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# Characterization and dynamic behaviour of Masonry Newly Concept Vault realized by 3d Printer

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**Abstract.** Masonry vaults represent one of the most recurrent types of horizontal structural elements in architecture in European countries, even in areas characterized by a high level of seismic risk. Therefore, their structural evaluation remains of primary importance. This paper proposes to apply already consolidated structural analysis methodologies on a contemporary vaulted space; in particular, on a small-scaled vault model. The aim, therefore, is to investigate the behaviour of a newly concept vault generated starting from the “Flat vault of Abeille” patented in 1699 starting by a scaled model made by 3D printer. The identification of this “new type” of vault is essential to design it correctly and to optimize the geometry for structural purposes. So, the dynamic identification of 1:8 small-scale vault model in 3D printing was studied. All the vault blocks were made in Polylactic Acid (PLA), have an infill of 70% and are assembled with dry joints. The paper describes the process to acquire the environmental and forced vibrations and to analyse them with the Operational Modal Analysis (OMA), identifying the natural frequencies and the mode shapes. Subsequently the results were compared with those of numerical distinct element method (DEM).

## 1. Introduction

The analysis of masonry vaults' structural behavior has emerged as a crucial and increasingly researched topic in the field of construction techniques. The vaults are prevalent present in traditional architecture and historic buildings throughout Europe, making them one of the most recurrent horizontal structural element types. However, masonry vaults are also one of the most vulnerable supporting elements to dynamic actions, as demonstrated by numerous post-earthquake inspection campaigns conducted on historic buildings in Italy and other European countries, all areas characterized by high seismicity. As a result, evaluating their structural safety and dynamic behavior has become of primary importance for engineering and architectural research.

Early important studies and experiments with small-scaled model in this field were conducted by Van Mele and Mc Inerney [1] for quasi-static tests, and by and by DeJong, De Lorenzis and Ochsendorf [2], [3] for dynamic tests. Since then, several studies have focused on investigating the dynamic behavior of masonry vaults [2], [4]–[8]. Computational methods, including Finite Element Method



(FEM) and/or Discrete Element Method (DEM) approaches, have been utilized in all these studies to investigate the strength of these vault forms.

Considering this topic, it was decided to increase the investigations to include new vaulted spaces. There are two reasons for this. The first is to experiment with new forms and the possibility of a more considered and suitable one for damaged buildings, typically those that are part of the historical heritage. The second is to revive traditional construction techniques, such as stone masonry, for new constructions. The second rationale is based on Valadier's well-known restoration theories, which favor the integrative intervention's ability to separate from the pre-existing elements. The notion of reinterpreting the Flat Vault of Abeille, patented in 1699, is studied from this perspective, or from the perspective of additions/integrations into existing historical structures. The overall research's goal is to investigate the structural behavior of contemporary vaulted spaces created by complex ashlar.

In particular, the paper focuses on the use of monitoring with ambient and forced vibrations and dynamic identification with the Operational Modal Analysis (OMA) on small-scale model to understand if it is possible to conceive a new investigation approach for complex structures. This would make possible to understand the real behavior and the effects of horizontal forces of these structures before their construction, and in particular the identification of their natural frequencies and mode shapes starting from the experimental campaign on the physical scale model and subsequently comparing the validated numerical small-scale model with the numerical one in real size.

The OMA is one of the most utilized techniques used to identify the unknown modal parameters of a –modern [9], [10] or historical [11], [12]– building from the monitoring output-only analysis [13]–[16]. This technique was already used in vaulted structures [17]–[20]. For this reason, it was decided to use it for the identification of the small-scale physical model.

