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The deviation angle for one-lane roundabouts: A general mathematical formulation and application



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HIGHLIGHTS

- The deviation angle is an effective measure of the deflection of trajectories and speed reduction at roundabouts.
- A mathematical formulation for linking geometric parameters with the deviation angle in one-lane roundabouts is proposed.
- The general framework is applicable for both rural/urban one-lane roundabouts, considering the presence of vulnerable users.
- The most influential parameters on the deflection of trajectories were discussed through a detailed sensitivity analysis.
- Some measures were proposed, alternative to the geometrically-induced deflection of trajectories, in different cases.

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ABSTRACT

Properly designed roundabouts may lead to safety improvements based on both reducing approaching speeds and controlling traffic. Measurements of deflection of vehicle trajectories are commonly used to estimate roundabout speed control. One of these measurements is the deviation angle, which is mentioned in both the Italian and Swiss road standards and, in specific conditions, can be more effective than other methods.

This article presents a general mathematical formulation for linking several geometric parameters with the deviation angle in different rural and urban one-lane roundabout configurations, which is currently missing in the literature. For urban roundabouts, refuge islands for pedestrians and cyclists were considered. Based on the proposed formulation, a sensitivity analysis of the influential geometric parameters was conducted. Results suggest that an insufficient deflection of trajectories (deviation angle less than 45°) is always present for roundabouts with inscribed circle diameter less than 25 m; for urban roundabouts with refuge islands for pedestrians and cyclists having inscribed circle diameter less than 34 m and orthogonal legs; and for roundabouts with angles between opposite legs smaller than 140°. The main parameters which are responsible for a decrease in the deviation angle are: a decrease in the inscribed circle diameter; a decrease in the angle between opposite legs; and an increase in the width of the circulatory lane. Some optimized procedures for

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roundabout design, the generalized application of the deviation angle method and alternative speed control measures in cases of small deviation angles are discussed.

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1. Introduction

Both the analysis of accident data and previous research (Al-Marafi et al., 2020; Chen et al., 2013; Elvik, 2017; Kim and Choi, 2013) show that roundabouts are an effective solution for reducing the severity of traffic crashes at intersections, with respect to conventional unsignalized intersections. However, in urban areas, the design of roundabouts should be carefully conducted by taking into account the presence of vulnerable users (Hydén and Vàrhelyi, 2000), who account for 54% of worldwide deaths in traffic crashes (pedestrians and cyclists) (WHO, 2018).

From the analysis of recent Italian data (Table 1), it emerges that 40.98% of crashes have occurred at intersections and, amongst them, about 10.77% at roundabouts. Most crashes at roundabouts were in urban areas (81.43% of all accidents at roundabouts). Similar percentages were found for the number of severe outcomes (fatal/injury) from road crashes at intersections: they were 40.92% of the total but, among them, only about 9.61% occurred at roundabouts, mostly in urban areas (78.73%). Whereas, different results have emerged for the deaths from traffic crashes: about 23.56% at intersections, and among them only about 7.66% at roundabouts (54.10% of deaths at roundabouts were in urban areas).

Italian data are coherent with similar American and European data. For example, the US Federal Highway Administration reports about 2.5 million accidents per year at intersections (40% of the total). However, about the 50% of serious collisions and 20% of fatal collisions were at intersections (NHTSA, 2018). Most of these intersection crashes were related to left turns, which may be potentially solved with roundabouts. Whereas, in Europe, in 2016, about 15% of deaths in traffic crashes occurred at road intersections and, among them, about 7% occurred at roundabouts (ERSO, 2018).

Hence, while roundabout crashes are generally less severe than those at conventional intersections, safety performances of roundabouts could still be improved, especially in urban areas (Montella, 2011; Polders et al., 2015), where vulnerable

3378

100

Death by traffic crashes

Percentage (%)

users necessarily cluster. In detail, an important safety-related aspect of roundabout design is speed control at the intersection. Thus, roundabout geometric parameters should be set in order to discourage speeds higher than those generally assumed as roundabout design speeds, usually varying in the range of 25–40 km/h (Rodegerdts et al., 2010), depending on the surrounding context. Speed control may produce positive safety effects, especially in urban areas and for vulnerable users.

For this reason, several international standards and guidelines prescribe specific requirements for roundabouts, aimed at controlling speeds of vehicle approaching and negotiating the roundabout (Kennedy et al., 2005; Montella et al., 2013). Most of these requirements relate to ensuring the deflection of trajectories, which allows speed to be controlled. The main parameters used for measuring the deflection of trajectories are as follows.

- The deflection radius, which is the radius of the vehicle path through the circulating lane. The reference value for this radius is usually assumed as the largest radius possible for circulating vehicles. This method was initially promoted by the UK and Australian guidelines (Arndt, 2008) and it is also used in France.
- The entry path radius, which is the measured radius of the path of a vehicle entering the roundabout circulatory lane. It is defined as the "minimum radius on the fastest through path before the yield line" (Rodegerdts et al., 2010). This control measure is used in practice in several countries (Afezolli and Paci, 2012; Ahac et al., 2016; Bezina et al., 2017).
- The deviation angle, which is the angle included between the two tangent lines to the offsets of the entry/exit radii and the central island (Fig. 1). Limiting this angle, as in Italy and Switzerland (Spacek, 2004), may help to ensure a curved vehicle path while entering the roundabout, thus controlling speeds.

Other international design standards provide different measures to control for the deflection of trajectories besides

33

61

1.80

Table 1 — Frequency and severity of Italian intersection crashes in 2017 (Source: Italian National Institute of Statistics, ISTAT).						
	Total	All intersection	Roundabout	Urban roundabout		
Accident frequency	174,933	71,694	7722	6288		
Percentage (%)	100	40.98	4.41	3.59		
Death/injury by traffic crashes	250,128	102,354	9844	7750		
Percentage (%)	100	40.92	3.93	3.09		

796

23.56

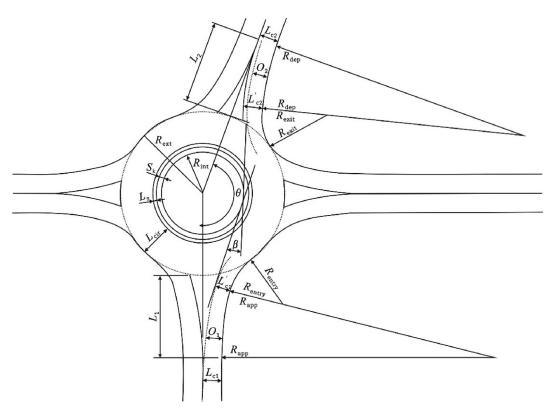


Fig. 1 – Geometric variables considered for the deviation angle calculation.

those listed above. For example, in the German standards the deflection is controlled by setting the radius of the central island as equal to at least twice the entry lane width. Different criteria used worldwide for the control of speeds/trajectories at roundabouts are summarized in Table 2.

