effective string tension $T = \frac{1}{\sqrt{2}}\sqrt{\lambda - \frac{1}{24}}$.

[36] The supercoset construction of this action was given in [60,61]. It can also be obtained by double dimensional reduction of the action for M2 brane in $AdS_4 \times S^7$ given in [62,63] (see [64]; this action in l.c. gauge was used in 2-loop cusp anomaly computation [30]). A better starting point may be the action in [65,66] that has a similar form to the $AdS_5 \times S^5$ action. The quadratic in fermions part of the action was discussed in [53]. To get the quartic fermionic term one may use the general form of expansion in θ of the GS action in a IIA supergravity background given in [67].

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