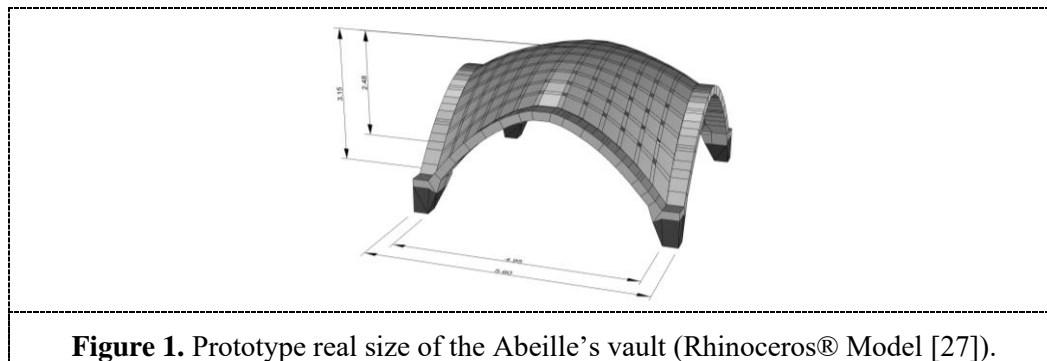
In detail, this investigation involved a first phase of monitoring that comprised collecting data by installing accelerometers in selected points on the Abeille's vault. In a second step, using the ARTeMIS program[21], the obtained data was subjected to OMA. In a third phase a numerical model was defined with the Distinct Element Method (DEM) and the 3DEC software [22]. The final step was the comparison of the numerical mode.

## 2. Physical model

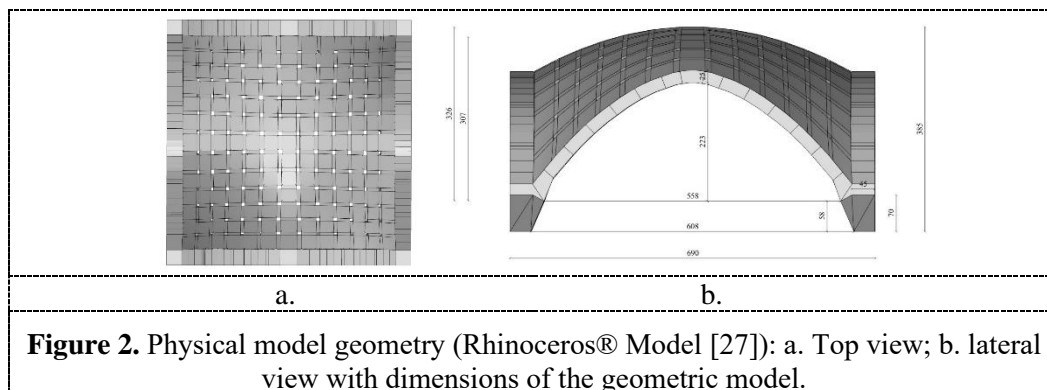
The "Abeille Flat Vault" [23], [24] served as a starting point for the creation of this vault's peculiar shape, which was then made possible through stereotomy examinations of actual stone and masonry vaults [25], [26]. The Abeille-Type standard ashlar, which is a polyhedron with two axial sections in the shape of an isosceles trapezoid that are oriented in opposing directions, was used to create this flat vault. It was then deformed on a spatially curved surface to resemble a sail/domed shaped vault typology. Once the assembly is complete, the geometry of the bidirectional plate created by the ashlar pattern ensures that each individual block of the vaulted system will support the others. Each block was therefore precisely planned in terms of shape and size to play a crucial purpose in the stability and static equilibrium of the vault.

According to Fig. 1, the prototype of Abeille's vault has a square base with a side of 5.60 m and a rise of 3.15 m.

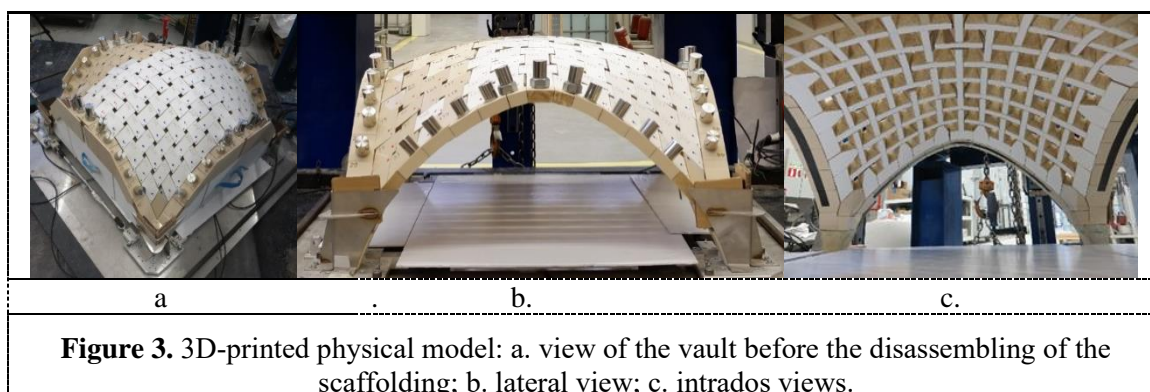
The vault is made up of 235 blocks, each of which is a formal and faithful reproduction of the prototype but differs in size and shape. The vault is twice symmetrical, however as can be seen in Fig.1, the two symmetrical axes have different ashlar. Although these blocks began as a spatial deformation of the *Abeille's vault*, they are comparable to trapezoidal prisms that behave in contrast due to their stereotomic nature.