 $L'_{\rm c1}/L'_{\rm c2}$ is entry/exit lane width, $R_{\rm app}$ is approaching radius, $R_{\rm entry}$ is entry radius, $R_{\rm exit}$ is exit radius, $R_{\rm dep}$ is departing radius, $R_{\rm int}$ is internal radius, $R_{\rm ext}$ is external radius, $L_{\rm cir}$ is circulatory lane width, Θ is opposite legs inclination, β is deviation angle, ICD is inscribed circle diameter, L_1 is approaching curvature length, L_2 is departing curvature length, $L_{\rm s}$ is left shoulder width, $S_{\rm k}$ is truck apron width.

A previous study (Berloco et al., 2018a) highlighted that the deviation angle method is more conservative (i.e., providing stricter deflection estimates) than other methods (e.g., the entry path radius) for some specific conditions, especially for roundabouts with non-orthogonal intersecting roads or with different entry and exit leg radii. In particular, the use of the deviation angle method reveals that most small roundabouts having an inscribed circle diameter (ICD) between 14 and 25 m do not comply with appropriate deflection requirements.

Moreover, the deviation angle can be considered as an important safety-related variable. For instance, Montella (2011) found that the lower the deviation angle is, the higher the likelihood of angle crashes between entering and circulating vehicles can be. Similarly, Hydén and Várhelyi (2000) pointed out that the more the lateral displacement of the central island is (clearly related to the deviation angle), the less the entering speeds in roundabouts will be (which may be in turn associated with less crashes). Novák et al.

(2018) included the entry angle (related to the deviation angle) in a prediction model for crashes at roundabouts (higher angles are related to lower crash frequencies). Moreover, a first preliminary analysis of 67 roundabouts (average ICD is 48 m) outside urban centers in which at least one fatal/injury crash (average crash frequency per roundabout is 0.2 crashes/year) was recorded in the period 2010-2017 in the Puglia region (Italy), has revealed that an appropriate deflection of trajectories is not guaranteed (i.e., the deviation angle β is less than 45°, based on MIT (2006), in about 75% of cases for at least one roundabout entry leg. The average critical deviation angle (the smallest angle among all the deviation angles which can be computed for different couples of opposite legs for each roundabout) was actually found to be far less than 45° for the roundabouts at which crashes have occurred.

Based on these preliminary assessments, in this study, the deviation angle method was used as a measure of the deflection of trajectories at roundabouts.

Objective

The importance of ensuring the deflection of trajectories at roundabouts for safety reasons has been stated above. The deviation angle method, considered in this study as a measure for deflection, is based on the determination of the angle β which depends on several geometric parameters, through a graphical method (Fig. 1). In contrast, mathematical formulations are provided for some other methods used for measuring the deflection of trajectories at roundabouts.

Method					Country	ry				
	France (SETRA, 1998)	Netherlands (CROW, 1998)	France Netherlands Sweden Spain UK USA (SETRA, 1998) (CROW, 1998) (Trafikverket, 2015) (MdF, 1999) (UKHA, 2007) (Rodegerdts et al., 2010)	Spain (MdF, 1999)	UK (UKHA, 2007)	USA (Rodegerdts et al., 2010)	Australia Switzerland Italy Germany (Aumann, 2015) (VSS, 1999) (MIT, 2006) (FGSV, 2006)	Switzerland Italy (VSS, 1999) (MIT, 2006)	Italy (MIT, 2006)	Germany (FGSV, 2006
Radius of	≤100 m	≤23 m	≤90 m or ≤30 m*							
deflection										
Entry path radius			≤90 m or ≤30 m*	≤100 m	≤100 m	\leq 52 m or \leq 66 m**	\leq 52 m or \leq 66 m** \leq 55 m** desirable			
Deviation angle								$\geq 45^{\circ}$ in case of $\alpha < 70^{\circ} ****$	≥45°	
Lateral										$\geq 2 L_c$
displacement										

Moreover, most of previous research on this topic was mostly oriented towards: a) finding empirical relationships between safety and roundabout features (e.g., Chen et al., 2013; Elvik, 2017; Montella, 2011; Novák et al., 2018; Polders et al., 2015) also considering geometric aspects; b) incorporating roundabout geometric aspects into more complex assessment which also include environmental and traffic aspects (e.g., Ahmed and Easa, 2021; Fernandes et al., 2020; Hydén and Várhelyi, 2000; Pilko et al., 2017); c) studying specific aspects related to roundabouts, such as conflicts with vulnerable road users (e.g., Dabbour and Easa, 2008; Vignali et al., 2020); d) considering in detail geometric aspects and their influence on design and operation, even if mainly using the entry path radius as the main reference parameter (e.g., Afezolli and Paci, 2012; Ahac et al., 2016; Arndt, 2008).

Hence, this study aims at filling a gap in both research and practice. In fact, as previously indicated, there is no previous study which considers in detail the possible relationships between the main input roundabout design parameters (also including specific aspects such as traffic islands parameters and conflicts with vulnerable users) and the deviation angle, here used as a reference parameter for measuring deflection at roundabouts. Moreover, a mathematical formulation of the deviation angle β may be of practical use for highway engineers, to check for appropriate deflection of trajectories solely based on the input design parameters. In fact, the design of roundabouts is usually a long iterative process, especially for enhancement projects of existing intersections, where space is limited and the inclination of entry/exit legs is not optimal. Knowing in advance the explicit relationships between the geometric parameters of a roundabout and the allowed deflection may help practitioners to optimize their design procedures.

For these reasons, this study has two main research objectives.

- Develop mathematical method for linking the deflection of trajectories to the several geometric parameters of roundabouts.
- Discuss the influence of the different geometric parameters (including detailed parameters related to traffic islands and pedestrians/cycle crossings) on the deflection of trajectories.