The physical model's overall dimensions (Fig. 2) were chosen so that it would fit on the testing apparatus of the University of Minho's Institute of Science and Innovation for Bio-Sustainability Laboratory (IB-S Lab). As a result, a physical model measuring 690x690x385 mm<sup>3</sup> was realized, with blocks' standard measurements equal to 30x70x25 mm<sup>3</sup>. The monitoring were therefore conducted at a scale of 1:8.

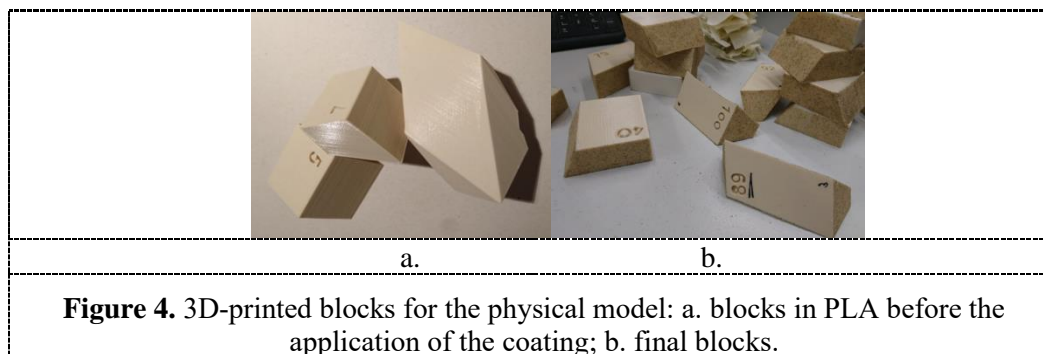


To consider the impact of embedment in the perimeter walls and stiff abutment, the model was truncated at the base. The failure of the vaults began at a somewhat higher level, as various studies of the damage to historic buildings during seismic events have shown, as was noted in [6], effectively being embedded into support parts to offset the outward thrust [28], [29]. Thus, this approximation is frequently used in the literature to understand and forecast how masonry vaults would react to seismic activity. The complete model is mounted on four rigid plastic abutments (shown in dark grey in Fig. 2.b), which are integrated component of the testing apparatus shown in Fig. 3.



The physical modelling was created by "CREA 3D S.R.L." (Italy) using different plastic blocks that were 3D printed and had dry joints.

The 3D-printed Polylactic Acid blocks (PLA) were shown in Fig. 4.a. This polymer, which is entirely biodegradable and compostable, was created by digesting dextrose-rich plants like corn. The S5 by Ultimaker was utilized as the 3D printer, and the printing resolution used for layer height was equal to 0.05 mm. In order to obtain the minimum acceptable weight required for tests, the vault blocks were made with an infill of 70% (with a specific weight  $\gamma = 7.6 \pm 0.2 \text{ kN/m}^3$ ), nevertheless, the low density of the plastic material could put at risk the model's stability in the accidental actions. Two operations were done on each block of the geometric model prior to 3D printing. Because that all the faces of the blocks are curved, the extrados faces were made flat to enable printing itself. They were also numbered to make it simple and accurate to reassemble the model in each test.