In particular, the development of a general mathematical method for both urban and rural one-lane roundabouts and the discussion of results based on its application, represents an extension of a previous study which was mainly focused on the rural environment and was limited to empirical observations (Berloco et al., 2018a). While the same basic hypotheses of the general mathematical method are set for both the urban and rural environments, the rural/urban difference is explicitly considered to take into account some peculiarities. For instance, in our study, we have considered splitter islands for all types of roundabouts, while they are also provided with refuge for pedestrians and cyclists, only in the case of relevant pedestrian/cyclist flows (which are typical in urban environments).

The remainder of the article is organized as follows. The methods used for developing the analytical model and for conducting the sensitivity analysis of all the parameters involved are described first. Then, the model equations obtained are presented, by differentiating them according to the cases considered. The results from the sensitivity analysis are also described and discussed in light of their possible usefulness for researchers and practitioners. Finally, some conclusions are drawn about the main results obtained from this study, including some practical remarks concerning current standards and the design of roundabouts.

3. Method

In this section, the methods used in this study are presented. In detail, the process which led to the mathematical model is described, as well as the criteria used for the further sensitivity analysis.

3.1. Assumptions and procedures for developing the mathematical model

The mathematical model should be developed so that the deflection of trajectories can be linked to all the relevant geometric parameters involved in the roundabout design through some equations. The list of all the geometric parameters taken into account is provided in Fig. 1. As previously mentioned, the deflection of trajectories is estimated using the deviation angle method.

Among the few country standards/guidelines which use the deviation angle method, the graphical method provided in the Italian standards (MIT, 2006) was taken into account as a guideline. The method for computing the deviation angle β according to the Italian standards is defined according to the following steps (Fig. 1).

- Take the entry and exit kerb radii and draw the offset curve at O₁=O₂=3.50 m from both.
- Plot the line simultaneously tangential to both the offset curves determined in the earlier stage and the nonmountable central island kerb.
- Measure the angle included between the tangent lines obtained as above defined.

In the both Italian and Swiss standards, it is recommended that the deviation angle β value be generally greater than or equal to 45° for each crossing leg. This requirement is valid for all types of roundabouts, independently of their diameter. Hence, this threshold is considered in this study as a minimum requisite to ensure a sufficient deflection of trajectories.

Three macro-categories of roundabouts were considered for the model development, listed as follows (Fig. 2).

- ullet "Category 1" roundabouts without approaching and departing radii and provided with entry/exit radii only: $R_{
 m entry}/R_{
 m exit}$.
- "Category 2" roundabouts with approaching and departing radii (R_{app}/R_{dep}) as well as entry/exit radii (R_{entry}/R_{exit}).
- "Category 3" roundabouts with refuge islands for pedestrians and cyclists.

The first two categories of roundabouts were so defined since precise guidance on whether or not to include the approaching/departing radii and on the values, which should

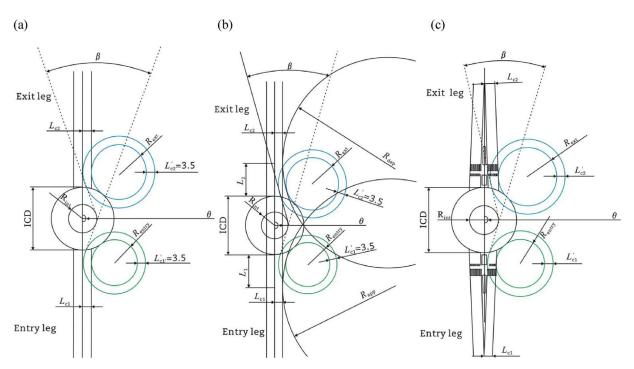


Fig. 2 — Categories of roundabouts taken into account. (a) With entry and exit radii only. (b) With approaching and departing radii. (c) With refuge islands.

be assigned to them, are often not included in standards/ guidelines. Hence, it was decided to take both cases into account whether or not they are present, to assess their influence on the deflection of trajectories. Both categories mainly reflect rural roundabouts. The first case refers to: a) roundabouts at which very-low speed roads intersect, or b) when approaching and departing radii cannot be designed due to the limited space available. The second case refers to roundabouts at which both approaching and departing radii can be implemented. These radii could be necessary in the case of high-speed intersecting roads and they may elicit correct driver behaviour, by providing an appropriate deceleration/acceleration distance independently of speed. In both cases, roundabouts may also be in urban areas, at which pedestrian/cyclist flows are not relevant (refuge islands are not needed).

The third category of roundabouts, which include the presence of splitter islands providing refuge for pedestrians and cyclists, was considered given its important safety implications (Vignali et al., 2020). In fact, splitter islands on roundabout legs may have the following positive effects for safety: a) drivers may better perceive their correct approaching behavior, b) speeds may be reduced, c) there is a gradual deceleration of approaching vehicles, d) approaching and departing flows are physically separated. Appropriately designed splitter islands may also provide refuge to pedestrians and cyclists by allowing a two-stage crossing. This is particularly important in urban areas (Jensen, 2015). For this aim, the refuge island considered in this study (Fig. 3) was designed so that the crossing is at a distance P = 5.0 m from the yield line markings, its length is equal to a distance F = 4.5 m to allow both pedestrian and bicycle crossings, its width is greater than or equal to 1.5 m (related to the dimensions of at least one crossing bicycle) and its shoulders are 0.50 m wide. In this way, a refuge is also provided for cyclists, for whom dedicated infrastructure is recommended at roundabouts (Dabbour and Easa, 2008). In fact, by summing the width of the island at the crossing with the two shoulders: the distance 0.50 \times 2 m +1.50 m = 2.50 m is obtained, while the distance occupied by cyclists is less than 2.0 m.

A triangular shape was assumed for the splitter island. The design of the total splitter island length and the related road markings were set in order to allow a gradual variation of the lane curvature close to the splitter island. The gradual variation was imposed according to the guidelines provided for the insertion of a central left-turn lane on two-way two-lane roads. As an example, the Italian standard for intersections was taken as a reference (MIT, 2006). The necessary length to allow the insertion of an appropriate refuge island is estimated as follows.

$$L_{v,a} = 0.6S_d \sqrt{d'} \tag{1}$$

where $L_{\rm v,a}$, in unit of meter, is the distance in which the appropriate width d' of the refuge island is reached, d' is the extra width necessary to ensure the insertion of an adequate refuge is land on the entry leg (in this case assumed to be equal to the extra width on the exit leg to give a symmetrical

configuration), set as equal to 1.25 m (half of the previously computed bicycle/pedestrian refuge), S_d is the design speed, two cases were considered in this study: 30 and 50 km/h, given that the focus of this configuration is on urban areas.

The standard refuge islands so obtained are depicted in Fig. 3. In this case, approaching and departing $R_{\rm app}/R_{\rm dep}$ were omitted from the design parameters, since it was assumed that the introduction of appropriately designed refuge islands can have similar or even greater effects on safety than approaching and departing radii.

3.2. Assumptions and procedures for conducting the sensitivity analysis

Since the deviation angle β was assumed as a synthetic parameter for measuring the deflection of trajectories, the sensitivity analysis was conducted by considering the deviation angle β as the response variable. The influence of the other geometric parameters of roundabouts was assessed by making them vary in given ranges and by observing the consequent variations of the deviation angle β .

The complete list of geometric parameters considered for developing the mathematical models are reported as follows (Fig. 1).

- Roundabout shape parameters: inscribed circle diameter (ICD), width of the circulatory lane $L_{\rm cir}$, left shoulder width of the circulatory lane $L_{\rm s}$, width of the eventual truck apron $S_{\rm k}$, angle θ formed by the opposite legs.
- Entry leg shape parameters: approaching and entry radii R_{app}/R_{entry} , width of the entry leg L'_{c1} , offset of the entry radius at the intersection O_1 , approaching curvature length L_1 .
- Exit leg shape parameters: departure and exit radii $R_{\rm dep}/R_{\rm exit}$, width of the exit leg $L'_{\rm c2}$, offset of the exit radius at the intersection O_2 , departing curvature length L_2 .
- Refuge islands shape parameters and design speed: S_d, d', P,
 F (Fig. 3).

Some of these parameters were fixed in the sensitivity analysis, given the several variables considered and their correlations. Moreover, a first hierarchical order of the most influential parameters was initially provided by Berloco et al. (2018a). This helped in selecting which parameters should have been necessarily considered as variable. However, fixing some parameters was necessary for the sake of the sensitivity analysis alone. In fact, all the listed parameters could be modified (i.e., are variable) in the developed mathematical models.

The fixed parameters are: the offsets of the entry and exit radii at the intersection O_1 = O_2 =3.5 m (set according to the Italian standards, taken as a reference for drawing the deviation angle), the left shoulder of the circulatory lane 0.50 m wide (a reasonable common measure for left shoulders in roundabouts), a truck apron 1.50 m wide (where needed), the approaching and departing curvature lengths $L_1 = L_2$, the geometric parameters of splitter islands having dimensions

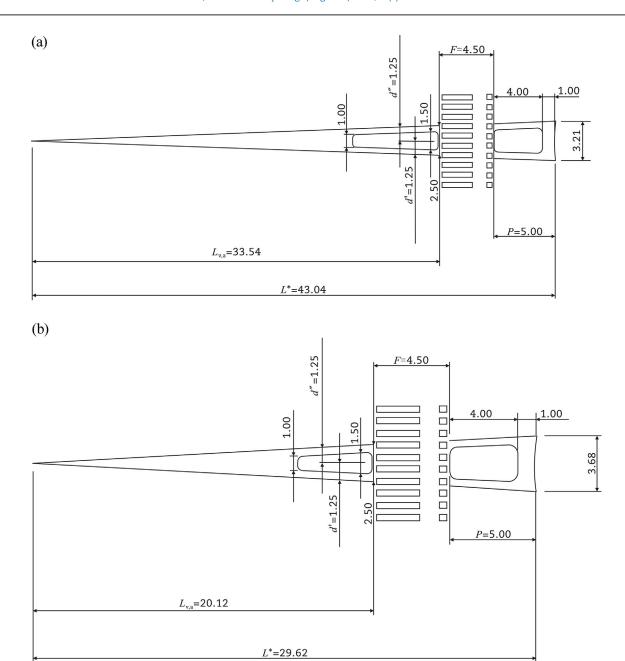


Fig. 3 — Measurements computed for refuge islands in case (measurements are in meters). (a) $S_d = 50$ km/h. (b) $S_d = 30$ km/h.

defined as in Fig. 3 to provide a safe refuge for pedestrians and cyclists. In this regard, the approaching and departing curvature lengths $L_1 = L_2$ were set to 15 m for the first two categories of roundabouts (a measure that allows an adequate transverse width of the splitter island equal to 3 m in correspondence with the circulatory roadway) and set as equal to L^* calculated for category 3 roundabouts (Fig. 3). The truck apron on the outer edge of the central island was only considered for ICD < 25 m. For ICD < 18 m, the central island is assumed to be fully mountable. Moreover, it is stressed again that this study is specifically focused on one-lane roundabouts.

The variable parameters are listed as follows, together with the assumed variation range.

- Width of the circulatory lane (L_c), variable according to the ICD (Italian standards taken as reference) (Table 3):
 - (a) 7–8 m for mini roundabouts (ICD = 14 25 m);
 - (b) 7 m for compact roundabouts (ICD = 25 40 m);
 - (c) 6 m for conventional roundabouts (ICD \geq 40 m).
- Angle θ between the opposite legs, variable between 120° and 180°, through subsequent increasing steps equal to 10°.
- Width of the lanes of the roads approaching and departing from the roundabouts L_{c1} and L_{c2} , variable between 2.75 and 3.75 m, through subsequent increasing steps equal to 0.25 m.
- ICD (related to the radius R_{int}), variable between 19 m (ICD value below which the central island is fully mountable) and 50 m (maximum ICD value according to the Italian

Table 3 — Widths of urban/rural single lane roundabout elements (in case of single lane entry legs) (MIT, 2006).						
Roundabout element	Inscribed circle diameter (m)	Lane width (m)				
Lane of the circulatory roadway	≥40 (conventional roundabouts)	6.00				
	25–40 (compact roundabouts)	7.00				
	14–25 (mini roundabouts)	7.00-8.00				
Entry leg		3.50				
Exit leg	<25 (mini roundabouts)	4.00				
	\geq 25 (conventional/compact roundabouts)	4.50				

standards) (Table 3), through subsequent increasing steps equal to $1\ \mathrm{m}$.