To make the vault model blocks behave as closely and similarly to a realistic stone masonry vault as feasible, a sand-based coating was put to them to increase the angle of friction between them. The material qualities, such as mass density, elasticity, strength, etc., do not affect the issue with similarity laws [1], [2], [30], and the mixture did not significantly deteriorate during the test campaign, thus this methodology has already been employed in others research, such as [31]. This coating was created using an epoxy laminating resin called "S&P Resin 55 HP" by "Simpson Strong-Tie" that has a maximum grain size of 0.18 mm, a transparent two-component epoxy resin with a specially formulated amine hardener, and fine sand that was sieved with Mesh 80 in accordance with the American Standard Test Sieve (ASTM) [32].

The sand was left in the drying oven for 24 hours in order to choose the right grain, especially taking the scale of the model into consideration, i.e. the fine grain. It was then sieved using sieves ATSM E 11-70. To get the appropriate particle size, use meshes 40, 60, and finally 80. The paper tape was placed to each face of the blocks (the vault and confining arches), which did not need this finish, before the coating was completed. The epoxy resin was then created, and a container containing the sieved sand was prepared in order to apply a very thin film of epoxy resin (0.5 mm) with a brush and tap the afflicted faces on the sand, being sure to remove any excess material in both cases.

The blocks were given 48 hours to rest. Sandpaper was run over all the interested faces in order to uniformly distribute the sand on all the blocks' faces and to remove the paper tape. The final blocks are displayed in Fig. 4.b. The physical model was modified to fit a component and a configuration that already existed in the IB-S Lab, as stated previously. Even with the utmost care taken, several calibrations and adjusts were made to the model, as shown below, following some assembly tests to try to match the original vault model to the setup in the modeling and printing phase:

- Several blocks that matched the original setup's abutments were firmly glued together with hot glue (Fig. 3.a);
- Applying insulation tape to the four confinement arches' intrados (Fig. 3.c);
- Placing steel elements on the confinement arches' blocks to increase weight and simulate stability without placing vertical elements (plexiglass or wooden walls) around the vault's four sides [6] (as shown in Fig. 3); in particular, an effort was made to simulate the weight of a potential overlying wall using cylindrical steel elements (with a diameter equal to 25 mm and a height equal to 30 mm) weighing 115.3 g;

- To increase the confinement of the system, the blocks of the confinement arches were rigidly linked two by two with hot glue, with the exception of the keystones, which are made up of three initial blocks; as a result, the arches at the end are built of five macro-blocks;
- The addition of additional elements to create the required contrast between the various blocks; in particular, these elements were constructed using corrugated cardboard sheets on which the same procedure used to coat the blocks with sand was applied.

### 3. Experimental Campaign and results

Within a broader experimental campaign –necessary to understand the structural behavior of the Abeille’s vault– which included quasi-static shear and tilting tests of the scale model, Ambient vibration monitoring were also performed for the dynamic identification and, in particular, of the natural frequencies and modal shapes of the vault.

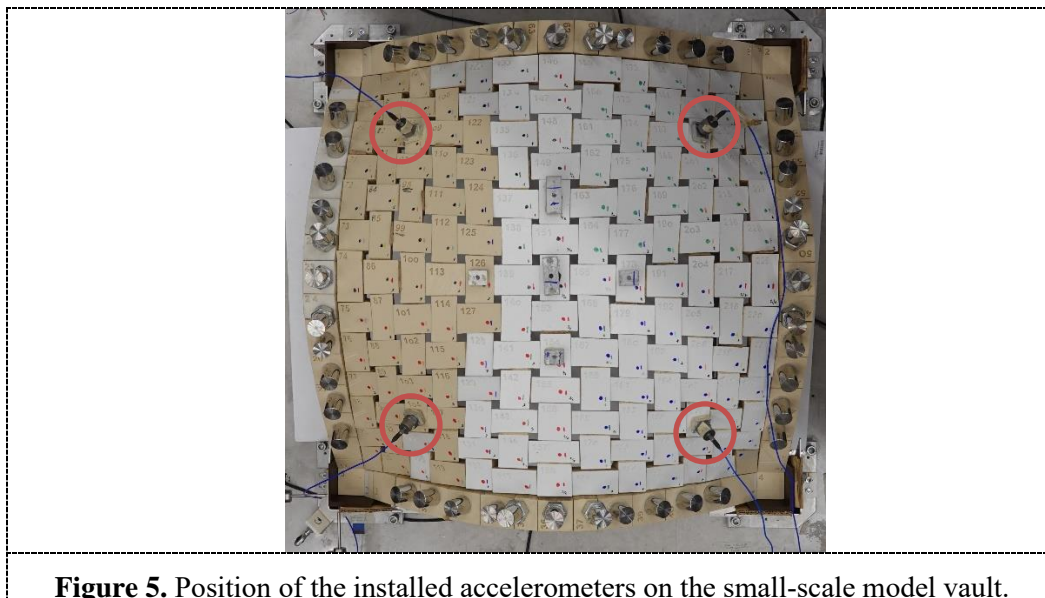
The monitoring envisaged two different types of acquisition:

- The first one with only environmental actions, for the total duration of 15 minutes and with an acquisition frequency of 196 Hz;
- The second one with undirect forced action, i.e. the application of finger tapping on the support base of the model for the duration of 5 minutes with an acquisition frequency of 196 Hz. In this case the reproduction of rhythmic sounds has been carefully avoided.

In both cases, no displacement constraints were applied to the vault at the base, as the supports were arranged on elements comparable to rollers. Only rotations were constrained.

#### 3.1. Test set-up

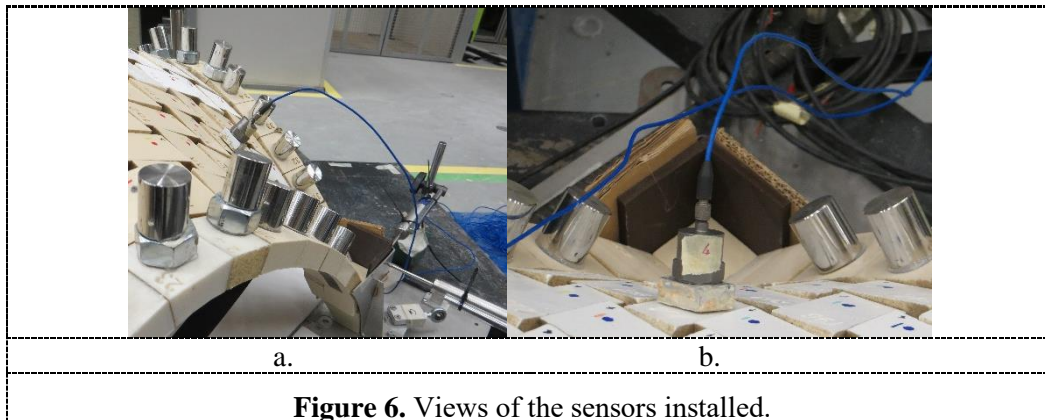
The positioning of the accelerometers was decided to the limited number of accelerometers and to obtain an adequate monitored points for each quarter of vault to estimate the mode shapes of the investigate structure. As shown in Figures 5, the accelerometers were four and were positioned one for each quarter, more or less, to the same level. Each point was monitored along only one direction orthogonal to the support surface (see Figure 7).



**Figure 5.** Position of the installed accelerometers on the small-scale model vault.

The testing setup was composed by n. 4 monoaxial accelerometers PCB GPTC03 (sensitivity 10 mV/g), n. 1 four-channel Dynamic Signal Acquisition Module and n. 1 chassis CompactDAQ 9178.