- Entry and exit radii R_{entry}, R_{exit}, for which three R_{entry}/R_{exit} combinations were selected: 10 m/12 m, 12 m/15 m, 15 m/ 18 m, according to best practices and guidelines from other European countries (e.g., VSS (1999) since they are not considered in the Italian standards).
- Approaching and departing radii $R_{\rm app}$, $R_{\rm dep}$, computed based on Swiss standards: $R_{\rm app} = 5R_{\rm entry}$ and $R_{\rm dep} = 4R_{\rm exit}$, thus resulting in the following three combinations 50 m/48 m, 60 m/60 m and 75 m/72 m, according to the above defined $R_{\rm entry}/R_{\rm exit}$ combinations.

The combinations of the fixed and variable parameters led to computing the deviation angle for 10,710 cases of roundabout scenarios. Since great Lcir values can lead to small deviation angles, especially for small ICDs, a further set of 10,080 combinations was simulated by setting L_{cir} equal to 6 m for all roundabouts (differently from the initial set of variable In this case, for compact roundabouts (ICD = 25-40 m), a truck apron 2.50 m wide was considered (increased by 1 m with respect to the standard value due to the 1 m decrease in the L_{cir}). Since the kerb should be realized in order to be easily mountable by heavy vehicles but unattractive for light vehicles, the lines needed for the deviation angle construction can be tangential to the truck aprons instead of central island edges. Moreover, several other combinations were simulated in order to assess the influence of the different geometric parameters on the deviation angle. These further combinations were obtained in this way: step 1, all the variable geometric parameters were fixed except for one parameter, step 2, the variable geometric parameter was made to vary. This process was repeated for each variable geometric parameter. In step 1, three different values were sequentially

assigned to the parameters: average, optimal and unfavourable fixed values. Hence, step 2 was repeated for the three different fixed values assigned in step 1. The definition of "optimal" and "unfavourable" values depends, for each parameter, on its influence on the deviation angle (i.e., optimal values mean that the considered values are those providing, ceteris paribus, the greatest deviation angle possible among all the values in the considered range; vice versa for the unfavourable values). The values used in step 1 are listed in Table 4.

In the step 2, the parameters were made vary, one at a time, according to the ranges included in Table 5. The values of Table 5 are with all the other variables fixed according to one of the average, optimal or unfavorable combinations in step 1, reported in Table 4.

4. Results

The results from the study conducted are presented in this section. In the first part, the developed mathematical method is described. Then, results from the sensitivity analysis are presented.

4.1. Mathematical method

The mathematical method was developed for three different categories of roundabouts: 1) those not provided with approaching and departing radii, but only with entry and exit radii $R_{\rm entry}/R_{\rm exit}$ (Fig. 2(a)), 2) those provided with approaching and departing radii $R_{\rm app}/R_{\rm dep}$ (Fig. 2(b)), 3) those provided with refuge islands for pedestrians and cyclists (Fig. 2(c)).

The equations for calculating the deviation angle β in the three different cases are reported as follows.

Table $4-$ Values of parameters used in the step 1.							
Parameter	ICD (m)	<i>⊕</i> (°)	R _{entry} (m)	R _{app} (m)	R _{exit} (m)	R _{dep} (m)	$L_{c1}=L_{c2}$ (m)
Average value	33	160	12	60	15	60	3.25
Optimal value	50	180	10	50	12	48	2.75
Unfavourable value	19	120	15	75	18	72	3.75

Table 5 — Values of the ranges used for the variation of parameters in the step 2.							
Parameter	ICD (m)	<i>Θ</i> (°)	R _{entry} (m)	R _{app} (m)	R _{exit} (m)	R _{dep} (m)	$L_{c1}=L_{c2}$ (m)
Minimum value	19	120	6	30	6	30	2.75
Maximum value	50	180	42	300	44	300	3.75
Increasing step	1	10	2	10	2	10	0.25

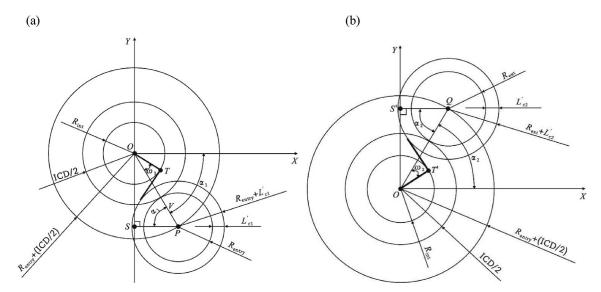


Fig. 4 – Geometric schemes of "category 1" roundabouts. (a) Entry leg. (b) Exit leg.

$$\beta(\text{category 1}) = T_1 + T_2 + \vartheta - \pi = \alpha_1 + \alpha_2 - \omega_1 - \omega_2 + \vartheta - \pi \tag{2}$$

$$\beta(\text{category 2}) = T_1 + T_2 + \vartheta - \pi = \mu_1 + \mu_2 + \delta_1 + \delta_2 - \omega_1 - \omega_2 + \vartheta - \pi$$
(3)

$$\beta(\text{category 3}) = T_1 + T_2 + \vartheta - \pi = \delta_1 + \delta_2 - \omega_1 - \omega_2 + \vartheta - \pi \tag{4}$$

where the angles $T_1, T_2, \vartheta, \mu_1, \mu_2, \delta_1, \delta_2, \omega_1, \omega_2, \alpha_1, \alpha_2$ are obtained from the geometric constructions defined in the following figures. Note that for all the angles, subscript 1 refers to the entry leg, while subscript 2 refers to the exit leg. Whereas, the angle between the opposite legs is indicated by ϑ . In case of orthogonal legs ($\vartheta=180^\circ$), the equations are reduced by eliminating the last two terms: $\vartheta-\pi$. In the following figures, specific details about how these equations were obtained are provided, by clarifying the meaning of all the geometric parameters involved in the formulations.

Fig. 4 shows geometric schemes of "category 1" roundabouts with entry/exit radii only, alongside with the equations based on the geometric parameters useful for the formulations.

"Category 1" roundabouts-entry leg

$$\alpha_1 = \cos^{-1}\left(\frac{R_{\text{entry}} + L_{c1}}{R_{\text{entry}} + \frac{ICD}{2}}\right) \tag{5}$$

$$\omega_1 = \cos^{-1}\left(\frac{R_1 + R_2}{d}\right) \tag{6}$$

with

$$\left\{egin{aligned} R_1 &= R_{\mathrm{int}} \ R_2 &= R_{\mathrm{entry}} + L_{\mathrm{c1}}' \ d &= R_{\mathrm{entry}} + rac{\mathrm{ICD}}{2} \end{aligned}
ight.$$

$$T_1 = T\widehat{O}X = \alpha_1 - \omega_1 \tag{8}$$

"Category 1" roundabouts-exit leg

$$\alpha_2 = \cos^{-1}\left(\frac{R_{\text{ext}} + L_{\text{c2}}}{R_{\text{ext}} + \frac{\text{ICD}}{2}}\right) \tag{9}$$

$$\omega_2 = \cos^{-1}\left(\frac{R_1 + R_2}{d}\right) \tag{10}$$

with

$$\begin{cases} R_{1} = R_{\text{int}} \\ R_{2} = R_{\text{ext}} + L'_{c2} \\ d = R_{\text{ext}} + \frac{\text{ICD}}{2} \end{cases}$$
 (11)

$$T_2 = \mathbf{T}' \hat{\mathbf{O}} \mathbf{X} = \alpha_2 - \omega_2 \tag{12}$$

Fig. 5 shows geometric schemes of "category 2" roundabouts with approaching/departing radii, alongside the equations based on the geometric parameters useful for the formulations.

"Category 2" roundabouts-entry leg

$$M = (q; -p) \tag{13}$$

with

(7)

$$\begin{cases} q = L_{c1} + R_{app} \\ p = \frac{ICD}{2} + L_1 \end{cases}$$
 (14)

$$\Delta = R_{app} - R_{entry} \tag{15}$$

$$\omega_1 = \cos^{-1}\left(\frac{R_1 + R_2}{d}\right) \tag{16}$$

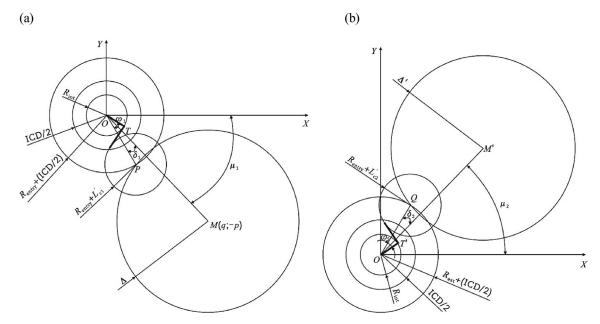


Fig. 5 - Geometric schemes of "category 2" roundabouts. (a) Entry leg. (d) Exit leg.

(18)

(20)

$$\delta_1 = tan^{-1} \left(\frac{\sqrt{4R_1^2R_2^2 - \left(d^2 - R_1^2 - R_2^2\right)^2}}{d^2 + R_1^2 - R_2^2} \right)$$

with

$$\begin{cases} R_1 = R_{entry} + \frac{ICD}{2} \\ R_2 = \varDelta \\ d = \sqrt{p^2 + q^2} \end{cases}$$

$$T_1 = T\widehat{O}X = \mu_1 + \delta_1 - \omega_1$$

"Category 2" roundabouts-exit leg

$$\mathbf{M}' = (\mathbf{q}'; -\mathbf{p}')$$

with

$$\left\{ egin{aligned} q' &= L_{
m c2} + R_{
m dep} \ p' &= rac{
m ICD}{2} + L_2 \end{aligned}
ight.$$

$$\Delta' = R_{dep} - R_{ext}$$

$$\omega_2 = \cos^{-1}\left(\frac{R_1 + R_2}{d}\right)$$

$$\delta_2 = \tan^{-1}\left(\frac{\sqrt{4R_1^2R_2^2 - \left(d^2 - R_1^2 - R_2^2\right)^2}}{d^2 + R_1^2 - R_2^2}\right)$$

with

(17)
$$\begin{cases} R_1 = R_{\text{ext}} + \frac{\text{ICD}}{2} \\ R_2 = \Delta' \\ d = \sqrt{p'^2 + q'^2} \end{cases}$$
 (25)

$$T_2 = T'\widehat{O}X = \mu_2 + \delta_2 - \omega_2 \tag{26}$$

Fig. 6 shows geometric schemes of "category 3" roundabouts with refuge islands; alongside the equations based on the geometric parameters useful for the formulations.

(19) "Category 3" roundabouts-entry leg

$$B = \left(\frac{ICD}{2} + L^*\right)\cos(\epsilon) + L_{c1}$$
 (27)

$$\epsilon = \tan^{-1} \left(\frac{L_{v,a}}{d'} \right) \tag{28}$$

(21)
$$\delta_1 = \pi - \epsilon - \sin^{-1} \left(\frac{B + R_{\text{entry}}}{\frac{\text{ICD}}{2} + R_{\text{entry}}} \right)$$
 (29)

(23)
$$\omega_1 = \cos^{-1}\left(\frac{R_1 + R_2}{d}\right)$$

$$T_1 = \widehat{TOX} = \delta_1 - \omega_1 \tag{31}$$

(24) "Category 3" roundabouts-exit leg

$$B' = \left(\frac{ICD}{2} + L^*\right)\cos(\epsilon) + L_{c2}$$
(32)

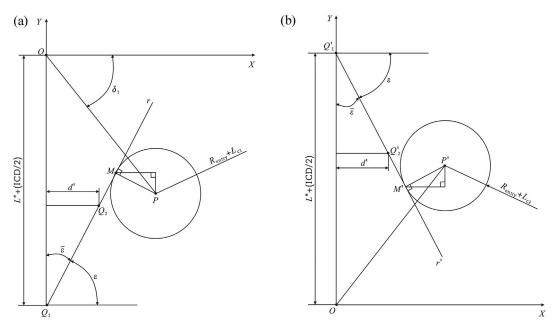


Fig. 6 – Geometric schemes of "category 3" roundabouts. (a) Entry leg. (b) Exit leg.

$$\epsilon = \tan^{-1} \left(\frac{\mathbf{L}_{\mathbf{v},\mathbf{a}}}{d'} \right) \tag{33}$$

$$\delta_2 = \pi - \epsilon - \sin^{-1} \left(\frac{B' + R_{\text{ext}}}{\frac{\text{ICD}}{2} + R_{\text{ext}}} \right)$$
 (34)

$$\omega_2 = \cos^{-1}\left(\frac{R_1 + R_2}{d}\right) \tag{35}$$

$$T_2 = T'\widehat{O}X = \delta_2 - \omega_2 \tag{36}$$

The developed mathematical method is useful to immediately obtain the deviation angle β , given the main geometric parameters of the roundabouts, for different roundabout categories. The formulation of the method is not dependent on any specific standard/guidelines, given that all the main geometric parameters can be modified. Some default parameters were used for the dimensions of the refuge islands in the "category 3" roundabouts (Fig. 3). However, the main shape parameters of the splitter islands could also be modified. Hence, the developed mathematical method could potentially be applied without country/regional restrictions.