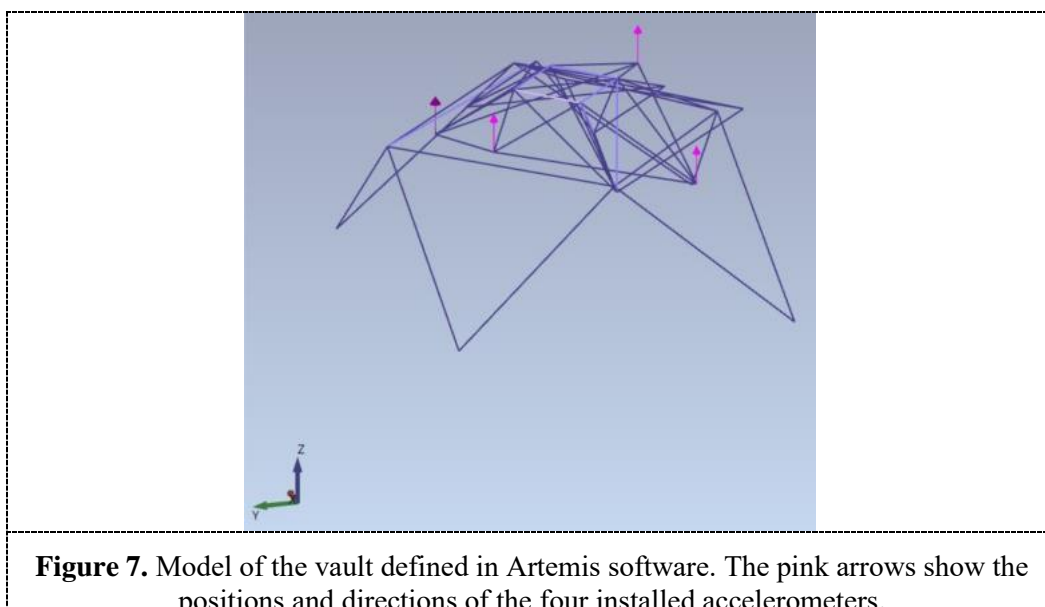
The chassis is connected to a modem by means of an ethernet cable. The data acquired by the control units are sent via cable from the model to the computer on which the data acquisition software is installed.



The 4 accelerometers were perfectly synchronised. The sampling rate available for each accelerometer (196 Hz) was utilised both in the acquisition phase and in the subsequent processing phase. Considering that the test of 15 minutes was carried out with a continuous acquisition in environmental conditions, 900 samples were acquired, instead the 5 minutes test, have a total of 300 acceleration data. All the acquired data were used in the identification phase. The acquisition system (accelerometers) was joined to the irregular ashlars of the vault with specific glues through steel elements with rectangular prism shape to facilitate their positioning as Figure show in Figs 5-6.

### 3.2. Identification results with Operational Modal Analysing (OMA)

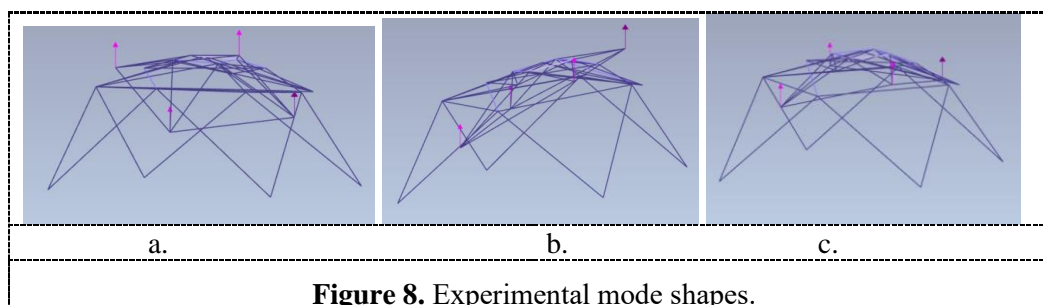
The acquired vibrations have been analyzed using the Artemis software [21]; thus, a simplified vault geometrical model has been defined, introducing the directions of acquisition of the four installed sensors, which are reported in pink in Fig.7.



The vibrations have been analyzed using the Stochastic Subspace Identification (SSI-UPC) method and the new Extended Unweighted Principal Component (SSI-UPCX) method. The first three identified frequencies are reported in Table 1, with a description of the corresponding mode shapes represented in Fig.8.

**Table 1.** Identified experimental frequency values.

Mode	Natural Frequency [Hz]	Mode Shape
1	16.05	Bending mode along y axis
2	16.48	Bending mode along x axis
3	29.89	Vertical motion mode



#### 4. Numerical Model

##### 4.1. DEM model

A non-linear DEM Model in 3DEC® [22]. was used to conduct the numerical analyses. The analyses was performed on a numerical model made up of rigid blocks; all the system deformability is gathered in the joints, which exhibit Mohr-Coulomb behavior. In order to simulate the interaction between the blocks, zero-thickness non-linear springs are used. Due to its ability to account for the discontinuous character and interlocking of the constituent elements of masonry structures subjected to quasi-static analysis, this modeling typology and approach are currently the most accurate and realistic.