4.2. Sensitivity analysis

The mathematical method presented in the previous section was transferred to a spreadsheet, in which the main round-about design parameters were used as input and the deviation angle as output. Hence, the previously cited simulations were performed by making the input in the spreadsheet (which includes the mathematical framework) vary systematically in order to observe the deviation angle variation. The systematic

variation of the input parameters, necessary for the sensitivity analysis, was run according to the procedure previously described in the Methods section.

Based on the 20,790 simulations performed, in which different combinations of the geometric parameters were attempted, the parameters which were most influential geometric on the deviation angle were identified. They are listed below, in descending order (from the most important to the least important parameters among the most influential ones).

- ICD diameter
- ullet Angle heta between opposite legs
- Circulatory lane width, Lcir
- Entry and exit radii Rentry, Rexit
- Width of entry and exit legs Lc1, Lc2
- Approaching and departing radii R_{app}, R_{dep}

From the results obtained from the simulations, synthetic diagrams were plotted to show the influence of the most important parameters on the deviation angle. In detail, it was chosen to show here the variation of the deviation angle with the main influential parameters: the ICD, the angle θ , the L_{cir} width and the connecting radii. These parameters were represented in two-dimensional plots in order to obtain diagrams useful for practitioners. Since they are only referred to the variation of the most influential variables on the deviation angle, the other parameters were fixed. In detail, the width of entry and exit legs was set as equal to 3.50 m (a common standard urban/sub-urban lane width), the approaching and departing radii ("category 2" roundabouts) were determined as a function of the entry and exit radii through the previously mentioned relationships ($R_{app}=5R_{entry}$, $R_{dep}=4R_{exit}$), the S_d was set to 50 km/h in Eq. (1) (a common urban speed).

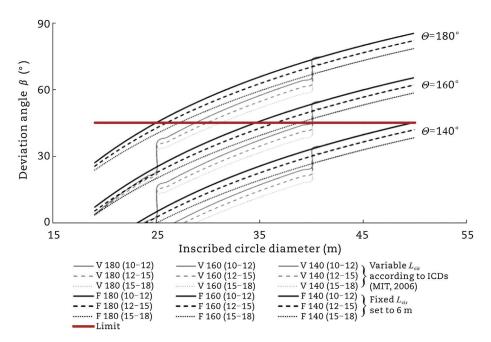


Fig. 7 — Variability of the deviation angle with the ICD, the θ angle, L_{cir} and connecting radii for "category 1" roundabouts, with entry/exit radii only.

Three diagrams are shown in the following figures, one for each of the considered categories of roundabouts.

In Figs. 7–9, V means variable, showing that $L_{\rm cir}$ varies according to the ICD. F means fixed, showing that $L_{\rm cir}$ is fixed and set to 6 m. The three numbers 140/160/180 refer to the considered θ angles. The three pairs of numbers 10–12/12–15/15–18 refer to the considered combinations of entry/exit radii. The threshold β angle (red horizontal "limit" line) is set to 45°.

In the diagrams, both conditions of variable $L_{\rm cir}$ according to the ICD (following the Italian standards) (Table 3), and $L_{\rm cir}$ fixed as equal to 6 m were taken into account (black and grey curves, respectively). Solid curves represent the case of $R_{\rm entry}/R_{\rm exit}$ equals to 10 m/12 m, dashed curves represent the case of $R_{\rm entry}/R_{\rm exit}$ equals to 12 m/15 m, and dotted curves represent the case of $R_{\rm entry}/R_{\rm exit}$ equals to 15 m/18 m. The calculations were repeated for three different values of the θ angle, namely equal to 180°, 160° and 140°. Smaller values of the

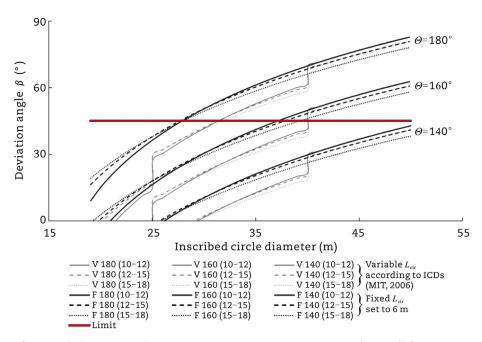


Fig. 8 — Variability of the deviation angle with the ICD, the θ angle, L_{cir} and connecting radii for "category 2" roundabouts, with approaching/departing radii.

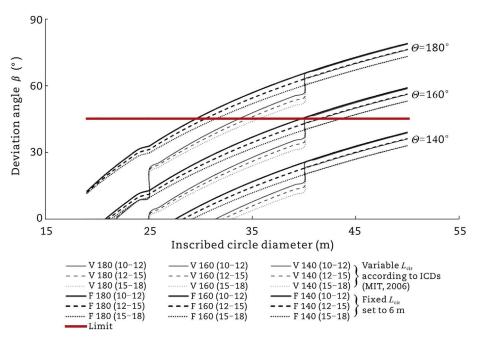


Fig. 9 — Variability of the deviation angle with the ICD, the θ angle, Lcir and connecting radii for "category 3" roundabouts, with refuge islands.

angle between opposite legs were not taken into account, as they may almost always result in insufficient deviation angles (i.e., less than 45° in this case). Configurations related to other θ angle values can be obtained through interpolation.