Finally, the joint strength is determined by the friction angle ( $\mu$ ), cohesion ( $c$ ), and tensile strength ( $f_t$ ), whereas the joint deformability is regulated by normal ( $jk_n$ ) and shear stiffnesses ( $jk_s$ ). In this specific vault, the tensile strength and cohesion were both set to zero, but the interface normal and tangential stiffness as well as the friction angle were determined through the characterization of the material in the laboratory. The mechanical characteristics of the numerical model are shown in Tab. 2.

**Table 2.** Mechanical properties adopted in the numerical small-scale model.

Parameter		U.M.	Value
$\rho_1$	Specific vault block mass	Kg/m <sup>3</sup>	767
$jk_n$	Interface normal stiffness	N/mm <sup>3</sup>	0.26
$jk_s$	Interface tangential stiffness	N/mm <sup>3</sup>	0.18
$\mu$	Friction angle	°	23.31

#### 4.2. Numerical Modelling of Modal analysis

The geometry was imported in the software and in a second step the material characterized was applied. After the boundary condition was imposed fixing the rotation in all directions of the four abutments to replicate the experimental circumstances.

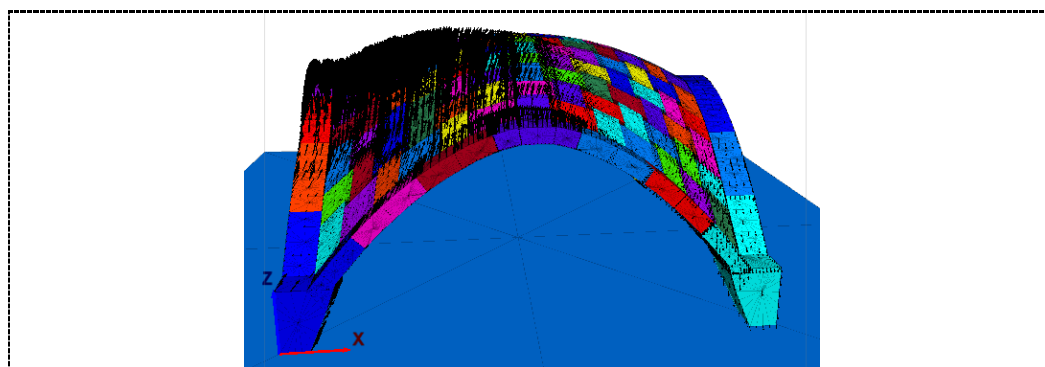
The gravity load was applied before starting the modal analysis code. The procedure followed is the one indicated by the software producer [22] only modified for the case study. So, the natural frequencies and modes of vibration for small amplitude vibrations was calculated in 3DEC® [22] considering block models subjected to dynamic excitation in the elastic range.

As demonstrated by Lemos [33] the global stiffness matrix is created by gathering the elementary stiffness matrix for each sub-contact. Assuming small displacements, unit displacements and rotations are considered for each of the 2 blocks in contact. At the end, the global stiffness matrix is obtained by assembling all the contact matrices and a simple vector iteration procedure is used to calculate the eigenvalues requested [22].

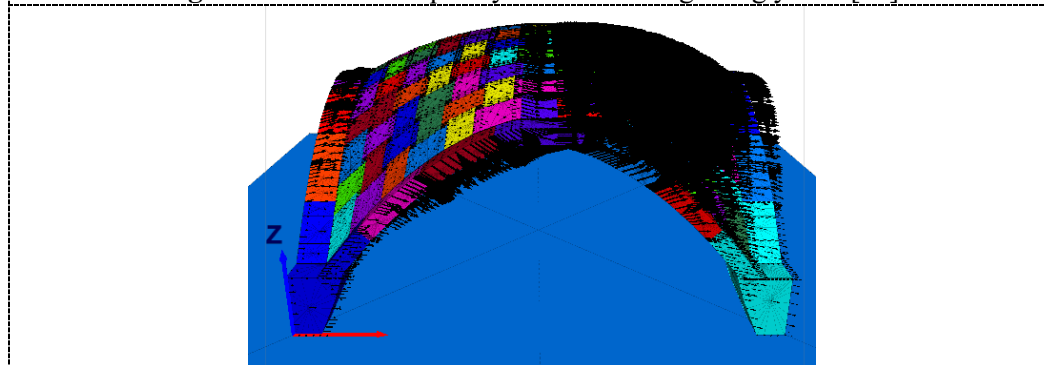
The first three natural frequencies were calculated. The participating mass is approximately 80% for all three modes. Tab. 3 reported the natural frequency values and the description for each modal shapes. The latter are shown in Fig.s 9-11.

**Table 3.** Identified Frequency values in the numerical small-scale model.

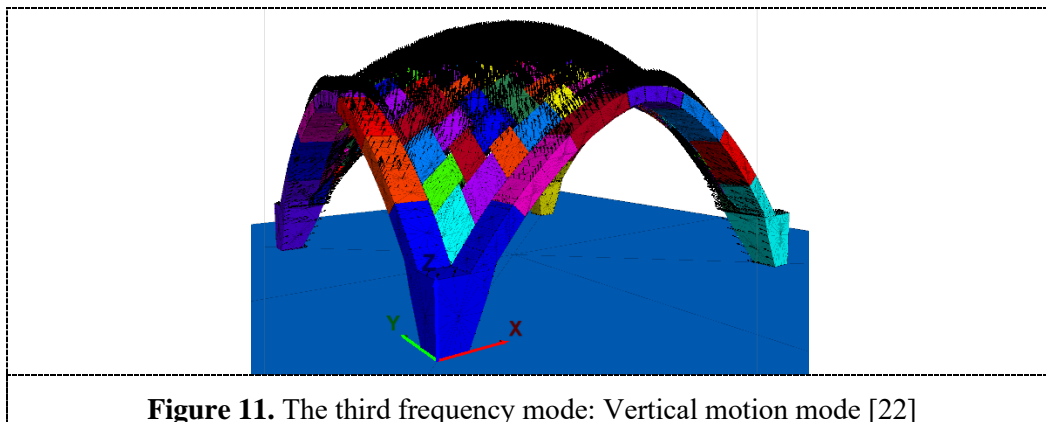
Mode	Natural Frequency [Hz]	Mode Shape
1	19.588	Bending mode along y axis
2	19.938	Bending mode along x axis
3	24.291	Vertical motion mode



**Figure 9.** The first frequency mode: Bending along y axis [22].



**Figure 10.** The second frequency mode: Bending along x axis [22].



**Figure 11.** The third frequency mode: Vertical motion mode [22]

### 5. Discussion and Conclusions

This research aims to identify the structural behavior of the small-scaled model of the new concept vault to understand if its modal analysis may effectively be an approach to understanding the dynamic behavior and mode shapes of complex structures before their construction. In particular, the case study was the Abeille's sail vault. This model was monitored considering the ambient vibration only and forced actions (drumming fingers on the base). The OMA technique was applied, and the natural frequency values resulting from the analysis were compared with the numerical model with Distinct Elements. The final values obtained are comparable, also considering the vibrating shapes. The first mode was a bending mode along y axis equal to 16.05 Hz from the OMA and 19.588 Hz for the DEM. The second was a bending mode along x equal to 16.48 Hz from the OMA and 19.938 Hz for the DEM and the last one respectively was equal to 29.89 and 24.291 Hz. It is possible to note from the images of the modal shapes the constant presence of a torsional component; this component is almost certainly due to structural irregularity.

Future works will include:

- Further optimization between small-scale physical model and numerical one;
- Monitoring with different set-ups;
- the possibility of producing a full-scale physical model to be monitored.

In any case, the values obtained allow us to hypothesize using this approach to understand the dynamic behavior –the values and shapes of vibrating typical– of a real-scale complex structure through the monitoring and analysis of small-scale model considering a subsequent validation with the corresponding numerical model.

### Acknowledgements

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