Based on the diagrams in Figs. 7-9, it is possible to appreciate that the ICD diameter, the θ angle, and the circulatory lane width L_{cir} are the three main parameters which can strongly influence the deviation angle β , all other conditions being equal. In the case of $\theta = 140^{\circ}$, the deviation angle is greater than 45° only for "category 1" roundabouts with entry/ exit radii and for ICDs close to 50 m. In the case of $\theta = 180^{\circ}$, and for "category 1" roundabouts, the deviation angle is greater than 45° for ICDs > 25 m, in the case of $L_{\rm cir} = 6$ m and only for R_{entry}/R_{exit} radii equals to 10 m/12 m. The presence of approaching/departing radii worsens the deviation angle (i.e., lower), with the ICD being equal. The variation of the entry/ exit radii within the considered possibilities (10 m/12 m, 12 m/ $\,$ 15 m, 15 m/18 m) can lead to significant changes in the deviation angle in the case of absent approaching/departing radii (Fig. 7). For example, the deviation angle limit of 45° is reached for ICD = 39 m, in the case of: R_{entry}/R_{exit} =15 m/18 m, no approaching/departing radii, Lcir = 6 m. Note that, other conditions being equal, a simple decrease in the R_{entry}/R_{exit} down to 10 m/12 m will lead to decreasing the ICD down to 35 m, to reach the same value of $\beta = 45^{\circ}$. This knowledge is crucial for conditions in which spaces are limited and land expropriation may be troublesome. On the other hand, for "category 3" roundabouts, it is possible to ensure an appropriate deviation angle (>45°) only for high ICDs. Note that, as already stated by Berloco et al. (2018a), the β angle is always less than 45° for very small ICDs (i.e., <25 m). Hence, in this case, alternative measures for controlling speeds should be implemented (Berloco et al., 2018b).

5. Concluding remarks

The limitation of the deviation angle β is one of the possible methods to be used for controlling speeds and ensuring the deflection of trajectories in roundabouts. Currently, this method is only used in Italian and Swiss standards, in which a deviation angle greater than or equal to 45° is recommended. In other words, the control of speeds and trajectories is deemed as effective when the paths of vehicles entering a roundabout are deflected to the right by 22.5°, by 45° to the left while navigating the roundabout and, finally, the paths of exiting vehicles are again deflected to the left by 22.5°.

This study was focused on one-lane roundabouts (both urban and rural) divided into three categories: a) those provided with approaching/departing radii besides entry/exit radii, b) those only provided with entry/exit radii, and c) those provided with refuge islands for pedestrians and cyclists. The main objectives of the study were: to develop a mathematical method to directly relate the deviation angle to the geometric parameters of the roundabout and to conduct a sensitivity analysis of the geometric parameters with respect to the deviation angle. Hence, the article was conceived to be useful for both researchers and practitioners. In fact, indicating the parameters which should be particularly designed for deflecting trajectories at roundabouts paves the way for dedicated research on this topic, by investigating the influence of these parameters on safety and operation. In fact, findings from this article are also useful to shed additional light on how to set optimal values for the entry/exit and approaching/departing radii, for which standards/guidelines are generally scarce. Moreover, engineers are provided with a practical calculation tool to be used before designing, once the initial geometric parameters are known.

Considering the application of the developed method and by taking into account the results of the sensitivity analysis, the main results from this study are reported as follows.

- An insufficient deflection of trajectories (based on the deviation angle method, i.e., $\beta < 45^{\circ}$) is always present for roundabouts with ICD < 25 m and very often for roundabouts with ICD = 25–40 m.
- An insufficient deflection of trajectories (as based on the deviation angle method, i.e., β < 45°) can be found for urban roundabouts with refuge islands for pedestrians and cyclists, in the case of ICD < 34 m and orthogonal legs.
- The main parameters which are responsible for a decrease in the deviation angle β are: a decrease in the ICD diameter; a decrease in the angle θ between opposite legs; and an increase in the width of the circulatory lane $L_{\rm cir}$.
- It is not possible to ensure an appropriate deflection of trajectories (based on the deviation angle method, i.e., $\beta < 45^{\circ}$) when θ angles are smaller than 140° .
- The other parameters which are responsible for a decrease in the deviation angle β are: an increase in the entry/exit radii, an increase in the width of the entry/exit legs L'_{c1}/L'_{c2} ; and the presence of and an increase in the approaching/ departing radii.

The study is particularly useful for the enhancement of existing intersections and their conversion into roundabouts. In fact, once the ICD has been determined based on the available space, the θ angle between opposite legs can beset as close as possible to 180°, the designer can then select the most appropriate values for the other parameters in order to achieve the highest deviation angle β possible.

Concerning the circulatory lane width $L_{\rm cir}$, the deflection of trajectories (measured through the deviation angle) is improved when the width is set to a limited constant value (i.e., when the width does not increase with the decrease in the ICD), also for small ICDs. In this study (in which widths were set, when needed based on Italian standards), the constant $L_{\rm cir}$ was set to the minimum possible value provided by the Italian standards: 6 m independently of the ICD. Hence, especially for small ICDs, limiting the $L_{\rm cir}$ width may result in the need for a truck apron for heavy vehicles, which however should not be attractive for light-vehicle drivers (Berloco et al., 2018a) otherwise trajectories will not be deflected. These remarks can be useful to address choices concerning the circulatory lane/truck apron width in future editions of standards/guidelines based on the deviation angle method.

Furthermore, some other measures should be taken into account for very small ICDs (i.e., <25 m), since in such cases the β angle is always <45°. In detail, the following measures are suggested:

- For $30^{\circ} \le \beta \le 40^{\circ}$, light traffic calming measures should be adopted such as horizontal and vertical markings, optical measures or rumble strips on the roundabout entry legs.
- For $\beta \leq 30^\circ$, more effective traffic calming measures should be implemented on the roundabout entry legs, such as speed humps, raised pedestrian crossings, speed cushions, chicanes and automated speed control. In the case of traffic calming measures implemented on low-traffic

roundabouts, raised splitter islands could be eventually replaced with zebra stripes, considering the decreased likelihood of both frequency and severity of crashes with vulnerable road users involved.

The conducted study is not without limitations. Clearly, even if the proposed mathematical model is general and it includes several geometric variables, it cannot account for all the possible geometric configurations and variables of roundabouts. Moreover, only one-lane roundabouts have been considered. Some of the geometric parameters were fixed for the sake of the sensitivity analysis, which could then be enlarged by considering the variation of other parameters. Some suggestions were provided for roundabout design optimization from a geometric perspective, which however should be integrated with delay estimates (Easa and Mehmood, 2004), which have not been considered in this study. However, in its present form, the mathematical model provided and the implications of the results from the simulations performed are useful for both highway engineers and road safety/geometric design researchers.

Conflict of interest

The authors do not have any conflict of interest with other entities or researchers.

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